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537

The Gift of

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THE
MATHEMATICAL PRINCIPLES
OF
GEOGRAPHY.

CONTAINING,

I. An Account of the various Properties and Affections of the EARTH and SEA; with a Deſcription of the ſeveral Parts thereof. And a Table of the Latitude and Longitude of Places.

II. The Uſe of the Artificial or Terreſtrial GLOBE, in ſolving Problems.

III. The Principles of Spherical and Spheroidical SAILING; with the Solution of the ſeveral Caſes in Numbers, by the common Tables, according to the Spheroidical Figure of the EARTH.

> ——— *Mount where Science guides,*
> *Go meaſure earth, weigh air, and ſtate the tides.*
> POPE's Eſſay.

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THE PREFACE.

HAVING treated of the world in its full extent under the title of Aſtronomy, we come now to that ſingle part of it called the Earth, *which is our deſtin'd habitation. This terreſtrial globe on which we are ſeated; tho' it ſeems large to us that live upon it, yet when compared to the magnitude of the heavens, is only like a point in it, being in a manner loſt in that vaſt fabric. But ſmall as it is, it is of the greateſt conſequence to us, for here we live and move and have our being, here our whole buſineſs and tranſactions are performed, here we muſt abide, and cannot remove into another planet, nor have any communication therewith, nor with the inhabitants thereof, if any ſuch there be; nor can we tell what they are doing, no more than they know what we are about. All we can do is to remove from one place to another upon this little ball, and paſs from one region to another, and by this means we get acquaintance with our fellow-creatures on different parts of the globe, with whom we can converſe, carry on a trade, or tranſact any buſineſs, by which we make a ſhift to get a living. And ſince we are confined to this globe, and are obliged to have various tranſactions with others, both at home and abroad; therefore* Geography *(which is the knowledge and deſcription of the earth) becomes a neceſſary branch of knowledge to us. For without it, we ſhould be ignorant of the ſituation of the ſeveral countries, with which we traffick, how they lye from one another, and the way thither.*

Geography teaches us the ſituation of all countries and kingdoms, and the limits thereof, how they are bounded by ſea and land, and by one another; into what provinces and diſtricts they are divided; the ſituation, latitude and longitude, of all cities and towns therein; what woods, foreſts, mountains, lakes, rivers, mines, &c. each contains; what commodities it affords; what matters of wonder, curioſity, or antiquity; what buildings, caſtles, towers are therein; what ports and havens; what rocks, ſands, ſhoals, and ſuch places of danger there are; and the manners and employment of the people. This art teaches us to draw maps, or repreſentations of the ſeveral countries of the earth, which gives a true idea of their ſituation, and magnitudes, by expoſing them all to our ſight. And theſe geographical maps are of very great uſe to travellers, as well as to ſailors; for by help of theſe they find the neareſt way from one place to another.

Without the knowledge of Geography, Hiſtory is very imperfect; for we can have but a very ſlender notion of any tranſaction, when we are ignorant of the place it was tranſacted in. Without it we cannot tell how the moſt memorable enterprizes of the world have been carried on and executed. Without it we are ignorant of the riſe, growth, flouriſhing, and fall of the ſeveral great monarchies in the world, which fix a beginning to all Hiſtory. Without it we cannot judge of the government, commodities, riches, and number of people in other nations; nor judge of the ſtrength of our enemies; nor underſtand the limits between one kingdom and another; nor diſtinguiſh between the names of places and the names of people. But by the help of Geography we can eaſily know all theſe things, and much more; without expoſing our bodies to long tedious travels and voyages.

How prejudicial the ignorance of Geography has been to Princes in foreign expeditions againſt their enemies; hiſtory gives many inſtances of their ill ſucceſs. And *theſe miſcarriages have been principally owing to their ignorance of the country they were going to invade. When any perſon goes upon ſuch an expedition, he ought to have a map of the country, with all the ſeveral paſſages in it from one place to another, the mountains, woods, rivers, marſhes; and what rocks or ſands lie near it. From which may be found where the ſafeſt place for landing is, where one may avoid this rock or that ſand, where ſuch a river may be paſt, and which is the moſt commodious place for giving his enemy battle, and what advantages and diſadvantages there is in his ſituation.*

And as this ſcience is ſo beneficial and uſeful, it is no leſs pleaſant and delightful. It at once pleaſes the eye and inſtructs the mind. It gives ſuch a vaſt variety of objects to contemplate, that we are ſtruck with admiration, with theſe beautiful ſcenes of nature, which this earth of ours affords. Princes have not thought it below them to make it their ſtudy, but to the great danger of their perſons, have travelled into foreign countries to make diſcoveries. And at great expence have ſent philoſophers and mathematicians to remote places to make proper obſervations; as upon the ſituation of places, the phænomena of the celeſtial bodies, and ſuch like; by which this art has been very much improved.

Yet this art at preſent is far from being at perfection. For there are very few places whoſe latitude and longitude are truly determined. And there are ſtill many countries which remain undiſcovered, and muſt wait for the induſtry of future ages to find out. There are likewiſe many continents of great extent, of which we know little more than the ſea coaſts. And even in places which we know, and daily travel over, there are many towns whoſe ſituation is very uncertain. This appears from the diſagreement of the ſeveral geographical maps made of the ſame country. Yet this art is daily improving;

for

for long voyages have made many new diſcoveries; and ſucceeding times will ſtill add more, and will increaſe our knowledge, and ſhew us the errors of former ages. The ancients thought the torrid zone uninhabitable, as well as the polar regions; and that there was no ſuch thing as antipodes, *or people ſtanding contrary ways on oppoſite ſides of the earth; for indeed they thought the earth flat. But later experience has taught us better. And ſo will the experience of future ages detect our ignorance in many things.*

I took notice that the earth compared to the heavens is no more than a point; the leaſt ſtar we can ſee, far exceeds it in magnitude. And yet this earth, ſmall as it is, is that which with fire and ſword is divided among ſo many nations; who are always contending, and never can agree, about their ſhares in it. And yet this is not ſo much owing to the ſmallneſs of it, as to the inſatiable deſires of men, after too large a ſhare of it; and their unquenchable thirſt and luſt after riches. For the earth produces enough for all its inhabitants, if people had but as much humanity as to ſuffer their fellow creatures to enjoy a reaſonable ſhare along with them. Inſtead of that, men are in perpetual war and ſtrife who ſhall get the moſt, till old Time *overtakes them, when all on a ſudden they drop into the boſom of our common* mother the earth; *and then the greateſt monarch is no richer than the pooreſt beggar.*

Navigation *is an art as uſeful and beneficial as Geography, and like two ſiſters, they go hand in hand, and one is ever aſſiſting to the other. By this art men can paſs from one country to another, or from one port to another with great eaſe and diſpatch. By this all trade is carried on, and goods exported and imported at pleaſure, to or from diſtant places as occaſion requires. It is a moſt uſeful art upon many accounts, for by this we can not only increaſe our knowledge by travelling to foreign parts, but our riches alſo by merchandizing. And therefore it is neceſſary to be known for the ſake* *of trade. But neceſſity may be underſtood two ways, either for* abſolute need, *without which a thing cannot be; or merely for a* conveniency, *without which a thing cannot well be. Now it is certain that many places are ſo poor, as not to be able to maintain a populous nation, without the help of foreign trade, at this time when the world is grown ſo full of people. In this caſe there is an abſolute neceſſity for Navigation, to carry on the buſineſs of merchandizing, without which the inhabitants could not live.*

Trading, which is neceſſary to ſome, is certainly very beneficial to all Nations; as is evident for many reaſons. For, 1. By this means one nation may traffick with another, by exporting ſuch goods as they have in too great abundance, and importing others that they want. For ſome commodities are ſo plentiful in ſome countries, that they are a mere drug; whilſt others have little or nothing of the ſort. Therefore tranſporting them to theſe deficient places, is a great advantage. 2. Goods cannot be conveyed from one place to another, ſo eaſily, nor ſo cheap, nor ſo ſoon, nor in ſuch quantities, by any methods, as by ſhipping. 3. By means of Navigation, arts and ſciences are promoted and improved, and conveyed to diſtant places. And for this reaſon, many famous Philoſophers have paſſed the ſeas and travelled far, to converſe with men of learning, for the improvement of arts. And hence the principal diſcoveries in Geography are much owing to Navigation. 4. Likewiſe when a nation is overburthened with inhabitants, colonies may by this means be tranſported to diſtant countries not ſo populous. And thus many idlers are frequently carried to Virginia and other places. 5. Navigation is of great uſe for the defence of a nation againſt foreign enemies, where a fight at ſea does not a quarter of the damage as at land. To which we may add the pleaſure of converſing with foreigners, and holding ſociety with men of learning, as well as *of mutual commerce with them; for ſociety is natural to mankind.*

Navigation has alſo been highly encouraged by many princes and ſtates; and great honours have alſo been paid to it. And for the encouragement of foreign trade, ſocieties have been formed in ſeveral nations, and endowed with ample privileges, by which many perſons have become immenſely rich.

It is time now to give ſome account of what is contained in this Tract. The firſt ſection treats of the figure of the earth, both as a ſphere and ſpheroid; of ſeveral properties of the ſea; of the origin of ſprings and rivers; of finding the diſtances of places; of making maps; a ſhort account of the ſeveral kingdoms of the world; and a table of the latitude and longitude of places. Here the longitude is reckoned from the iſland of Ferro, *according to the antient geographers, which is about the ſame longitude as the weſtermoſt part of the continent of* Africa, *and therefore is a very proper and fit place for the beginning of longitude. And was fooliſhly altered by later Geographers, by which they have confuſed all reckonings.*

The ſecond ſection contains the uſe of the terreſtrial globe in ſolving problems of the ſphere. The practice of this is very eaſy and delightful.

The third ſection is Navigation, and ſhews the principles both of ſpherical and ſpheroidical ſailing. And as ſpheroidal ſailing has made ſo much noiſe in the world, I have given the ſolution of all the common caſes in numbers by my method.

This Treatiſe is moſtly mathematical, the hiſtorical account of kingdoms, &c. not being my deſign.

Of all the books of Navigation that have come into my hands, I have met with few or none, that give a true notion of departure; the moſt of them confounding it with meridional diſtance. And ſome of them tell us that it is quite uſeleſs in navigation; and yet at the ſame time, they cannot find the longitude without it, *or ſomething equivalent to it, but not ſo ſimple as it; and therefore uſe it for that purpoſe. For this reaſon I have made ſome remarks in the Schol. of Prop.* I. *Sect.* III.

As to ſpheroidal ſailing, I have given my thoughts of it in my book of Navigation, and have ſhewn that in a day's run, the difference between that and the ſphere is quite inſenſible. And a ſhip muſt reckon her way every day, and ſo day after day, thro' the whole voyage. And when an obſervation is had, this ſweeps away all irregularities from every cauſe, and ſets all right, as far as there is a poſſibility to do it; and ſurely an obſervation is the only thing to be depended on in a reckoning, and ought never to be neglected. And therefore as no apparent advantage is got by this way of ſailing, I ſet it aſide, and kept to the ſimple and eaſy way by the ſphere. For who will think it worth their while to ſpend a deal of ſuperfluous time and labour, to obtain a degree of accuracy, which can never be wanted? No body will, but ſuch as are fond of novelties, and therefore they prefer ſuch things, becauſe they are new, tho' they have no advantage above other methods, but a manifeſt diſadvantage, of embarraſſing the calculation, and making more work for the ſailor; for which I believe, he will never thank them.

But altho' this method is really of little uſe in Navigation, yet becauſe ſome people may think otherwiſe, and may ſuppoſe there will reſult a greater difference between the two methods, than there really is in practice; I have therefore laid down the principles of that method, and ſolved all the caſes thereby; and if it ſerve for nothing elſe, it will ſerve for an amuſement to ſuch people as are delighted with mathematical enquiries.

Theſe caſes are all reſolved according to Maupertuis's *figure of the earth, which is by far the flatteſt earth that has ever been ſuppoſed. That of* d' Juan *and* Ulloa *are not half ſo flat, and differ very little from*

from Sir J. Newton's *earth. But after all the experiments, that have been made to determine the figure of it, the reſult is, that its figure is inconſiſtent with that of any ſpheroid. And if the earth is not a ſpheroid, theſe* Virtuoſi *that want to be exact, will have new rules to ſeek out, or elſe be forced to take up with imperfect ones. Laſtly, if it ſhould happen that the earth is nearly in the form that* Juan *has determined it; then thoſe that follow* Maupertuis *will be farther from the truth, than thoſe that ſuppoſe it a ſphere; and we know nothing to the contrary. For which reaſon we may well content ourſelves with the ſpherical figure of the earth as heretofore, in the practice of Navigation.*

W. Emerſon.

CON-

THE CONTENTS.

SECT. I.

OF the figure of the earth and ſea. Of finding the latitude and longitude of places. The phœnomena of the celeſtial motions, to the inhabitants of different ſpheres and zones. Of meaſuring a degree of the earth. The meaſure of the earth. To find the polar and equinoctial diameters. A table of the length of a pendulum in all latitudes. Of the ebbing and flowing of the ſea; finding its depth. Of finding the diſtances of places. The origin of ſprings and rivers. Of drawing maps. Of gaining or loſing a day in going round the earth. Properties of the earth and ſea. Of the changes that have happened in the earth. Its diviſion into kingdoms. A table of the latitude and longitude of places.

SECT. II.

The uſe of the terreſtrial globe, where 20 problems of the ſphere are ſolved by it.

SECT. III.

The theory of Navigation, relating to plane ſailing, parallel ſailing, middle latitude ſailing, Mercator's ſailing, and that by the log. tangents. Three uncommon caſes

in Navigation ſolved. The elements of ſpheroidal ſailing. Three propoſitions containing rules, for the ſolution of the caſes of ſailing by the ſpheroid. The actual ſolution of the 5 caſes of ſailing by the ſpheroid, and 3 of parallel ſailing. Correcting a reckoning at ſea. Of finding the meridian by two equal altitudes. To take an altitude without a horizon.

ERRATA, Geography.

page	line	read
4	17	length of days
5	9	*Aſcii*: At
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23	1	whoſe diſtance
40	9b	pole, A and B
57	8	oblate ſpheroid
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82	6	as the land of
123	2b	long night be
128	14	dif. lat. AR.
	4b	velocity A*k*.
130	13	$ts + sr$, &c.
132	16	*ſer* than Az;
137	10	whence $z = 2.30$
144	10	$HI = BE \times \overline{1 + \frac{1}{2}qss}$,
162	22	fig. 18.

GEOGRAPHY.

DEFINITIONS.

DEF. I.

GEOGRAPHY, is a ſcience which teaches the deſcription of the earth and its ſeveral parts; as it is a globe conſiſting of land and water.

DEF. II.

The *axis* of the earth is a line paſſing thro' the center of it, upon which it is ſuppoſed to turn round.

DEF. III.

The *poles*, are the two extremities of the axis, where it cuts the ſurface of the earth; the one the *north*, the other the *ſouth pole*.

DEF. IV.

The *equinoctial* is a great circle of the earth, 90 degrees diſtant from the two poles. This divides the globe into two hemiſpheres, the *northern* and *ſouthern*.

DEF. V.

Meridians or *hour circles*, are great circles of the earth paſſing thro' the two poles, and cutting the equinoctial at right angles. The *firſt meridian* is that from which the Geographers have agreed to reckon from, which anciently went thro' the iſle of *Ferro*.

DEF. VI.

The *horizon* is a plane touching the earth at the place where we ſtand; which plane extended to the heavens, divides the upper from the lower hemiſphere; this is called the *ſenſible horizon.* And a plane drawn thro' the center of the earth parrallel to the former, and extended to the heavens, the great circle made thereby, is called the *rational horizon.*

DEF. VII.

Parallels of latitude, are leſſer circles parallel to the equator; of which the principal are the two *tropics*, and the two *polar circles.* The tropics are 23½ deg. from the equinoctial, the northern tropic is called the *Tropic of Cancer*; the ſouthern, the *Tropic of Capricorn.* The polar circles are 23½ deg. from the poles. The northern is called the *Artic circle*, and the ſouthern the *Antarctic.*

DEF. VIII.

Latitude of a place is the diſtance from the equator to that place, reckoned upon the meridian of the place. If the place be in the north hemiſphere, it is *north latitude*; if in the ſouth, it is *ſouth latitude.*

DEF. IX.

Longitude of a place, is the diſtance from the firſt meridian to the meridian of the place, and is counted on the equinoctial. Formerly longitude was counted from the firſt meridian eaſtward, quite round the globe.

DEF. X.

Rhombs, are the diviſions of the horizon, into ſeveral parts, which are 32 in number; theſe are the ſeveral points of the compaſs; all of theſe have particular names, expreſſed on a compaſs card. The 4 *cardinal points* are eaſt, weſt, north and ſouth.

If you ſet your face to the north, as all geographers do, then the eaſt is on your right hand, the weſt on your left; the north before your face, and the ſouth behind your back. Theſe points of the compaſs are equivalent to ſo many azimuths.

DEF. XI.

A Rumb line, is a ſpiral, drawn or ſuppoſed to be drawn upon the earth, which cuts all the meridians at the ſame angle, which is the proper angle of that rumb. This being continued never returns into itſelf, except it happen to be eaſt or weſt, and then it coincides with ſome parallel circle.

The circles here defined being extended, coincide with their reſpective circles in the heavens, or the celeſtial circles of the ſame name, which have been defined in Sect. II. of the Aſtronomy. And thither I refer the reader for the definition of the reſt of the circles, which more properly belong to Aſtronomy. We muſt next add ſome things in regard to the inhabitants of the earth.

DEF. XII.

A parallel ſphere, is that poſition the earth has, when the horizon coincides wiith the equator. Here the poles are in the zenith and nadir; and the inhabitants of this ſphere live juſt at the poles.

DEF. XIII.

A right ſphere, is that poſition of the earth, where the equinoctial paſſes thro' the zenith, and the poles are in the horizon; and the inhabitants here live juſt at the equator.

DEF. XIV.

An oblique ſphere, is that poſition of the earth, where the equator and horizon make an oblique angle. Here one of the poles is elevated above the horizon, and the other depreſſed below it. The inhabitants here live any way between the poles and the equator. There is alſo a diviſion of the earth according to the ſituations of different inhabitants; as,

D E F. XV.

Antæci, are thoſe that live under the ſame ſemi-meridian, and on different ſides of the equinoctial, and equally diſtant from it. Therefore their longitudes are the ſame, and the latitudes equal but contrary. Noon and midnight, and all the hours of day and night, are the ſame to both. The length of the days to one is always equal to the length of the nights to the other. The ſeaſons of the year are contrary, being ſummer in one, when 'tis winter to the other. They have the ſame ſeaſons, the ſame heat in ſummer, the ſame cold in winter, the ſame length to days and nights; but all at different times of the year.

D E F. XVI.

Periæci, are thoſe people that live in the oppoſite points of the ſame parallel. And therefore when it is noon to one, it is midnight to the other. They have the ſame ſeaſons, the ſame temper of the air, the ſame ſummer, the ſame winter, the ſame length of days and nights, all at the ſame time.

D E F. XVII.

The *Antipodes*, are thoſe inhabitants that live in places of the earth diametrically oppoſite. Here both their latitudes and longitudes are contrary; conſequently when 'tis noon to one, 'tis midnight to the other; when one has the longeſt day the other has the longeſt night, and when one has the ſhorteſt day the other has the ſhorteſt night. The ſeaſons of the year are all contrary in both. They have the ſame ſeaſons, the ſame length of days and nights, but at different times. When they ſtand, their feet are towards one another, and their heads oppoſite. The inhabitants are alſo conſidered in regard to their ſhadows; as,

DEF. XVIII.

Amphiſcii or *Amphiſcians*, are thoſe people that caſt their ſhadows both north and ſouth, at different times of the year. Theſe people live any way between the tropics. When the ſun is in their zenith, they are called Aſcii. At that time the Aſcians caſt no ſhadow at all.

DEF. XIX.

Heteroſcii, are thoſe inhabitants that caſt their ſhadows all one way, either north or ſouth. The Heteroſcians live between the tropics and the polar circles.

DEF. XX.

Periſcii, are thoſe people whoſe ſhadows turn round about them. The Periſcians live within the arctic or antarctic circle, where their ſhadows in one day are directed to all points of the compaſs.

DEF. XXI.

A *Zone* is a portion of the earth's ſurface contained between two parallel circles, or at leaſt within one parallel. Thus the whole ſpace contained between the two tropics, is called the *Torrid zone*. The ſpace between the tropic and polar circle, either north or ſouth, is the *Temperate zone*, theſe are two. The ſpace contained within the polar circle is the frozen or frigid, theſe are alſo two.

The ancients thought the torrid zone not habitable, by reaſon of the great heat of the ſun, but experience has ſhewn the contrary. For the heat there is much diminiſhed on account of ſea breezes, long nights, frequent rains, &c. In the middle of this zone, is the equinoctial circle; and thoſe that live there have perpetually their days and nights

equal.

equal. But here the twilight is very ſhort, being little more than an hour. As the ſun goes twice in the year over their heads in this zone, it cauſes two ſummers and two winters every year, which does not happen in the other zones.

The frozen zones are ſo cold, that the ancients did not think them habitable; and indeed theſe places muſt be exceſſive cold in the winter, where they want the ſun for almoſt half a year. But then to make amends, he ſtays with them almoſt half a year in ſummer. And this ſo warms and nouriſhes the earth, that it is able to reſiſt the cold the moſt part of winter. So that we find that even the coldeſt of theſe places are inhabited.

DEF. XXII.

The Climates, are certain parts of the earth's ſurface, contained between parallels of latitude, in ſuch manner, that at every ſucceeding parallel, reckoning from the equinoctial, the longeſt day ſhall increaſe half an hour. Whence from the equator to the polar circle there will be 24 of theſe climates. And later geographers have added 6 more, where the longeſt day is ſuppoſed to encreaſe by a month at each climate. So that now there are 30, or in all 60, conſidering the whole globe of the earth.

A Table

A Table of Climates, ſhewing in what Latitude each ends.

clim.	lat.		lon. day
1	8	25	12½ h.
2	16	25	13
3	23	50	13½
4	30	20	14
5	36	28	14½
6	41	22	15
7	45	29	15½
8	49	1	16
9	51	58	16½
10	54	27	17
11	56	37	17½
12	58	29	18
13	59	58	18½ h
14	61	18	19
15	62	25	19½
16	63	22	20
17	64	6	20½
18	64	49	21
19	65	21	21½
20	65	47	22
21	66	6	22½
22	66	20	23
23	66	28	23½
24	66	31	24

Climates continued to the Pole.

clim.	lat.		lon. day
25	67	21	1 mon.
26	69	48	2
27	73	37	3
28	78	30	4 mon
29	84	5	5
30	90	0	6

It is evident, this diviſion into climates is very irregular, for near the polar circle, they are ſo ſmall, that they are hardly diſtinguiſhable. It had been better to have divided them at the diſtance of every 5 degrees quite to the pole. However the climates as here ſet out, ſerve to ſhew what length the longeſt day is of, by having the climate given; and that is by taking half the number of climates, and adding that to 12. Or knowing the length of the longeſt day, the climate may be found, by ſubtracting 12 from it, and doubling the remainder.

An Explanation of Terms.

Bay, a ſmall part of the ſea, encompaſſed with the land, having only a narrow paſſage into it.

Bed of a river, is the hollow or channel in which the water runs.

Borough, a ſmall corporate town.

Brook, *Bourn*, or *Beck*, a narrow current of water, that runs continually.

Canal, a deep ſtream or current of water, incloſed on both ſides by banks either natural or artificial.

Cape, or *head land*, a high part of the land ſtanding by the ſea ſide.

Caſcade, a fall of water in a river, which may be either natural or artificial.

Cataract, a high fall of water in a large river, that makes a great noiſe, ſo as to be heard ſeveral miles.

Champion country, a flat open country of great extent.

Channel, a ſtreight or narrow place of the ſea, leading from one part of the ſea to another.

Chorography, the deſcription or repreſentation of any country or kingdom.

City, a large corporate town, with ſeveral privileges.

Cliff, a high ſteep rock on the ſea ſide.

Continent, a large continued tract of land, containing ſeveral countries or kingdoms.

Coaſt, that part of the land which is next the ſea.

Country, a tract of land under a king or prince. This is oppoſed to town or city.

Creek, an arm of the ſea running into the land.

Current, a rapid motion of the ſea.

Deſart, a quantity of ground uninhabited.

Downs, hills of ſand by the edge of the ſea.

Ebb, the ſettling of the ſea after the tide.

Fens, places full of bogs or ſtanding waters.

Flood, the riſing of the ſea in the tide.

Ford, a ſhallow place in a river to go or ride through.

Foreſt, a large extent of ground with trees in it, and ſometimes wild beaſts.

Grove or *Thicket*, a ſmall place ſet with trees, made for pleaſure.

Gulph, a large part of the ſea running a great way into the land.

Hamlet, a ſmall village, or ſome diviſion of a large town.

Harbour, a place where ſhips may lye at anchor ſafely.

Haven, an entrance of the ſea within the land, at the mouth of ſome river or creek, where ſhips may lye.

Hill, a high piece of ground.

Hydrography, the deſcription of water, as the ſea, rivers, lakes, &c.

Iſle, or Iſland, a part of the earth encompaſſed round by the water.

Iſthmus, a neck or narrow piece of land running into the ſea.

Lake, a great collection of ſtanding water, in the land.

Map, a geometrical deſcription of any country upon paper, &c.

Marſhes, ſee Fens.

Monſoons, periodical winds, which blow for a certain time one way, and as long the contrary way.

Mountain, a very high part of the land in any country.

Ocean, that great collection of water that ſurrounds the whole earth.

Pariſh, ſuch an extent of land as belongs to a church.

Park, a parcel of ground encloſed for the keeping of deer.

Peninſula, a piece of land encompaſſed by the ſea, except a narrow place or entrance into it.

Pond, a ſmall collection of ſtanding water in the land.

Port, a place where ſhips lye, or where they load and unload.

Precipice, a very high and ſteep place, as the brow of a hill.

Promontory, a high land ſtretching into the ſea.

Reflux, ſee ebb.

Region, a great ſpace of land containing many inhabitants, under ſome king, &c.

Rill, a ſmall rivulet of running water.

River, a large current of water, able to carry ſhips, &c. eſpecially where it runs into the ſea.

Rivulet, a ſmall river.

Road, a place near the coaſt, where ſhips may lye at anchor.

Sea, a part of the ocean that lyes between one country and another.

Shelves, rocks and ſands lying under water.

Shoar, dry land next the ſea.

Sphere artificial, an artificial globe, with the countries of the earth drawn upon it.

Spring, a ſmall current of freſh water riſing out of the earth.

Staple town, is a port town where merchants traffick.

Strand, that part of the coaſt which the ſea covers and uncovers, as it flows and ebbs.

Streight, a narrow paſſage between two lands, in going into a larger ſea.

Thicket, ſee grove.

Topography, a repreſentation of a ſmall particular place of the earth.

Town, a collection of houſes near together.

Valley, *vale or dale*, a low or hollow place of the earth between ſeveral hills.

Village, a ſmall town, whoſe inhabitants have no particular privileges.

Whirlpool, a very deep place, where the water turns round and draws every thing into it, and ſinks it.

Wilderneſs, a large uncultivated piece of ground, growing nothing but uſeleſs ſhrubs and buſhes.

Wood, a piece of ground planted full of trees.

SECT. I.

Of the figure and magnitude of the Earth. The ebbing and flowing and depth of the Sea. To find the diſtances of Places. Of Springs and Rivers. Of drawing Maps. Adjuncts and properties of the Earth, and the changes that have happened in it. The diviſion of it into Kingdoms and Countries. A Table of the latitude and longitude of Places.

PROP. I.

The figure of the earth as compoſed of land and water is nearly ſpherical.

THAT the earth is ſpherical, or nearly ſo, will appear from the following particulars.

1. Perſons that are a ſhip-board ſailing northwards obſerve the pole ſtar to be higher and higher the further they go. Likewiſe they obſerve new ſtars continually aſcending above the horizon which were hid before, and theſe ſtars continue to riſe higher and higher the farther north they go. Likewiſe, ſuch ſtars as are ſouthwards are obſerved to grow continually lower and lower, till by degrees they diſappear below the horizon. Likewiſe ſuch ſtars as were at firſt vertical, move gradually towards the ſouth, appearing ſtill lower and lower.

On the contrary, when a ſhip is ſailing towards the ſouth, the pole ſtar is obſerved to grow lower and lower; and all ſtars that are northward are obſerved to deſcend gradually, and that in proportion to the diſtance ſailed. And at the ſame time, ſuch ſtars as are ſouthward, continually aſcend; and new ſtars, that were hid before, are obſerved continually to emerge above the horizon, and to riſe higher and higher, ſo long as the ſhip's courſe is continued towards the ſouth. Now theſe phænomena, can be owing to nothing but the ſpherical figure of the earth; for if the earth was a plane, all theſe ſtars would appear always of the ſame height. But as the earth is round, new ſtars muſt continually come into the zenith, as the traveller goes north or ſouth. Therefore the earth is round from north to ſouth.

2. The ſame is evident from the different longitude of places; for when any eclipſe of the moon happens, it is obſerved ſooner, by thoſe that live eaſtward, than by thoſe that live weſtward. Conſtant experience ſhews, that for every 15 degrees difference of longitude, an eclipſe begins ſo many hours ſooner in the eaſt, or later in the weſt; which ariſes from the ſpherical figure of the earth. But if the earth was flat, the eclipſes would happen at the ſame time in all places; alſo day and night would alſo be at the ſame time. But as theſe things are contrary to experience; it is plain the earth is round from eaſt to weſt.

3. Again, a ſhip ſailing in any part of the world, and upon any courſe; at leaving any coaſt, all high towers, or high mountains, gradually diſappear; firſt the bottom part becomes inviſible, then the middle part, and laſtly the top; appearing to ſink gradually under the horizon, till they be quite out of ſight. The ſame is true of ſhips at a great diſtance off at ſea; firſt their top maſts appear, and

Fig. as they draw nearer, their lower maſts and rigging, and laſtly the whole ſhip. And this can be owing to nothing but the ſpherical figure of the earth.

4. In all lunar eclipſes, that ever were obſerved, and in whatever poſition the earth is at that time, ſtill the ſhadow of the earth upon the moon's diſk, always as to ſenſe, appears circular. And therefore it follows that the earth, which is the body that caſts the ſhadow, muſt be round on all ſides; that is, it muſt be ſpherical. And further, ſince all calculations of eclipſes, and of the planets places, are made upon this ſuppoſition; and all anſwer to the true times; which they do not upon any other ſuppoſition; therefore this confirms the propoſition, that the earth is round.

5. Several people have ſailed round the earth, ſetting off weſtward, and continuing their courſe weſtward continually, till they arrived again at the place they departed from. And nothing but a round figure will admit of this.

Cor. 1. *Hence the ſea has the ſame convexity as the earth, and both together make one globe; and therefore it is called the terraqueous globe.*

Cor. 2. *If the difference of longitude of two places, be 15 degrees; the people in the eaſt will reckon the time of the day ſooner by an hour, than thoſe in the weſt. And for every 15 degrees dif. longitude, thoſe in the eaſt will reckon their time ſo many hours ſooner, and theſe in the weſt ſo many hours later, than the others.*

This is an immediate conſequence from the ſpherical figure of the earth. For it is 12 a clock in any place, when the ſun is in the meridian of that place. And as the ſun apparently moves from eaſt to weſt, thro' 360 degrees in 24 hours; it will be an hour of moving from the meridian of any place, to the meridian of any other place which is

15 degrees more weſterly. That is, it is 12 a clock Fig. at the eaſtermoſt place, an hour before it is 12 a clock at the weſtermoſt; and the like for the other hours. And ſo the difference of apparent times, will be proportional to the difference of longitude.

SCHOLIUM.

It is not here meant that the earth is a perfect geometrical ſphere. For experience ſhews, that its ſurface is full of mountains and valleys. And beſides it is higher at the equinoctial than at the poles by a 230th part, which amounts to 17 miles, which is far more than the higheſt mountains. And yet none of theſe irregularities are diſcoverable to ſenſe. And they no more hinder the earth from being reckoned ſpherical, than the roughneſs of an orange or a lemon, hinders it from being eſteemed round.

PROP. II.

The latitude of any place on the earth, is equal to the height of the pole above the horizon.

Let HO be the horizon of the place, Z the zenith, EQ the equinoctial, P the pole, Pp the earth's axis, then (Def. 8) EZ is the latitude of the place. But ECP = a right angle = ZCO, from which taking the common angle ZCP, there remains the latitude ECZ = height of the pole PCO. 15

Cor. *Hence the height of the equinoctial* ECH = *complement of the latitude* ZCP.

For ECH = comp. of ECZ = comp. OCP = ZCP.

Fig.

PROP. III.

The direction of motion of heavy bodies at the earth's ſurface, is perpendicular to the ſurface, or plane of the horizon, in that place.

Here the earth's ſurface is ſuppoſed to be perfectly even and regular in that place, like the ſmooth ſurface of ſtanding water. Suppoſe then a body deſcending to the earth by the force of gravity, and that it is covered with water at that very place; it is plain, if the line of direction of the falling body is not perpendicular to its ſurface, the water would not reſt in that poſition, but will run down continually towards that ſide which is loweſt, that is, where the angle is obtuſe; or from the acute ſide to the obtuſe ſide; till ſuch time as the fluid makes right angles on all ſides with that line of direction. And by the laws of hydroſtatics, it can never reſt till it get into that poſition. And therefore this line of direction muſt be perpendicular to that ſurface, or to the plane of the horizon in that place.

Cor. *If a body be ſuſpended by a ſtring; the ſtring continued to the earth, is perpendicular to the horizon.*

PROP. IV.

In the globe of the earth, it is probable there is more ſolid earth than water; and more ſuperficial water than earth.

The former ſeems to be evident from the motion of the tides, requiring generally 3 hours for the time of high water, after the moon's ſouthing; which could not be, if the ſea was of a very great depth; for then the ſea would have nothing to do but to riſe perpendicularly under the moon. For Fig. in any column of water, reaching near the center of the earth, the gravity of every particle of it is leſſened by the approach of the moon; therefore every particle of it will endeavour to riſe at once, which therefore it will do inſtantly, requiring no ſenſible time; the water coming in laterally from the other columns at that depth, to aſſiſt the motion; which is done in a very little time, as the water has but a few feet to riſe in that direction. But in ſhallow ſeas the caſe is quite contrary, for this tide of water cannot now be ſupplied from an abyſs below; but muſt come in laterally, from all parts around, tending to the place where the moon is vertical; and having many miles to go, muſt create a great current, and require a deal of time to perform the motion in, juſt as we ſee it does in fact. And this proves the ſhallowneſs of theſe ſeas where ſuch currents happen to be. That the ſea is of no great depth, appears alſo by the great number of Iſlands diſperſed all over the broadeſt ſeas. And ſome have ſuppoſed the depth of the ſea to anſwer to the height of the mountains.

That there is more water than land in the ſurface, is evident by inſpecting the terreſtrial globe. And ſome who pretend to have meaſured both, or rather to have weighed them, tell us, that there is near three times as much water as land.

Cor. *Hence the depth of the ſea may be judged of, by the motion or current of the tides.*

For the deeper the ſea is, the leſs the current will be; as more of the water is ſupplied from the abyſs below.

PROP. V. *Prob.*

To find the latitude of any place on the earth.

Altho' this has been ſhewn in the Aſtronomy (*Prob.* I. *Sect.* IV.) yet I ſhall ſhew other methods

Fig. of doing it here, as it is a very neceſſary requiſite in Geography.

1 *Way.*

With a very good inſtrument take the height of the pole ſtar, which note down. Then at twelve hours diſtance, take its altitude again, which alſo note down. Then add theſe two altitudes together, and take half the ſum, and this will be the latitude of the place.

Inſtead of the pole ſtar, any other ſtar will do, that is not too far from the pole; for if it be low at one obſervation it will be affected with refraction, which muſt be allowed for, or elſe the lat. will not be exact.

2 *Way.*

If you know the declination of a ſtar; take the meridian altitude of that ſtar; from this ſubſtract the declination if it be north; or add it, if ſouth; and you have the height of the equinoctial. Subtract the height of the equinoctial from 90 degrees and you have the latitude.

Inſtead of a ſtar, you may make uſe of the ſun, if his declination be known. But refraction muſt be allowed for in both.

PROP. VI. *Prob.*

To find the longitude of any place upon the earth.

This Prob. is very neceſſary in Geography, tho' it has been ſolved before in the Aſtronomy.

1 *Way.*

Having the time of the moon's ſouthing at any place, whoſe longitude is known; the longitude at any other place may be found thus. Having a meridian line, and a clock or watch exactly ſet to the time; obſerve the time of the moon's ſouthing at your place; then by ſubtraction, find the difference of times, at this place and that of known longitude. Then as 48 minutes is the difference of the times of ſouthing of the ſun and moon, in 24 hours. Say, as 48 minutes to 360° : : ſo is the ſaid difference of times, to the difference of longitude. And this difference added to the longitude of the known place, when your time of ſouthing was ſooner, or ſubtracted, when later; gives the longitude of your place.

2 *Way*.

Having the time of the moon's ſouthing at any place whoſe longitude is known, as alſo the time of the ſouthing of ſome ſtar near the moon; the longitude of any other place may be found. Obſerve by a clock or watch, by your meridian, how long after or before the ſtar, the moon ſouths. By ſubtraction find the difference of times at the firſt place; and having the difference of times at your place, ſubtract one from the other for the ſecond difference. Then ſay, as 48 minutes, to that ſecond difference; ſo 360 degrees, to the difference of longitude.

If the ſtar ſouths firſt, that place is eaſt of the other, whoſe difference of times is the leaſt.

If the moon ſouths firſt, that place is eaſt, whoſe difference of times is greateſt.

But the moon may alſo ſouth the firſt at one place, and the laſt at the other. In that caſe you muſt take the ſum of the times, inſtead of the ſecond difference before mentioned; and that place is eaſtward where the moon ſouths firſt. Therefore in any caſe your latitude will be known; by adding or ſubtracting your lat. to or from the known lat. as the caſe requires.

SCHOLIUM.

In this laſt method, the times of ſouthing at the place whoſe longitude is known, may be found either by obſervation, or by the aſtronomical tables. If by obſervation, then you need not know the preciſe times of their ſouthing, but only the difference of times, by a correct clock. And therefore if proper obſervations be made at the two places, on any night agreed on; that ſecond difference will be found, and from thence the diff. longitude. And both ways, to be more exact, inſtead of 48 min. the motion of the moon from the ſun in 24 hours, ought to be taken from the Aſtronomical tables.

PROP. VII. *Prob.*

To deſcribe the phænomena happening to the inhabitants of the ſeveral ſpheres and zones.

I. *In a right ſphere.*

1. Here the people live under the equinoctial, and therefore have no latitude. 2. The inhabitants here enjoy a perpetual equinox, their days and nights being equal thro' the whole year. 3. All the conſtellations are viſible to them, and every ſtar is 12 hours above and 12 hours below the horizon. 4. The ſun is twice in the year directly over their heads; and twice in the ſolſtices; in one, declining 23½ degrees towards the north; in the other, as far towards the ſouth. 5. They have two winters and two ſummers in a year, as alſo two ſprings and two autumns; the ſummers being when the ſun is vertical.

II. *In a parallel ſphere.*

1. Here the people live under the poles, and conſequently their latitude is 90 degrees, which is

the greateſt that can be. 2. They have no eaſt or weſt, or any other point but north and ſouth. Thoſe that live under the north pole have nothing but ſouth; and thoſe under the ſouth pole, nothing but north. 3. They have but one day and night in the year, and each of them is 6 months long. 4. Thoſe at the north pole have day and ſummer, when theſe at the ſouth pole have night and winter; and *vice verſa.* 5. The ſtars never riſe and ſet, but move round them in circles parallel to the horizon; and therefore are always at the ſame height. 6. The ſun moves continually round about them, riſing higher and higher every day, till he be 23½ deg. high, or at the tropic. And afterwards deſcends again, by the ſame degrees, till he ſets in the horizon. Fig.

III. *In the oblique ſphere.*

1. Here, there is equal day and night only twice in the year, and that is when the ſun is in the equator. 2. In ſummer the days continually increaſe, and the nights decreaſe, till the ſun is at the tropic towards their pole, after which the days continually decreaſe and the nights increaſe, till the ſun be at the other tropic in winter. 3. The greater latitude any place is in, the longer are the longeſt days, and the ſhorter the ſhorteſt. 4. Some ſtars about their pole are always above the horizon, and therefore always viſible; and ſome ſtars about the contrary pole, are always below the horizon, and therefore never ſeen. 5. To the inhabitants of the oppoſite hemiſpheres, the ſeaſons are all contrary. 6. To all people in the ſame parallel, the days and nights are of the ſame length at all times of the year. 7. The greater the latitude of the place is, the longer twilight continues.

Fig.

IV. *In the torrid Zone.*

1. Thoſe that live in the torrid zone, have the ſun twice over their heads in a year. 2. They have two ſummers and two winters in the year, but of unequal lengths. 3. Their days and nights are unequal; and the more ſo, the further they are from the equator. 4. Thoſe that live under the tropic, have the ſun but once a year over their heads, and have only one winter and one ſummer; and to them all the ſtars within their polar circle always appear, but none of theſe in the oppoſite polar circle.

V. *In the frigid Zone.*

1. Here the ſun continues above the horizon for ſeveral days in ſummer, and below the horizon for as many days in winter; and the nearer the pole, the more days it continues. 2. Here is but one ſummer, and one winter in the year. 3. In contrary zones, the one has ſummer, when the other has winter. 4. Thoſe that live under the polar circle, ſee the ſun going quite round them, when he is in their tropic; and when he is in the contrary tropic, he does not riſe for a day. To them the ſtars between the tropics riſe and ſet: the ſtars between their own tropic and the pole never ſet, and thoſe between the other pole and tropic never riſe.

VI. *In the temperate Zone.*

1. The meridian ſun is always one way, viz. ſouth in north latitude, and north in ſouth latitude: and therefore never comes over their heads. 2. They have but one winter and one ſummer in the year. 3. The days and nights are always of unequal lengths except at the equinoxes. 4. Thoſe ſtars about their own pole never ſet, which are

within that parallel whoſe diſtant from the pole is equal to the latitude. And thoſe about the oppoſite pole never riſe. 5. In the oppoſite zones, the ſeaſons are contrary, being winter in one, when it is ſummer in the other. Fig.

PROP. VIII. *Prob.*

To find the length of a ſmall arch of the earth, as a degree, &c.

1 *Way.*

Chuſe two ſtations A and B on the tops of two mountains, and as far diſtant as poſſible, ſo as to be ſeen from one another. And having ſet two marks at A and B, which may be ſeen by help of an inſtrument with teleſcopic ſights. Place yourſelf at one ſtation A, and with the inſtrument take the angle BAC contained between the perpendicular AC, and the viſual ray AB. Then remove to B the other ſtation, and in like manner take the angle ABC contained between the perpendicular BC and the viſual ray BA. Then by adding theſe angles CAB and CBA together, you have the ſum, which taken from 180 degrees, gives the angle ACB. Laſtly, meaſure the line AB, leading directly from one ſtation to the other, and you will find how many miles, &c. of the earth's circuit correſpond to the angle ACB. If any part of the line AB be impaſſable, and cannot be meaſured directly, ſuch part or parts muſt be meaſured trigonometrically, by taking off ſtations, as is uſual in meaſuring inacceſſible diſtances. 2.

2 *Way.*

Chuſe two places on the Earth ſituated under the ſame meridian, in a level and even country, and as far off as poſſible, ſo that one may travel from

Fig. 2. from one to the other in a right line; take the latitude of theſe two places ſeparately (Prop. V.) which note down; and ſubtracting one from the other, gives the difference of latitude. Then meaſure the diſtance of the two places in a direct line; and you will know what length in miles, &c. belongs to that arch, or difference of latitude.

3 *Way.*

Take two places as before in a very level country, and in the ſame meridian, and as far diſtant as you can. At theſe two places, let the meridian altitude of ſome noted ſtar be exactly taken, the difference of theſe altitudes ſhews the diſtance of theſe places in degrees, &c. Then if the diſtance of the two places be exactly meaſured, it will be known how many miles, &c. anſwer to that arch on the earth; and conſequently how many to a degree.

Cor. *Hence at a medium, the length of a degree upon the earth is about* 69.31 *Engliſh miles.*

In the year 1635 Mr. *Norwood* meaſured the diſtance between London and York, by which he found the length of a degree to be 367196 feet or 69.54 miles.

Mr. *Picard* found a degree in the middle of France to be 57060 French toiſes. And Mr. *Caſſini* afterwards found the ſame length.

Mr. *Muſſchenbroek* found a degree in Holland to be 57033 toiſes.

Mr. *Maupertuis*, afterwards correcting Picard's meaſures, makes only 56926 toiſes to a degree.

In the year 1736 Mr. *Maupertuis* and his company were ſent by the French king into the gulf of *Bothnia*, to meaſure a degree of the meridian. And they found the length of a degree under the polar circle to be 57438 toiſes.

At the ſame time Mr. *Bouguer* and *Condamine* were ſent into New Spain, and there they found a degree under the equator to be 56753 toiſes. Fig. 2.

But Bouguer in his Navigation ſtates it thus,
a degree at the equator 56748 toiſes
——— in lat. 45 57000
—under the polar circle 57422.

From theſe meaſures, one may reckon the mean length of a degree to be about 57050 toiſes, or 365586 feet = 69.24 Engliſh miles; and the mean between this and Norwood and Picard is about 69.31 Engliſh miles.

Note, a toiſe is reckoned equal to 6.4080 feet Engliſh, or only to 6.4 by ſome writers.

PROP. IX. *Prob.*

To find the circumference and diameter of the earth.

Having the length of a degree by the laſt Prop. If that be multiplied by 360, the product gives the circumference; and this divided by 3.1416 gives the diameter; and half of the diameter is the radius. All this ſuppoſes the earth to be a ſphere, or very near it. 2.

Cor. *Hence the circumference of the earth is 24951.6 Engliſh miles, the diameter 7942, and the radius 3971, or thereabouts.*

PROP. X.

Suppoſing the body of the earth to be every where of equal denſity; its figure is nearly that of an oblate ſpheroid.

Let APEG be its figure, PG the axis, ACE the equinoctial. And let FPHG be the inſcribed ſphere. 3.

Fig. ſphere. Draw BID parallel to AC. If it was not for the diurnal revolution, the equal gravitation on all ſides, would cauſe it to have a ſpherical figure, ſuch as FPHG. But by its rotation round its axis, a part of the gravity is deſtroyed, by the centrifugal force; and the diminution of the gravity in every place, will be proportional to the centrifugal force in each place; and therefore if we ſuppoſe the earth in form of a fluid, that centrifugal force will cauſe the fluid parts to riſe higher about the equator, in order to keep an equilibrium among all the parts of it.

Now ſince AF $=$ AC — PC, the part AF is ſuſtained by the centrifugal force. And to find how much is ſuſtained at B thereby, we have (by Cor. 3. Prop. III. Centrip. forces) the centrifugal force at A : centrifugal force at B in the plane DB : : AC : BD : : AF (the effect of the cent. force at A) : BI (the effect of it at B). And by diviſion AC : BD : : FC or PC : ID. Therefore (by Prop. XIX. Ellipſis) GAPE is an ellipſis, and the earth a ſpheroid.

Cor. 1. *The increaſe of gravity, in going from the equator to the pole, is as the ſquare of the coſine of the latitude.*

Draw IC, and BL parallel to it or perp. to IL. Then ſince BI repreſents the centrifugal force of B, acting in direction IB perpendicular to the axis PC; if this be divided into the two forces IL, BL; the force IL is no way oppoſed to the force of gravity, but the force LB is directly oppoſed to it. Therefore the diminution of gravity at A : to that at B : : as AF : to BL. But AF to BL is compounded of the ratio of AF to BI or IC to ID, and BI to BL or (by ſimilar triangles) as IC to ID. Therefore AF : BL : : IC^2 or FC^2 to ID^2. Therefore the diminution of gravity at A : is to the dimi-

nution at B : : $FC^2 : ID^2$. Whence the diminution Fig. 3.
at any place B being as ID^2, it is continually diminiſhed (in that ratio) from P to A. But a diminution from P to A, is the ſame thing as an augmentation in going from A to P.

Cor. 2. *The gravity of a body at the equinoctial* A, *is to the gravity of the ſame body at the pole* P : : *reciprocally as* PC *to* AC.

For the gravities of the whole columns AC, PC, are equal; and therefore a certain quantity Q, will weigh at A, $\frac{Q}{AC}$; and at P, will weigh $\frac{Q}{PC}$. And theſe are as $\frac{1}{AC}$ and $\frac{1}{PC}$, or as PC to AC.

Cor. 3. *Inſtead of ſuppoſing all the earth in the ſtate of a fluid, if we ſuppoſe there is in the middle of it, a ſphere of ſolid matter, or terrella, as* GFPH, *encompaſſed by a fluid. The figure of the earth will then approach neater to a ſphere; ſuppoſing the denſity every where equal.* 4.

For let AFPQ be a part of the water or fluid; then the diminution of gravity, in the columns CA, CB; going directly to the center, ariſes from the centrifugal force as before. But the centripetal force does not affect the parts of the columns CF, CL, below; but only the upper parts AF, BL. Therefore only the parts AF, BL, are increaſed in the ratio of the coſine ſquare of the latitude (by Cor. 1.), the parts CF, CL remaining equal; and conſequently the wholes AC, BC, muſt be nearer a ratio of equality; that is, the earth approaches nearer to a ſphere. And the more ſo, the leſs the fluid matter, or the greater the terrella at the center.

Cor. 4. *And therefore the earth is the moſt oblate it can be, when the whole of it is in a fluid ſtate.*

PROP.

PROP. XI. *Prob.*

Having the length of a degree in two ſeveral latitudes of the earth; to find the polar and equinoctial diameters of it.

5. Let AC be the radius of the equinoctial, CP the ſemiaxis, draw BD parallel to AC, and BR perpendicular to the curve in B. And put CA $= a$, CP $= e$, CD $= z$, BD $= y$. S. lat. DBR $= s$. Coſ. $= c$, BR $= \pi$, d = length of a degree at B. Then by the nature of the ellipſes BD² $= yy = aa - \frac{aa}{ee} zz$.

Then (by Cor. 1. Prop. XI. Ellipſis) DR $= \frac{aaz}{ee}$. And BD : DR : : $c : s$, or $aa - \frac{aa}{ee} zz : \frac{a^4}{e^4} zz : : cc : ss$; that is, $ee - zz : \frac{aa}{ee} zz : : cc : ss$. Whence $zz = \frac{e^4 ss}{ccaa + ssee}$. Again, s : DR $\left(\frac{aa}{ee} z\right)$: : Rad. (1) : BR or $\pi = \frac{aaz}{ees}$. But the radius of curvature in B is equal $\frac{4\pi^3}{bb}$, b being the parameter. (See my Fluxions, Prob. V. Sect. II. Schol.)

Now in the ſame figure, the length of a degree in any place, is as the radius of curvature in that place; therefore d is as $\frac{4\pi^3}{bb}$, or as $\frac{4a^6 z^3}{bbe^6 s^3}$, that is, as $\frac{z^3}{s^3}$; b, a and e being conſtant. But $\frac{zz}{ss} = \frac{e^4}{ccaa + ssee}$, and $\frac{z^3}{s^3} = \frac{e^6}{\overline{ccaa + ssee}|^{\frac{3}{2}}}$. Therefore

d is as $\frac{e^5}{\overline{ccaa + ssee}|^{\frac{3}{2}}}$ or as $\frac{1}{\overline{ccaa + ssee}^{\frac{3}{2}}}$, e being conſtant; and $d^{\frac{2}{3}} \propto \frac{1}{ccaa + ssee} \propto p$, putting $p = d^{\frac{2}{3}}$. Fig. 5.

For the ſame reaſon, in any other latitude, $P \propto \frac{1}{CCaa + SSee}$. Whence $p : \frac{1}{ccaa + ssee} :: P : \frac{1}{CCaa + SSee}$. Therefore $PCCaa + PSSee = pccaa + pssee$, and $\overline{PCC - pcc}.aa = \overline{pss - PSS}.ee$; and $\frac{aa}{ee} = \frac{pss - PSS}{PCC - pcc} = \frac{PSS - pss}{pcc - PCC}$; and $\frac{a}{e} = \sqrt{\frac{PSS - pss}{pcc - PCC}} = f$, for ſhortneſs.

Let r = radius of curvature in B, then $r \times .01745 = d$, and $r = \frac{d}{.017453}$. But we had $r = \frac{4\pi^3}{bb} = \frac{e^2}{a^4}\pi^3 = \frac{ee}{a^4} \times \frac{a^6}{e^6} \times \frac{z^3}{s^3} = \frac{aa}{e^4} \times \frac{z^3}{s^3} = \frac{aa}{e^4} \times \frac{e^6}{\overline{ccaa + ssee}^{\frac{3}{2}}} = \frac{aaee}{\overline{ccaa + eess}|^{\frac{3}{2}}}$; and $r \times \overline{ccaa + ssee}^{\frac{3}{2}} = aaee$; or (becauſe $a = fe$), $r \times \overline{ccffee \times ssee}|^{\frac{3}{2}} = ffe^4$. And dividing by $\overline{ee}^{\frac{3}{2}}$ or e^3, we have $r \times \overline{ccff + ss}|^{\frac{3}{2}} = ffe$, and $e = \frac{r}{ff} \times \overline{ccff + ss}|^{\frac{3}{2}}$. Whence $a = fe = \frac{r}{f} \times \overline{ccff + ss}|^{\frac{3}{2}}$.

If the two latitudes be at the pole and the equator, $s = 0$, and $C = 0$; and $S = 1$, and $e = 1$; then $\frac{a}{e}$ or f becomes $= \sqrt[3]{\frac{D}{d}}$; and $e = \frac{\sqrt[3]{ddD}}{.01745}$, and $a = \frac{\sqrt[3]{dDD}}{.01745}$.

Take

Fig. 5. Take the two latitudes of 66° 31′, and 48° 50′, and the meaſures of the degrees 57438 and 56926, according to *Maupertuis*, and *Picard* corrected. And there comes out $\frac{a}{e}$ or $f = 1.0109$, and $e =$ 3237300 toiſes = 3929 miles, and $a = 3972$, the difference is 43 miles, that the earth is higher at the equator than at the poles, which is $\frac{1}{92}$ part of the radius.

Again, if the meaſure of a degree at the equator and polar circle be taken, which are 56753 and 57438, as ſet down in the VIII Prop. Then the difference of the ſemidiameters will be $\frac{1}{216}$ the radius of the earth.

If we take the meaſures at the equator and in France, lat. 0, and 48 50, which are 56753 and 56926 toiſes, the difference of the ſemidiameters will be $\frac{1}{577}$ of the mean ſemidiameter. Or if *Picard*'s own meaſure be taken, 57060; the difference will be $\frac{1}{320}$.

If we take *Bouguer*'s meaſures 56748, 57000 and 57422; for the latitudes 0, 45°, 66½. The firſt two give $\frac{1}{339}$; the firſt and laſt, $\frac{1}{215}$; and the two laſt, $\frac{1}{139}$, for the difference of the radii.

In the Aſtronomy (Sect. V. Prop. 25. Cor.) I have ſhewn from the theory of gravity, that the difference of the ſemidiameters is $\frac{1}{285}$. And Sir J. Newton makes it $\frac{1}{230}$. And other people purſuing different methods and meaſures, have found other numbers, all differing from one another. Particularly *Bouguer* by uſing ſeveral numbers, finds $\frac{1}{178}$ for a mean. Whilſt *Juan* (who accompanied them to the ſouth) makes $\frac{1}{236}$ for the difference of the radii. Fig. 5.

Scholium.

In a caſe of ſuch uncertainty as this, what can we conclude on? Only this in general, that the earth is flatter at the poles than at the equator, or higher at the equator than at the poles. For ſo great is the difference reſulting from theſe ſeveral obſervations, that ſome of theſe numbers are twice or thrice or even four times as great as others. And who knows which is right, or which to truſt to? Sir *I. Newton*'s, calculated from the theory of gravity, ſeems to be about a mean among them. The meaſures under the polar circle and in *France*, make the height at the equator more than twice as much as Sir Iſaac. Whilſt the ſame meaſure under the polar circle, and that under the equinoctial, makes the height about the ſame as Sir Iſaac does. Now all this inconſiſtency muſt ariſe from ſome of theſe cauſes. 1. From the defects of meaſuring. 2. The ſhrinking or lengthening of their poles. 3. Errors in taking the latitude. 4. The attractive force of mountains. Or 5. That the earth is not a true ſpheroid. 1. As to the defects of meaſuring, according to the account *Maupertuis* gives, all imaginable care was taken about it. Yet ſome errors might happen, by not laying the rods in a right line from right to left. Or ſome might happen, by not laying them exactly horizontal. For if in any caſe they made any angles, tho' very ſmall, but being often repeated, they would a-

Fig. 51 mount to ſomething; and then the diſtance of their ſtations would meaſure to more than it really was. And it would be difficult enough to lay the rods truly horizontal, among ſo much ſnow as they had to wade thro'. Add to this, that theſe rods might bend in ſome places and ſo become ſhorter. 2. As to the alteration in the length of the rods, Maupertuis tells us they could diſcover none. But I never yet met with any ſort of bodies, that were not affected with heat and cold. And I can never think that wood, eſpecially fir, is ſecure againſt it. For how could they be ſure of the contrary; for if they brought their rods out of a cold into a warm place, where their thermometers were kept, they would immediately expand with the heat, and during the time of meaſuring would be reduced to the ſame ſtate as their ſtandard. In like manner, if the rods were brought out of a warm place to be meaſured in a cold one by their ſtandard, the cold would certainly contract them, and reduce them to the ſize of their ſtandard. And this is the more probable, becauſe they took a great deal of care to have them nicely adjuſted, which would require the more time to perform the operations in; and being all the time in one place, they muſt partake of one ſtate. So that if the rods did really lengthen and ſhorten, yet I cannot ſee that they could have any certain way to find it out. And I take this to be the principal cauſe of a degree meaſuring to ſo many toiſes, happening thro' the contraction of the rods, by ſuch an extreme degree of cold. But if *Maupertuis* will have it, that heat and cold have no effect upon them; then the curious mechanic need ſeek no longer for a metal to endure heat and cold without any alteration; for here wood will do the buſineſs. However, we know that wood will be contracted or dilated by drought or moiſture; and then this quality may be as pernicious as the other. 3. The errors that may happen in taking the latitude, may be the incorrectneſs of the inſtrument, or the want of care in uſing it. But neither of theſe has any place here, as they had the beſt of inſtruments for this uſe, and the ableſt hands to uſe them. 4. The irregular attraction of mountains may have ſome effect here; for all their operations were performed among mountains. And a like effect happened to the ſouth mathematicians; for they found the attraction of a great mountain, had ſo much influence on a plumb line, as to cauſe it to deviate 7 or 8 ſeconds from the perpendicular. But ſuch an effect was not expected in the north, and ſo no experiments were made about it. What is ſaid of mountains is applicable to ſuch parts of the earth as are denſer than the reſt, which will have a like effect, tho' they are quite inviſible to us. And a ſmall matter will deſtroy the accuracy of an experiment ſo nice and curious as this is. We may obſerve that for every error of 1 ſecond, there will be an error of 16 toiſes; and in 20″, will amount to 320 toiſes, which makes a ſenſible difference. Add to this, that the very ſhort baſes that are made uſe of in ſuch caſes, is another cauſe of error. For a very ſmall error made there, will be greatly multiplied at the far end. And yet ſhort baſes will perform better than long baſes, for a great part of theſe will be upon uneven parts of the ground, which cannot be truly meaſured. 5. But if the earth be an irregular ſolid; no meaſures can ever conſpire to make it regular. And what ground have we to think it is regular. The internal parts of it are unknown to us. It may happen to be denſer towards the poles, or it may happen to be denſer towards the equator. But theſe two different ſuppoſitions will cauſe different figures. And as the internal conſtitution of the earth is unknown to us, the true form of it will always remain a ſecret; tho' it cannot be far from that of a ſpheroid. Fig. 5.

Fig. 5. And tho' theſe eminent perſons that were ſent to meaſure the earth, have made ſurpriſing diſcoveries, and have executed their deſign as far as human art could go: yet an operation ſo nice, ſubtle and critical, can never by any art be performed to the required exactneſs. So that I think it better to acquieſce in Sir I. Newton's numbers, for the figure of the earth, which he found by the theory of gravity; at leaſt till we can get ſomething better. Eſpecially as it is a mean among the different figures, which theſe inconſiſtent and various meaſures have afforded us. And upon this footing I will put down the following table, after I have conſidered the lengths of pendulums.

Cor. 1. *The length of a pendulum vibrating ſeconds, increaſes from the equator to the pole, as the force of gravity increaſes, that is, as the ſquare of the coſine of the latitude.*

For (by Prop. XXVII. Mechanics), the length of the pendulum, is as the force of gravity; and therefore the increaſe of the length, is as the increaſe of gravity, that is (by Cor. 1. Prop. X.) as the ſquare of the coſine of latitude.

Cor. 2. *The length of a pendulum at the equator is to the length of a pendulum at the pole, as the axis of the earth to the equatorial diameter.*

For (by Cor. 2. Prop. X.) the gravity is in that ratio. Yet by experiments, it appears that the length of a pendulum is ſtill leſs at the equator; and therefore the gravity is leſs there than in that ratio; and conſequently the earth is higher at the equator than we ſuppoſed. For the gravity in any place muſt be reciprocally as the height of the earth in that place from the center; becauſe the ſeveral columns of the fluid, reaching to the earth's center, exactly ballance one another. But the lengthening of a pendulum in different latitudes is ſo ſmall a matter, (being but the 230th part), that it will be very difficult to meaſure it exactly; and eſpecially in different countries, where this experiment is diſturbed by heat and cold. However all theſe experiments tend to prove this, that the earth is higher at the equator than at the poles. But to find exactly how much higher, will never be known from ſo nice an experiment as the different lengths of pendulums, no more than it can be from the different meaſures of a degree upon the earth.

A table of the lengths of a pendulum to vibrate ſeconds; and the length of a degree, to every 5th degree of latitude.

degrees.	length degree.	length pendulum.
	miles.	inches.
0	68.723	39.027
5	68.730	39.029
10	68.750	39.032
15	68.783	39.036
20	68.830	39.044
25	68.882	39.057
30	68.950	39.070
35	69.020	39.084
40	69.097	39.097
45	69.176	39.111
50	69.256	39.126
55	69.330	39.142
60	69.401	39.158
65	69.467	39.168
70	69,522	39.177
75	69.568	39.185
80	69.601	39.191
85	69.620	39.195
90	69.628	39.197

Fig.

PROP. XII.

The ebbing and flowing of the ſea is cauſed by the attractions of the ſun and moon.

6. Let ABCD be the earth covered with water, L the ſun or the moon. Then ſince the attractive force of the body L is reciprocally as the ſquare of the diſtance, the water under L being nearer than the center T of the earth, the water at A will be more attracted than the parts about the center T; and therefore the water will riſe at A and ſubſide at D. For the diſturbing forces of the ſun and moon (explained in Cor. 3 and 4, Prop. XXVII. Centr. forces) will accelerate the water from the quadrature D to the ſyzigy A (by Cor. 8. ib.), and retard it from A to the quadrature B. Now as the earth turns round its axis from D thro' A and B; the water, from D to A being continually accelerated, will move faſter than the earth at A. And being continually retarded from A to B, and drawn back; it will move ſlower at B than the earth. Therefore at ſome intermediate point N, the water will be at reſt in reſpect of the earth. Therefore ſince it moves on all ſides towards N, it muſt be accumulated there; and therefore it will be full tide at N. Alſo at ſome point Q in the quadrant DA, the water will alſo be at reſt; but as it moves on all ſides from Q, therefore the water will be depreſſed at Q; that is, it will be ebb or low water at Q.

Likewiſe in the oppoſite hemiſphere BCD, the attraction being greater at T, than at C, by the body L; T will endeavour to leave C behind, which comes to the ſame, as if C was attracted from T in direction TC, juſt as A was attracted from T; by the difference of the attractions. The conſequence will be that the ſame effects will happen Fig. 6. in the hemiſphere BCD, as in the other DAB; the water will be accelerated from B to C, and retarded from C to D; and at ſome point P (oppoſite to N) it will be accumulated, where it will be flood; and at ſome point O (oppoſite to Q) it will be depreſſed, and there it is ebb. So that there are two floods and two ebbs in the ſpace of 24 (lunar) hours. The point N where it is flood is always paſt the moon's meridian at A; and the farther paſt it, the ſhallower the water is, or the quicker the earth's motion. Generally, it is about 3 hours paſt.

Cor. 1. *At the time of the new and full moon, the tides are the greateſt; and in the quarters the leaſt. The former are called ſpring tides, the latter neap tides.*

For at the new and full, both ſun and moon are in the line AC, and both raiſe the tides at A and C by their joint forces. But in the quarters, one raiſes the tide at A and C, and the other at B and D. In the firſt caſe the height of the tide will be as the ſum of the forces; in the ſecond caſe, as their difference. But as the moon's force is four times as great as the ſun's; the ſun will always make ſome tide tho' leſs.

Cor. 2. *But the ſpring tides do not happen on the day of the change or full. Nor the neap tides on the days of the quarters; but about 3 days after.*

For the waters will retain the motions impreſſed upon them for ſome time after; tho' the forces of the luminaries ſhould quite ceaſe. But as their forces are very little diminiſhed in 2 or 3 days, the tides will ſtill increaſe for a time.

Cor. 3. *Theſe are the greateſt tides, when the moon (as alſo the ſun) is neareſt the earth.*

For

Fig. 6. For then the diſturbing force is greater.

Cor. 4. *The tides are higher when the moon (and alſo the ſun) is in the equinoctial.*

For the effect will be greater in a great circle, and leſs in a leſſer. At the pole it would be nothing; there would be no reciprocation.

Cor. 5. *The tides are greater in leſſer latitudes, than in greater.*

For the motion is leſs in a leſſer (or parallel) circle, than in a greater. At the pole it would be nothing.

Cor. 6. *The times of the tide happening in particular places, may be very different according to the ſituation of theſe places. And likewiſe the height.*

For the motion of the tide is propagated ſwifter in the open ſea, and ſlower thro' narrow channels or ſhallow places. And being retarded by ſuch impediments, they cannot riſe ſo high.

SCHOLIUM.

As the moon is the cauſe of a tide in our ſea, ſo the earth will alſo raiſe a tide in the lunar ſea, if there is any. But that tide will be greater in proportion to the greater force of the earth (which is about 40 to 1.); and leſs in proportion to its leſſer diameter (which is 1 to 3.65). Therefore if the moon raiſes our tide 8.6 feet, our earth will raiſe their tide about 93 feet. But as ſhe turns the ſame face always towards the earth, that tide will ſtand there, without any reciprocation. And the moon will put on the figure of an oblong ſpheroid.

PROP.

Fig.

PROP. XIII. *Prob.*

To ſound the depth of the ſea.

Take a narrow cylindrical tube *ab*, 2 feet long 12.
or more, the longer the better; cloſe at the top *a*, and open at the bottom *b*. Fit a cork to the end *b*, to go pretty eaſy in. Then immerſe the open end *b* into a veſſel of treacle, ſo that the treacle may aſcend about half an inch in the tube. Then whilſt it is in the veſſel, put the cork in at *b*, which will force the treacle up to *c*, and keep it there. Then this tube muſt be incloſed in a ſtrong braſs veſſel or cover DE, of great weight, that it may ſink far enough; and full of holes in the ſides, to let the ſea water go freely in and out. After the tube is put into this braſs cover, fill it full of wool on all ſides, to hinder the tube from breaking againſt the cover. Then the cap G muſt be ſcrewed on to keep all faſt. At F is a ring fixed to the braſs cover, and a long cord is to be tied to this ring; and ſo the whole machine ſunk into the ſea, till it reach the bottom, if the cord will reach ſo far. Then it muſt be drawn up by the cord; and obſerving how far the inſide of the tube is daubed with the treacle, as at *r*; the preſſure of the water; and conſequently the depth will be known: and the depth is $\frac{ab}{ar} - 1 \times 10$, in yards.

For 10 yards deep of the ſea is about equal to the weight of the atmoſphere; and at all depths, the preſſure, as alſo the depth, is reciprocally as the ſpace *ar*, that the air is compreſſed into. Therefore $\frac{ab}{ar}$ = number of atmoſpheres preſſing upon the cork at *r*. But one of theſe atmoſpheres is

ſpent

Fig. ſpent in keeping it at *b*, or keeping the included
12. air in the ſtate *ab*, when the depth of the water is nothing. Therefore the number of atmoſpheres preſſing it from *b* to *r* is $\frac{ab}{ar}-1$; and conſequently the depth $= \overline{\frac{ab}{ar}-1} \times 10$.

SCHOLIUM.

It may be ſuppoſed that the length of the cord is ſufficient for meaſuring the depth, without this apparatus. But this will not hold, for theſe reaſons; 1. becauſe the ſhip being in motion, the weight does not ſink perpendicular; and 2. where there is any under current, it will alſo carry it out of the perpendicular, tho' the ſhip was to be at reſt.

PROP. XIV. *Prob.*

To find the diſtance of any two places upon the earth whoſe latitudes and longitudes are known.

If the places have the ſame longitude, or be under the ſame meridian, there is no more to do but to ſubtract one latitude from the other, when they are both on one ſide of the equinoctial; or add them, if on different ſides; and the difference or the ſum is the diſtance in degrees.

7. In general, let EPQ be the meridian of one place, EQ the equinoctial, P the pole, A and D the two places whoſe diſtance is required, PB the meridian of B. Thro' A and B draw the great circle AB. Then ſince the longitudes are given, their difference of longitude or angle APB, is found by ſubtracting one from the other. And the latitudes being given, the complements thereof will be known, or their diſtances from the ſame pole P, that is AP and BP. Therefore in the triangle

APB, there are given the two ſides AP, BP, and Fig. the included angle APB; to find the ſide AB, by 7. Caſe 8 of Spherical Trigonometry.

From the end of the leſſer ſide AP, let fall the perpendicular AD, upon the longer ſide PB. Then,

Radius :
Coſ. APB : :
Tan. AP :
Tan. DP.

Then DP taken from BP, when the angle APB is leſs than a right angle; or added to it when greater, gives the ſegment BD. Then,

Coſ. PD :
Coſ. PA : :
Coſ. BD :
Coſ. BA, the diſtance required in degrees; which multiplied by the number of miles in a degree (found in the table under Prop. XI. for the middle latitude), gives the diſtance in miles.

If AP and PB happen to be equal, as when the places are in one parallel of latitude; the ſolution will be eaſier. The proportion is this,

As Radius :
Coſ. latitude of either place AP : :
S. half the diff. longitude APB :
S. half the diſtance AB, which doubled gives the diſtance ſought, in degrees, on the arch of a great circle.

PROP. XV. *Prob.*

Having the longitude of two places in one parallel; to find their diſtance on the parallel.

Finding the diſtance in the arch of a great circle, was ſolved in the laſt Prob. And to find the diſtance along the parallel; ſubtract the longitude of

Fig. 7. of one from that of the other place, to get the difference of longitude. Then ſay,

As Radius :
Coſ. latitude : :
Difference of longitude :
Diſtance, in degrees of the equinoctial; which multiplied by 69.31, gives the diſtance in miles Engliſh. Or multiplied by 60 gives the diſtance in geographical miles.

Otherwiſe thus,

From the following table take the length of a degree for the given latitude, which multiply by the difference of longitude in degrees, give the diſtance in the parallel, in geographical miles.

A Table ſhewing how many geographical miles Fig. are contained in one degree of any parallel, for all latitudes.

degr. lat.	geog. miles.	deg. lat.	geog. miles.	deg. lat.	geog. miles.	deg. lat.	geog. miles.
0	60.00	23	55.23	46	41.68	69	21.50
1	59.99	24	54.81	47	40.92	70	20.52
2	59.96	25	54.38	48	40.14	71	19.53
3	59.91	26	53.93	49	39.36	72	18.54
4	59.85	27	53.46	50	38.57	73	17.54
5	59.77	28	52.97	51	37.76	74	16.53
6	59.67	29	52.47	52	36.94	75	15.53
7	59.55	30	51.96	53	36.11	76	14.51
8	59.41	31	51.43	54	35.27	77	13.50
9	59.26	32	50.88	55	34.41	78	12.47
10	59.09	33	50.32	56	33.55	79	11.45
11	58.89	34	49.74	57	32.68	80	10.42
12	58.68	35	49.15	58	31.79	81	9.38
13	58.46	36	48.54	59	30.90	82	8.34
14	58.22	37	47.92	60	30.00	83	7.31
15	57.95	38	47.28	61	29.09	84	6.27
16	57.67	39	46.62	62	28.17	85	5.22
17	57.38	40	45.96	63	27.24	86	4.18
18	57.06	41	45.28	64	26.30	87	3.14
19	56.73	42	44.58	65	25.35	88	2.09
20	56.39	43	43.88	66	24.40	89	1.05
21	56.01	44	43.16	67	23.44	90	0.00
22	55.63	45	42.42	68	22.47		

PROP. XVI.

The origin of ſprings and rivers is entirly owing to rain and vapours.

The ancients had very unaccountable notions about the cauſe of fountains and rivers; and many of

Fig. of them abſurd and even impoſſible. Some of them imagined large caverns and receptables of water within the earth, placed there from the beginning, which were to keep running for ever without any ſupply. Others ſuppoſed vaſt quantities of ſteam or vapour to be raiſed within the earth by the force of ſubterraneous fires, which perſpiring thro' the earth, and condenſed near the earth's ſurface, might be ſufficient for the production of ſprings and rivers; as if there were a vaſt number of boilers or natural alembicks, placed within the earth, on purpoſe to generate ſteam, to be converted into water for ſprings. Others ſticking by the words of ſcripture, would have the water of the ſea, contrary to its nature, to aſcend to the tops of mountains, and breaking out from thence, to form ſprings and rivers; as if there was no other way but this, for rivers to be derived from the ſea. Others again would have water conveyed from the ſea by ſome miraculous and ſupernatural powers; as if God had not been able to do this in a natural way, but in the firſt inſtitution of nature, ſhould impoſe a perpetual violence upon nature. Others recur to the influence of the celeſtial bodies, for attracting the water to the higher parts of the earth. All which ſuppoſitions are too weak to endure any examination. But leaving ſuch whims and chimera's, let us aſſign the true cauſe. The true cauſe then, is this. The heat of the ſun draws vaſt quantities of vapour from the ſea, which being carried by the wind to all parts of the globe; and being converted by the cold into rain and dew, it falls down upon the earth; part of it runs down into the lower places; where it immediately forms rivers; and the earth drinks up the reſt; part of which ſerves for the purpoſes of vegetation, and the reſt deſcending into hollow cavern and places within the earth, is lodged there for a while;

which breaking out by the ſides of the hills, form little ſprings, which will continue to run till theſe ſources be exhauſted, or more rain comes for a new ſupply. Many of theſe ſprings running into the vallies, form little rivulets; and ſeveral of theſe meeting together make a river; and ſeveral of theſe meeting again, if the way be long enough, form large rivers, which at laſt empty themſelves into the ſea. Fig.

That the ſun exhales as much vapour as is ſufficient for rain is paſt diſpute; it has been proved by actual experiments; and beſides it is the only proceſs that nature affords. The only difficulty is to know certainly whether the rain falling in a year, be ſufficient to keep the rivers running for a year. In order to know this the area of ſome particular country muſt be meaſured in ſuperficial degrees. Then all the rivers running from thence into the ſea muſt be noted, and their tranſverſe ſections taken juſt above the tide mark, at a mean depth, and a mean velocity. Then it will eaſily be known how much is diſcharged in a year at all theſe ſections of theſe rivers. Then the depth of the rain falling in a year being known by experiment, being in ſome places 16 inches, in ſome places 20, in others 30. Then a quantity of water whoſe baſe is the whole area of the country and of this depth, is the fund for a year's conſumption. Therefore comparing theſe two quantities together, the queſtion will be decided. A French author has made a computation this way, for the country lying about the river *Seyne* in *France*, and finds that 6 times as much water falls in a year as is ſufficient to keep that river always running. Five parts out of ſix are employed in feeding of trees, herbs, and expence of vapours, and extraordinary floods, and other waſting of the water, by ſinking into the ground.

Dr.

Fig. Dr. Halley ſays, the vapours that are raiſed copiouſly from the ſea, and carried by the winds to the ridges of the mountains, are conveyed to their tops by the current of the air; where the water preſently precipitates, gleeting down by the clefts of the rocks; and part of it entering the hills, is collected into baſons or receptacles; which being filled, the water breaks out at the ſides of the hills, where it can find vent; and forms ſprings; ſeveral of which form rivulets, and many of theſe make a river, ſuch as the Rhine, the Rhone, the Danube, &c.

Thus one part of the vapours, is returned again by rivers into the ſea; another part, by cold nights, fall again into the ſea, in dew or rain, which is much the greateſt part: a third part falls on the lower lands, and is the nouriſhment of plants.

All the phenomena of rivers and ſprings confirm this account. For in a long drought, rivers and ſprings grow very low, and are ſometimes dry; growing leſs and leſs as their magazine is more and more exhauſted. In mountainous places, the rivulets at the beginning are very ſmall, but grow bigger the further they go, by the acceſs of other ſmall parcels of water. Therefore the heads of all rivers are at the mountains. In places where it never rains, and yet have rivers; theſe rivers come from places where it does rain, but run thro' theſe countries that have none, in their way to the ſea; but receive no augmentation in their way; and in ſuch countries there are few rivers, only ſuch as are generated from dews and miſt. That the ſea is no bigger for all the rivers running into it, is becauſe the water returns back to the land, in vapours. It is found by digging into the ground, that all places at any great depth are full of water. If it was not ſo, there would be no need of engines to draw the water out of mines. For the rains that fall upon the

Fig.

the earth, not being immediately ſent off by the rivers, a part of it lying upon the ground muſt gradually ſink into it, deſcending lower and lower, thro' narrow fiſſures and creviſes; from whence none of it can return, but what breaks out in ſprings, according to this theory.

Cor. 1. *There is a continual circulation of water from the ſea to the land, thro' the air in vapours and rain; and from the land to the ſea, upon the earth, by the rivers.*

Cor. 2. *Hence, thoſe are the largeſt rivers that run thro' the largeſt tract of ground.*

For the longer the tract of ground, the more rain falls, and conſequently the greater any river will be. The further they go, they are perpetually gathering. Such is the *Danube* running over a length of above 800 miles, and at laſt falls into the *Euxine* ſea.

Cor. 3. *Wherever we find a large river, in any country known or unknown, we may be ſure of this, that it runs over a large tract of land. But where only ſmall rivers are to be met with, it indicates a ſmall country.*

This may be of ſome uſe to thoſe that are ſeeking after new countries.

PROP. XVII. *Prob.*

To draw a map of the world.

A *map* is a geometrical draught or repreſentation of any country, ſhewing the true ſituations of all places contained therein, as to longitude and latitude. And likewiſe all mountains, rocks, rivers, lakes, woods, foreſts, mines, ſands, havens, and whatever is remarkable therein; and alſo how it is bounded

Fig. bounded by ſea or land, on the eaſt, weſt, north, and ſouth.

It is an eſtabliſhed rule among all Geographers, that the top or upper part of any map is the north, for Geographers always turn their faces towards the north pole, in viewing or deſcribing the ſeveral regions of the earth. On the contrary, Aſtronomers, obſerving the twelve conſtellations of the zodiac, look towards the ſouth. Prieſts look towards the eaſt, and Poets towards the weſt.

There are ſeveral methods of drawing maps uſed by different people; the principal whereof we ſhall here deſcribe.

1 *Method.*

A map of the world muſt repreſent two hemiſpheres; and both muſt be drawn upon the plane of that circle which divides the two hemiſpheres. The firſt way is to project either hemiſphere upon the plane of the firſt meridian, by the rules of orthographic projection, laid down before. And then upon another equal circle, project the other hemiſphere by the ſame rules. Here the meridians and parallels are to be projected to every ten degrees difference of latitude, and longitude; or if you pleaſe, to every 5 degrees. Upon the plane of the meridian, the meridians will be ellipſes, and the parallel circles all right lines. Upon the plane of the equinoctial, the meridians will be right lines, and the parallels of latitude will be circles. The fault of this way of drawing maps, is, that near the outſide, the circles will be too near one another; and therefore equal ſpaces on the earth will be repreſented by unequal ſpaces upon the map.

2 *Method.*

Another method is to project all the meridians and parallels by the rules of ſtereographic projection, likewiſe laid down before. And this is to be done for both hemiſpheres; and may be either upon the firſt meridian or upon the equinoctial. And by this method all the parallel circles will be repreſented by circles, and the meridians by circles or right lines. Here again equal ſpaces on the earth are repreſented by unequal ſpaces in the map. For on the outſide, the circles are too far diſtant, and near the middle too near together. To remedy theſe things, another method may be uſed. Fig. 8.

3 *Method.*

This may be done either upon the firſt meridian, or upon the equinoctial. For the firſt meridian; draw the circle PENQ for the meridian, thro' the center C, draw the equinoctial EQ, and NP perpendicular to it; then P and N are the poles. Divide the quadrants PE, EN, NQ, and QP into 9 equal parts, each repreſenting 10 degrees, beginning at the equinoctial EQ. Likewiſe divide CP and CN, into 9 equal parts; beginning at EQ. Thro' the correſpondent points, draw the parallels of latitude. Again divide EC and CQ into 9 equal parts; and thro' the points of diviſion, and the two poles N and P, draw circles or rather ellipſes for the meridians. And another like projection being made for the oppoſite hemiſphere, the map is ready for inſerting the ſeveral places and countries of the earth.

For the projection on the plane of the equinoctial, draw AQBE, for the equinoctial; divide it into the 4 quadrants EA, AQ, QB, and BE; and each quadrant into 9 equal parts repreſenting 10 degrees of longitude. From the points of diviſion, draw lines to the center C, for the circles of longitude. Divide any circle of longitude, as the firſt meridian EC, into 9 equal parts, and thro' theſe points, deſcribe circles from the center C for the parallels of latitude. Then number them as in the 9.

Fig. the figure. Likewiſe draw the oppoſite hemiſphere,
9. after the ſame manner, and the projection is finiſhed.

In this laſt method, equal ſpaces on the earth are repreſented by equal ſpaces on the map, as near as any projection will ſuffer; for a ſpherical ſurface can no way be repreſented exactly upon a plane. Then the ſeveral countries of the world, ſeas, iſlands, ſea coaſts, towns, &c. muſt be placed therein, according to their longitudes and latitudes, for which the circles already drawn are to be your guide.

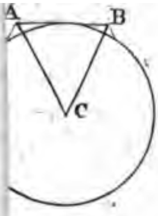
All places repreſenting land are filled with ſuch things as the countries contain; but the ſeas are left white; the ſhores adjoining to the ſea muſt be ſhaded. Rivers are marked by ſtrong lines, or by double lines, drawn winding in form of the rivers they repreſent; and ſmall rivers are denoted by ſmall lines. Different countries are beſt coloured with different colours, or at leaſt their borders ſhould be ſo. Foreſts are repreſented by trees; and mountains ſhaded to make them appear. Sands are denoted by ſmall points or ſpecks. Rocks under water are known by a croſs. In any void place draw the mariners compaſs with the 32 winds. Theſe general maps, or maps of the world, are called *planiſpheres.*

PROP. XVIII. *Prob.*

To draw a map of any particular country.

1 *Method.*

In order to this, you muſt firſt know its extent as to latitude and longitude; as ſuppoſe it lies between the north latitudes 36 and 44; and extends from the longitude of 10 to 23 degrees; ſo its extent from north to ſouth is 8 degrees, and from weſt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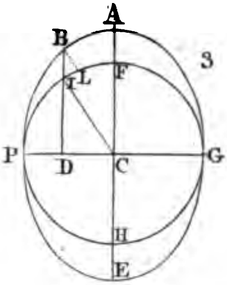
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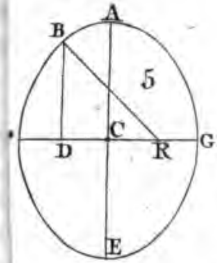
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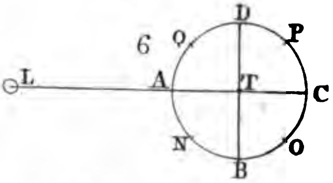
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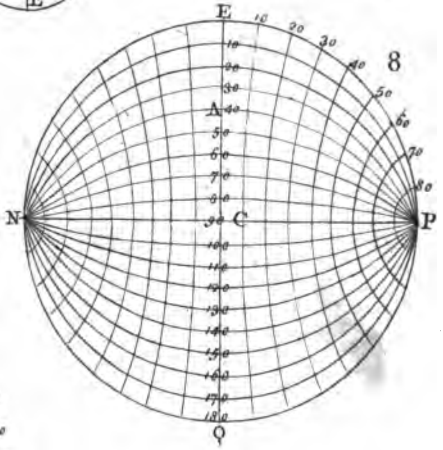
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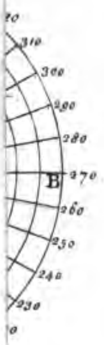
B

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weſt to eaſt 13 degrees. This will be a map of *Spain*.

Fig. 10. To proceed, draw the line AB for a meridian, paſſing thro' the middle of the country, on this ſet off 8 degrees from B to A, taken from any convenient ſcale. Let A be the north point, B the ſouth. Thro' A and B draw the perpendiculars CD, EF, for the extreme parallels of latitude. Divide AB into 8 parts or degrees, thro' which draw the other parallels of latitude, parallel to the former.

Next, to draw the meridians; divide any degree in AB into 60 equal parts or miles. Then as the length of a degree in each parallel decreaſes towards the pole; from the table Prop. 15, take the number of miles anſwering to the lat. of B, which is 48.54 or 48½, and ſet from B, 7 times to E, and 6 times to F; ſo EF is divided into degrees. Again from the ſame table, take the number of miles anſwering to the lat. of A, which is 43.16; this makes a degree in the lat. 44. Therefore ſet off 43 minutes, 7 times from A to C, and 6 times from A to D. Then from the points of diviſion, to the correſpondent points in the line EF, draw ſo many right lines, for the meridians. Then number the degrees of latitude up both ſides of the map; and the degrees of longitude on the top and bottom. In ſome vacant place make a ſcale of miles; or of degrees, if your map repreſent a large part of the earth; this will ſerve for finding the diſtances of places upon the map.

Then make the proper diviſions and ſubdiviſions of the country. And having the latitude and longitude of the principal places of the country, it will be eaſy to put them down in the map. For the place any town will be where the circles of its latitude and longitude interſect. Thus *Gibraltar*, whoſe lat. is 36° 11′; and longitude 12° 27′, will be at G. And

Fig. *Madrid*, whoſe lat. is 40° 10′, and long. 14° 44′,
10. will be at M. And thus the boundaries will be deſcribed, by ſetting down the remarkable places on the ſea coaſt, and drawing lines from one to another. In like manner the mouth of a river muſt be ſet down; but to deſcribe the whole river, the lat. and long. of every turning muſt be ſet down, and the towns and bridges by which it paſſes. And ſo for woods, foreſts, mountains, lakes, caſtles, &c. This way is very proper for ſmall countries.

2 *Method.*

Maps of particular places are but limbs of the globe, and therefore may be drawn after the ſame manner as the whole is drawn. Therefore a map of any particular country may be drawn either by the *orthographic* or *ſtereographic* projection of the ſphere, as in the laſt Prob. But as this is difficult to do in ſuch partial maps, we ſhall ſhew ſome eaſier ways, as follows.

10. Having drawn the meridian AB, and divided it into equal parts as in the laſt method; draw thro' all the points of diviſion lines perpendicular to AB for the parallels of latitude; EF, CD being the extreme parallels. Then to divide theſe parallels, ſet off the degrees in each parallel, diminiſhed according to the table in Prop. 15, as was done for the two parallels EF and CD in the laſt method. And thro' all the correſpondent points draw the meridians, which will be curve lines. In the laſt method theſe meridians were right lines, becauſe only the extreme parallels were divided by the table. This method is proper for a large country as *Europe*, &c. And then the parallels and meridians need only be drawn to every 5 or 10 degrees. This method is much uſed in drawing maps. And here all the parts will be nearly of their due magnitude,

but a little diſtorted towards the outſide, from the Fig. oblique interſections of the meridians and parallels. 10.

3 *Method.*

Draw PB of a convenient length, for a meridi- 11. an; divide it into 9 equal parts, and thro' the points of diviſion, deſcribe ſo many circles for the parallels of latitude, from the center P which repreſents the pole. Suppoſe the height of the map to be AB; then CD will be the parallel paſſing thro' the greateſt latitude, and EF will repreſent the equator. Divide the equator EF into equal parts, equal to thoſe in AB, both ways, beginning at B. Alſo divide all the parallels into the ſame number of equal parts, but leſſer, in proportion to the numbers for the ſeveral latitudes, as given in the table of Prop. XV; juſt as the rectilineal parallels were divided in the laſt method. Then thro' all the correſpondent diviſions, draw curve lines, which will repreſent the meridians: the two extreme ones are EC and FD. Then number the degrees of latitude, and thoſe of longitude, ſuppoſing EC to be the firſt meridian. And a ſcale of equal parts, either of miles or degrees, may be laid down for meaſuring diſtances. Maps are very commodiouſly drawn this way, and it is called the *globular projection.* Here alſo the parts of the earth will be repreſented of their due magnitude, excepting that they are a little diſtorted on the out ſides.

When the place is but ſmall that a map is to be made of, as if a county was to be map'd; the meridians as to ſenſe, will be parallel to one another, and the whole will differ very little from a plane. Such a map will be made more eaſily than by the foregoing rules. It will be ſufficient to meaſure the diſtances of places in miles, and by the help of

Fig. trigonometry, to lay them down in the map. And
11. this belongs more properly to ſurveying.

Cor. 1. *Hence, any place may be found which is in the map; if its longitude and latitude be given.*

This is done by carrying your eye along the parallel of latitude in which the place lies, till you meet with its circle of longitude; and there the place will be found.

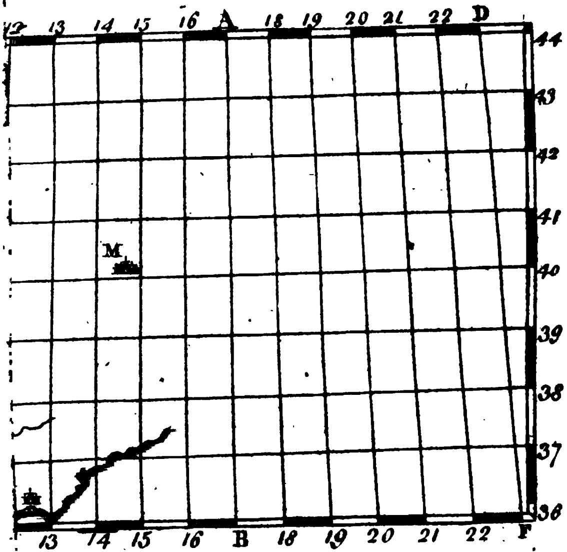
Cor. 2. *Hence alſo, the diſtance of any two places in the map may be found.*

To do this, take the diſtance of the two places in your compaſſes, and apply it to the ſcale, and it will ſhew the diſtance. If there is no ſcale apply it to the ſide of the map, and it will ſhew the diſtance in degrees.

If your diſtance be large, take ſome even number, in your compaſſes from the ſcale, as 100; and turning it over and over from one place to the other, going in a right line; the number of theſe turns, will give the diſtance. If any thing remains, apply it to the ſcale, which will ſhew how much the odd part comes to. But after all, the diſtance is moſt exactly known by ſpherical trigonometry, as was ſhewn in Prop. XIV.

SCHOLIUM.

Since no maps deſcribed upon a plane ſurface, can exactly repreſent any part of the earth's ſurface, which is ſpherical, but the parts will be in ſome meaſure diſtorted; and the more ſo, the larger the portion of the earth's ſurface is, which is to be repreſented. Therefore *Geographers* have invented the terreſtrial globe, on which they delineate all the parts of the earth's ſurface; which therefore will give a true and exact repreſentation of every part of it. And the ſame may be done for any ſegment of a ſphe-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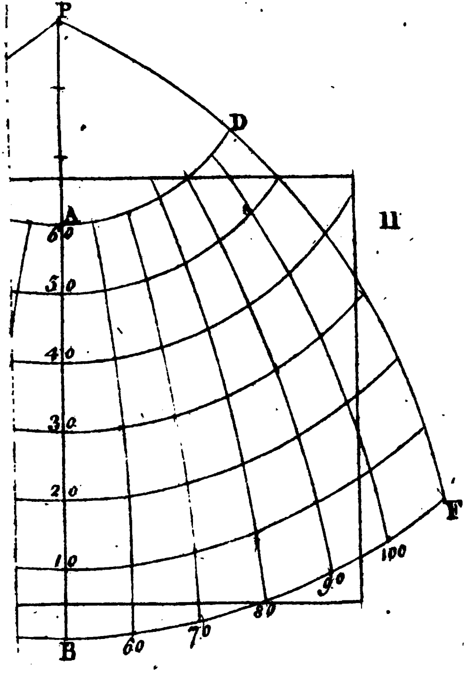
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II

Pl. II. pa. 54.

Fig. 11. a ſpherical ſurface; and then a map drawn upon ſuch a portion of the ſurface of the ſphere will alſo give a true repreſentation of the country deſcribed. And ſuch a part of a convex ſurface is capable of being made to a much larger diameter than a whole globe can. Theſe may be made of thin convex ſhells of paſteboard, formed to the intended radius, and the map paſted on, piece by piece. And each piece will be beſt made in form of a zone of 10 degrees in breadth. And each zone will be nearly a portion of a conic ſurface including, and touching, the ſphere in the middle of that zone. And that part of the conical ſurface being unrolled or extended into a plane, is the bottom part of the ſector of a circle. And therefore ſuch pieces are eaſily deſcribed; for the radius thereof is the tangent of the diſtance of the greater circle of the zone from its pole; and its breadth, that of the zone (10 degrees); and its length, the circumference of the zone. Theſe zones thus cut out, muſt firſt be printed, whilſt in plano, and then paſted on the ſegment of the globe, one after another. And this will conſtitute a kind of ſpherical map.

PROP. XIX.

If a man travels weſtward (or ſails in a ſhip) quite round the earth, till he comes to the place he departed from; he will loſe a day in his reckoning. But if he travels eaſtward round the globe, he will gain a day, when he comes to the ſame place again.

For ſuppoſe him that travels weſtward, continually to keep pace with the ſun, till he arrives at the firſt place; it is evident he has continual day, and therefore it is the ſame day to him all the while. But theſe people that remain at the place departed from, have had night in the mean time; conſe-

Fig. 11. quently they reckon a day more than he does. And the caſe is the ſame if he does not keep pace with the ſun, provided he ſtill goes weſtward; for by doing ſo he gets continually into more weſtern longitude, by which means every day is lengthened, ſo much as he changes his longitude. And at laſt arriving at the firſt place, he has changed his longitude to 360 degrees, which amounts to 24 hours or 1 day, that they have all been lengthened; and therefore he has loſt ſo much in his reckoning.

Again, ſuppoſe him that travels or ſails eaſtward, to perform his journey round the earth, in a very ſmall time; ſo that when he arrives at the place departed from, the ſun is ſtill up. It is plain, the ſame day continues to the inhabitants he left; but in the mean time, he has had one night; and that, when he was oppoſite to the ſun; and is now come into day-light again; and therefore he muſt reckon a day more than theſe inhabitants; that is, he has gained a day. And it is the ſame thing if he travels or ſails ſlowly eaſtward; for he continually comes into more eaſtern longitude, by which every day is ſo much ſhortened, which in the whole circumference amounts to 24 hours or one day. And his days being ſhortened by 24 hours, the number of them muſt be ſo much greater; that is, the ſailor reckons one day more than the inhabitants.

Cor. *Hence if one man ſails or travels weſt, another eaſt, and a third remains in the ſame place; what is friday to the weſtern ſailor, will be ſaturday to the inhabitant, and ſunday to the eaſtern traveller, when they all meet again.*

For he that ſails weſt has loſt a day, ſo ſaturday will be friday to him. And he that ſails eaſt, has gained a day, and therefore ſaturday will be ſunday to him. The inhabitant remaining in the ſame place, neither gains nor loſes; ſo that ſunday is ſtill ſunday to the inhabitant. Fig. 11.

PROP. XX. *Prob.*

To deſcribe the general affections and properties of the earth, &c.

It has been ſhewn before that the figure of the earth is ſpherical, or rather of a figure inclining to the ſhape of an oblong ſperoid; and that the earth and ſea together compoſing one body, both partake of the ſame figure; and that is, without taking notice of the ſmall irregularities upon its ſurface, inſenſible in reſpect of the whole. We have likewiſe ſhewn the origin of rivers, and the cauſe of the ebbing and flowing of the ſea. Likewiſe the various phænomena peculiar to the ſeveral inhabitants of the globe, in different zones of the earth, have been deſcribed. Its diurnal motion round its axis, and annual motion about the ſun have been ſhewn in the Aſtronomy. But there are ſeveral adjuncts or peculiarities belonging to the earth, which we muſt ſpeak of in this propoſition. And,

1. The body of the earth is every where ſurrounded by a body of air called the *Atmoſphere.* This body of air reaches to the height of 50 miles or more above the earth, as is gathered from the beginning of twilight, which is when the ſun is 18 degrees below the horizon. But notwithſtanding this great height of the atmoſphere, it grows continually thinner and rarer, the higher we aſcend. This fluid body of air is extremely light, being at a mean denſity 800 times lighter than water. It is likewiſe extremely elaſtic, as the leaſt motion excited in it, is propagated to a great diſtance. It is likewiſe inviſible,

Fig. 11. viſible, its exiſtence being only ſenſible from the effects it produces.

Air is ſo neceſſary to the life of man and other living creatures, that no animal on the earth can live without it. If any animal is deprived of this breath of life, it inſtantly dies. And even if the ſame air be breathed over and over again, it loſes its vivifying ſpirit, and becomes uſeleſs; ſo that an animal breathing ſuch air, without a ſupply of freſh air, will die in a very little time. So neceſſary is freſh air to the life of animals, that there is no living without it; and to be deprived of it, is immediate death.

Air is likewiſe the vehicle of *ſound*; and this ariſes from its elaſticity. For a body being ſtruck or put into any tremulous motion, communicates the ſame to the air, which, by its great ſubtilty and elaſticity, is ſuſceptible of any motion. This motion is then conveyed, from that body thro' the intermediate ſpace, to the ear, where it ſtrikes the *tympanum*, and excites the like motion in the auditory nerve, and gives the ſenſation of ſound; and particular motions, thus excite particular ſorts of ſound. One ſort is diſcourſe, another is muſic and harmony, and another diſcord and noiſe, &c.

To this ſubtle, elaſtic, property of the air, is owing that peculiar privilege we enjoy above all other creatures, that we can converſe with one another. For by the help of articulate ſound, thus conveyed to the ear of another perſon, we can at the ſame time convey to him our thoughts and ſentiments; and he reciprocally can return his. And thus we can converſe with one another on all ſorts of ſubjects with extreme eaſe and delight. So that this property of the air, is entirely the cauſe and origin of that noble ſenſe of *hearing*; and of that ineſtimable faculty of converſation. They that are deprived of the ſenſe of hearing are in a miſerable condition;

Fig. 11. condition; and eſpecially if they have been always in that ſtate. For people born deaf are quite deſtitute of any language, as they cannot be acquainted with any ſort of words; and conſequently all their other faculties will be almoſt uſeleſs to them.

Another property of the air is, that it is expanded by heat, and condenſed or contracted by cold; and from hence is the origin of *winds*. The wind is a current or ſtream of air moving from one place to another; and the principal cauſe thereof, is the heat of the ſun. This heat acting upon ſome part of the air, cauſes it to expand and grow lighter, conſequently it muſt aſcend, whilſt the air adjoining, which is denſer and heavier, will preſs forward towards the place where it is rarified and aſcending, to keep up the equilibrium; and that motion of the air conſtitutes the wind. Hence in any place where the wind blows the air is rarified and lighter, as is evident from the falling of the barometer in high winds. As the air is by its own nature extremely fluid, it yeilds to any the leaſt motion exerted on it, and therefore it is hardly ever at reſt, or continues any conſiderable time in ſuch a ſtate. As the ſun moves from eaſt to weſt, it rarifies the atmoſphere as it goes round the earth; and hence in the torrid zone, the motion of the wind is from eaſt to weſt. In the heat of the day gentle breezes blow from the ſea to the land, becauſe the earth is more heated than the ſea, and rarifies the air over it. Several other cauſes may likewiſe concur to the production of winds, as hot exhalations, rarefaction, or deſcending of clouds, or any thing that condenſes the air.

Tho' a great deal of miſchief is ſometimes done by high winds and tempeſts; yet the wind is exceedingly uſeful in the affairs of life, as long as it keeps within due bounds. Without the wind there would be no navigation, and without navigation

Fig. 11. there would be no trading from one country to another, and conſequently no commerce, no merchandize. Nor would it be poſſible to get from one country to another far diſtant country. The wind alſo clears the air, and rids it of theſe noxious vapours that are ſo prejudicial to our health, and the cauſes of almoſt all diſtempers. And thus by keeping the air in continual motion it keeps it pure and wholeſome. A ſtrong wind will move through 10 yards in a ſecond. And a tempeſt will run over 45 miles in an hour.

Another property of the air is, that it is capable of ſuſpending theſe vapours from which rain is generated. For the ſun's heat rarifying the particles of water, and transforming them into vapour; theſe vapours being ſpecifically lighter than the atmoſphere; they will aſcend, till they come at ſuch a region of the air, where the ſpecific gravity is the ſame. There they will remain in the form of clouds; where being toſſed to and fro with the wind, they will at length be reduced to ſmall particles or drops of water, and afterwards into larger drops, which will then deſcend in miſt, dew, or rain. Hence without the air we ſhould have no vapours, no clouds, no rain; and conſequently no produce from the ground. And the earth would be nothing but a uſeleſs, dry, barren heap of matter, uncapable of producing any fruit for the ſupport of our lives.

From the fluid nature of the air it likewiſe follows, that the atmoſphere muſt alſo be ſubject to the diſturbing forces of the ſun and moon; and therefore theſe forces will generate a tide of air, or the flux and reflux of the atmoſphere, twice in 25 hours, after the ſame manner, and for the ſame reaſon, as they cauſe the flux and reflux of the ocean. And conſequently, theſe tides of the atmoſphere muſt be bigger at the full and change, and at that time raiſe a ſenſible commotion in the atmoſphere, which at thoſe times may ſenſibly affect valetudinarians and weak conſtitutions. Fig. 11.

2. As to the ſurface of the earth, it is not perfectly ſmooth and even, but is every where interſperſed with *hills*, *mountains*, *vallies*, *rivers*, &c. If the earth was perfectly level, there could be no ſuch thing as any rivers, for no water can flow, but from a higher to a lower place; and therefore upon level ground it could not run away; the rain that falls in any place muſt always remain there, till it ſinks into the earth or be dried up; in ſome places it would ſtand and putrify, and fill the air with unwholeſome ſteams and vapours. The face of nature having no variety, would have no beauty, and would appear with no pleaſing aſpect. On the contrary, experience ſhews us that all countries abound with mountains and rivers, which is not only the greateſt ornaments thereof, but the moſt advantageous for the inhabitants. Upon plain ground, a man's proſpect is very much limited, but from the top of a hill or mountain a large proſpect lies open to his ſight. Here he can view with pleaſure cattle feeding on the hills, rivers running in the vallies, the vaſt variety of trees and plants with which the ground is enriched, and all the houſes, towns, caſtles, &c. that are placed there, for a great many miles. This makes a grand appearance, and gives a juſt idea of the extent and beauty of nature. The mountains by their height ſecure the vallies from cold, from winds and tempeſts; and by keeping them warm, make them fruitful; whilſt the rivers by their turnings and windings become uſeful to many places.

As all rivers are upon the deſcent, therefore in going from the mouth of a river, the ground muſt continually riſe from the ſea, all the way to the ſpring head. And therefore the longer any river is, Fig. 11. is, the higher the ground will be at the head of it. And therefore ſuppoſing for every mile a deſcent of 2 feet; in ſuch a river as the *Danube*, which runs thro' more than 800 miles of ground, the ſpring head will be more than 500 yards above the level of the ſea; and the mountains will be ſtill higher. So that the middle of any continent muſt always be a great deal higher than the ſea coaſts.

That there has always been mountains from the beginning, is as certain as that there has always been rivers. For they were as neceſſary at firſt for every purpoſe, as they are at preſent. Alſo all mountains and high places have always been decreaſing and growing lower. For rivers running near mountainous places, do by degrees eat away the feet of them, where parts of them being undermined, tumble into the river, and are waſhed down the ſtream. Rain falling on the tops of mountains waſh away the looſe parts, and undermine the ſolid parts which in time will tumble down. Thus old buildings on the tops of mountains are obſerved to have their foundations laid bare, by the gradual waſhing away of the earth; and at laſt fall down. In plains and vallies we find it quite contrary. For the matter waſhed down from the high places, fill up the low places, which by that means are raiſed higher. Whence it is, that ancient houſes built in vallies ſeem to have ſunk a great way into the earth, occaſioned by the earth being raiſed higher than at firſt. For the ſame reaſon a deal of the mud, ſlime, ſand, and earth, that is continually waſhed down from the higher places into the rivers, is carried down by theſe rivers; by which means the mouths of the rivers are by degrees choaked up, to the great detriment of navigation; and at the ſame time a part of this ſand and rubbiſh is carried into the ſea. Upon the whole, as the higher parts are always deſcending, and the lower parts

filling up and riſing, the earth is continually ap- Fig.
proaching to a level, tho' by very ſlow degrees. 11.

In former times many mountains were covered with large and thick woods, which ſheltered them and the adjoining plains from the rigour of the cold. In later times, men having much occaſion for timber, for the building of houſes and ſhips, &c. have cut moſt of them down; and large woods are not ſo common now as in former times. It is obſerved, that trees thrive beſt and grow largeſt, where there is plenty of heat and plenty of moiſture.

As to the height of mountains there are various accounts. The ancients reckoned their heights a great deal more than they really are, making ſome of them many miles high. But later obſervations have ſhewn us, that few or none of them are above 3 miles high. The *pike* of *Tenerif*, one of the *Canary* iſlands, has been meaſured to 2½ miles. And *Snowden hill* to ¾ of a mile. It is extremely cold on the tops of all mountains, even under the equinoctial; for the tops of them are for the moſt part covered with ſnow; except it be in ſuch of them as have any *Volcano's* within them. And theſe Volcano's are generally in ſome mountain, as that in mount *Etna*, mount *Veſuvius*, &c. It has been obſerved, that there are mountains in the moon 3 times as high as any on the earth.

3. We have already ſhewn, that the *ſea* together with the earth makes one globe, that it ebbs and flows twice in near 25 hours, that it furniſhes matter for vapours and rain. But it is likewiſe of infinite ſervice to trade and commerce: for if all the globe was earth, it would be impoſſible to trade to far diſtant countries. Such long journies could not be performed by land; much leſs could any traffic be carried on, or any goods tranſported to or from places ſo far diſtant; all which is now done by ſhipping, with incredible eaſe. So that if there

Fig. 11. was no ſea to ſail on, there would be no ſhips to ſail; and conſequently no trading to foreign parts in any kind.

The *ſaltneſs* of the ocean is the peculiar property of ſea water. How it comes by ſuch a quality has been a matter of diſpute. Dr. Halley is of opinion, that it may acquire that property gradually. For he imagines the ſea was freſh at firſt, but by the rivers running into it, and carrying with them ſuch ſaline particles, as they have imbibed from the earth, they paſſed thro'; theſe particles of ſalt are then depoſited in the ſea, where they muſt remain; for nothing evaporates from the ſea but freſh water. So that the ſaltneſs will continually increaſe, as long as the rivers keep running into it. This is his opinion, or rather his hypotheſis, which he propoſes to have tried. Others imagine, that there are a great number of rocks of ſalt diſperſed thro' the earth at the bottom of the ſea, and from theſe rocks it acquires its ſaltneſs. But whatever be the cauſe, the fact is ſo; and this quality may ſerve to keep the water of the ſea pure and good, to which the motion, by the ebbing and flowing of it, may contribute.

Another property of ſea water is, that when it is abſolutely ſalt it *never freezeth*. This property is proved by manifold experience; and is ſaid to ariſe from ſomething of a hot ſpirituous nature which is contained in the ſalt. In the north ſeas, ſailors frequently meet with huge rocks and iſlands of ice. But theſe don't conſiſt of ſalt water frozen; for they frequently get large pieces of the ice, and thaw it, and always find it freſh, which therefore they take for the uſe of the ſhips. This ice therefore is frozen in the rivers, and in ſummer when a thaw comes, the ice is all carried down the rivers, and driven into great heaps, where they float in the ſea, as the ſailors find them. It may be obſerved,

ſerved, that ſea water is denſer and heavier than Fig. 11. freſh water; which is owing to the ſalt it is pregnant with.

To find out a paſſage to the Eaſt Indies by the north eaſt, or north weſt, has long been wiſhed for, and often attempted, but without any ſucceſs; for the quantities of ice that float up and down in the northern ſeas, makes the navigation very dangerous, and the ſea almoſt impaſſable. And this ſeems to make it improbable that there is any ſuch paſſage to be found. For great quantities of ice, argue great quantities of freſh water, and theſe denote great rivers, and theſe again denote a large extent of land. And therefore if there be any ſuch paſſage, it is only thro' ſome narrow ſtraits; and conſequently could be of little uſe if known; ſince in that part of the world, ſuch ſtraits muſt almoſt always be blocked up with the ice, which would render ſuch a paſſage either uſeleſs or very dangerous. Hence it will he the beſt way for thoſe that would attempt ſuch a voyage, to endeavour to get into an open ſea, where there is no ice, and purſue their voyage therein as far as they can; for then they may be aſſured they are far from land. But theſe heaps of ice are certain ſigns of an adjoining country.

Concerning the iſlands every where diſperſed thro' the ſea, we ſhall ſpeak afterwards; as likewiſe of the ſeveral continents, and the countries they contain.

4. As to the *internal parts* of the earth, we are not ſo well acquainted with theſe, as with the external. For it is but a very little way we can penetrate into it, to make any obſervations. In the digging or working of mines, we meet with all ſorts of earths, loom, marl; all ſorts of ſtones, as pebbles, freeſtone, marble, alabaſter, ſpar, ſlate, chalk; alſo ſand, clay, coal; oars of all ſorts and metals,

Fig 11. gold, ſilver, lead, iron, copper, tin; all ſorts of minerals, nitre, ſulphur, ſalt, alum, boles, oker, vitriol, borax, cinnabar, antimony, arſenic, biſmuth, bezoar, and infinite others which are contained in the body of the earth, ſometimes pure, and ſometimes mixt.

All theſe different ſorts of ſubſtances within the earth, are diſpoſed of in certain layers, beds, or ſtrata, which compoſe the body of the earth; each kind in its own bed. In ſome places, they are ſeated nearly according to their ſpecific gravities; in others not at all ſo, but lye in a very confuſed order.

In ſeveral parts of the earth, there are found ſhells reſembling the ſhells of fiſhes, and their bones, and likewiſe vegetables, ſometimes very deep in the ground. As to the ſhells of fiſhes, ſome are of opinion that they have been brought there at the time of the flood, when all things were in a ſtate of confuſion, and promiſcuouſly mixt with any other bodies that happen to be there; and that they have lain there ever ſince. Others ſuppoſe that theſe ſhells are generated in the beds wherein they are found, and that the earth may be as fit for the production of ſuch bodies as the ſea. And this ſeems to be confirmed from the ſubſtance of the ſhells being the very ſame with the ſubſtance of the ſtone, or other bodies they adhere to; and likewiſe of the ſame ſpecific gravity therewith; which would not be ſo, if theſe were real ſhells. Some people will likewiſe have ſuch trees as are buried deep underground, to have laid there ever ſince the flood, a thing quite improbable, and the more ſo, becauſe other reaſons may be aſſigned for their coming there; as has been proved in ſeveral inſtances beyond contradiction.

5. The ſurface of the earth is not only made habitable, but enriched with all ſorts of herbs, plants, fruits, roots, &c. neceſſary for the life and ſupport

of all animals, of which, man is the principal. Upon this earth we are born, we live, and we die, it is our deſtined habitation. When we are born, the earth like a tender mother receives us into her lap; ſhe feeds and cloaths us whilſt living; and at laſt kindly receives us again into her boſom, and covers us, where we reſt till the diſſolution of all things. Fig. 11.

The animals that inhabit the earth are divided into four general kinds. 1. Men. 2. Beaſts, theſe are reckoned land animals. 3. Birds, whoſe element is the air. 4. Fiſhes, which dwell in the ſea or element of water. To theſe, if you pleaſe, you may add all the tribes of inſects. The ſeveral ſorts of animals are ſo well known as to need no deſcription of mine; and beſides I am not writing a book of natural hiſtory, but a mathematical treatiſe.

PROP. XXI. *Prob.*

To give account of ſuch remarkable changes as have happened to the earth in ſeveral ages of the world.

That the earth has undergone ſeveral changes and alterations, all hiſtory will teſtify. To paſs by ſuch changes as happen by the deſcending of the earth, from the hills, and filling up the vallies, and ſtopping the mouths of rivers and ſuch like matters, which are gradual, and much the ſame in all ages, and natural to the earth: I think, the more remarkable changes may be reduced to two general cauſes, and theſe are floods and earthquakes.

1. As to floods, there have been three remarkable ones recorded in hiſtory. The firſt and moſt terrible is that of *Noah*, which is ſaid to have overflowed the whole earth, and drowned all the living creatures thereon, except ſuch as ſaved themſelves in an ark. Concerning the cauſe of this flood,

Fig. 11. ſome ſuppoſe it brought about by natural cauſes; and others, by nothing leſs than a divine power. Theſe that are for natural cauſes, imagine a comet to have paſſed near the earth at that time; and by its approach, to have raiſed a very ſtrong tide, which would increaſe as the comet approached the earth. The effect of this would be, that this great tide would lay all places under water; and would conſequently drown all the inhabitants ſo far as it reached. That ſuch a cauſe of this is capable of producing this diſmal effect, is very evident. For if ſo ſmall a body as the moon, at the diſtance of 60 of the earth's ſemi-diameters, be able to raiſe a ſtrong tide in the ocean, of 12 or 15 feet high; a comet as big as the earth, and coming very near it, would raiſe a prodigious tide, capable of overflowing all that ſide of the earth which is next to the comet, and alſo the oppoſite ſide. But then this could not drown all places at once; for at the quadratures, or in thoſe places, which have the comet in their horizon, they would have as great an ebb. But then it would have this effect, to overflow and drown all places ſucceſſively. For this huge ſpheroid of water, always pointing towards the comet, would by the earth's rotation paſs over all the countries of the world; and therefore in the ſpace of 24 hours, the whole earth would be involved in water, and all animals as effectually deſtroyed as if the water ſtaid 150 days upon the earth; eſpecially as the earth muſt needs make ſeveral rotations after this manner, before it could get clear of this diſturbing force of the comet. The natural and neceſſary effect of all this would be, that by ſuch a prodigious and rapid motion of this vaſt body of water round the earth in 24 hours, all plants and trees muſt be torn up by the roots, and carried along with the current; all buildings demoliſhed; the rocks, hills and mountains daſhed in pieces,

and torn away; all the product of the ſea, as Fig. 11. fiſhes, ſhells, teeth, bones, &c. carried along with the flood, and thrown upon the earth, or even to the tops of mountains, promiſcuouſly with other bodies; hardly any thing could be found ſtrong enough to withſtand its force. In ſuch a caſe as this it would be impoſſible for any ark to live (be preſerved) at ſea, or the ſtrongeſt man of war that ever was built. The like vaſt tides would alſo be raiſed in the atmoſphere, attended with the moſt violent commotion of all the body of air; the conſequence whereof would be continual rains. In this caſe no place of ſafety could be found for any animals, except they had the good luck to be upon ſuch a high mountain as was without its reach.

Thoſe that ſuppoſe the water to be over all the face of the earth at once, are forced to call in the power of omnipotence to effect it; for it is a thing impoſſible by any natural cauſe, to produce ſuch an effect. Therefore in order to preſerve mankind and other animals, God was pleaſed to order an ark to be built, to contain a few of every ſpecies of animals, which were afterwards to repleniſh the earth. As this ark, with its contained animals, was to ſwim upon the water of the deluge, all the time of its overflowing the earth; it was neceſſary that this flood of water ſhould be perfectly calm, and free from all ſtorms and tempeſts. For if the ark came to be toſſed about in a tempeſtuous and raging ſea, or deluge of water; from its ſtructure and magnitude, it muſt inevitably periſh, with all its freight of animals. But I believe it would be a perplexing affair to make out how ſuch a tumultuous concourſe of water ſhould be ſo very quiet and ſtill, ſo clear of winds, ſtorms and tempeſts, as is here required. And if this was granted, it would ſtill be equally difficult to account for another phænomenon; that is, how all ſhells and marine bodies, ſhould be

Fig. 11. thrown upon the land, or even to the tops of mountains by ſuch a ſtill water; and many of them buried deep in the earth; this effect is not at all reconcileable with ſuch a ſuppoſition. Therefore it does not appear that both theſe hypotheſes can be true. For the calm ſea, neceſſary for preſerving the ark, could move none of the ſhells; and the rough ſea, neceſſary for tranſporting the ſhells, would deſtroy the ark. The reconciling theſe things is not eaſy; and perhaps here, as in other ſuch caſes, our only refuge is, when we are pinched with any difficulty, to cry out for a divine power. Yet the Almighty generally brings about his purpoſes by natural cauſes. But be that as it will, it is certain, that ſome time or other this earth has ſuffered a moſt violent ſhock, concuſſion or agitation by ſome cauſe or other, unknown to us; but which manifeſtly appears by its effects.

The next flood we have an account of, is that of *Ogyges*; this flood overflowed all *Attica*, a country in *Greece* joining to the Mediterranean. This flood happened in the time of *Ogyges*, who was king thereof, and dwelt in *Thebes*, a city of his own building. This country is a part of what is now called *Achaia*, being a part of *Greece*.

The next remarkable flood was that of *Deucalion*, celebrated by the poets, eſpecially by *Ovid*. This flood overflowed all *Theſſaly*, where *Deucalion* was king. This country of *Theſſaly* is a part of *Greece*, and joins upon the ſea.

Beſides theſe floods, there is no doubt but great floods have happened in other countries; but the people being ignorant of the uſe of letters, no account thereof is tranſmitted to us. All theſe floods muſt needs make great havock and devaſtation in all the countries where they happen; by demoliſhing towns, plants, trees; by beating and waſhing down mountains; and levelling the low grounds;

by filling up the channels of rivers, and changing their courſes; by covering all the country with ſlime and mud, and giving a new face to the earth; by breaking off part of a country, and reducing it to an iſland; and by inundations breaking into the land and forming gulphs therein. Thus ſome have ſuppoſed that *Sicily* has been divided from *Italy*; *Spain* from *Barbary*; and, as ſome ſay, *England* from *France*. Fig. 11.

The violence of the wind alſo much increaſes the depth and turbulent motion of a flood, eſpecially in ſuch inundations as proceed from the ſea. Thus high winds and tempeſts frequently cauſe the ſea to tranſcend its ordinary bounds, and treſpaſs on the land, to the great loſs and danger of the inhabitants. Thus it happened in *Zealand* and *Holland*, where ſeveral towns with multitudes of people were ſwallowed up by the ſea; occaſioned by the violence of the north weſterly winds.

Several inundations or floods have alſo happened in Egypt, by the overflowing of the Nile, as is reported by ſeveral hiſtorians, and the like in a great many other places.

2. Earthquakes are another great cauſe of the changes made in the earth, and the deſolation of ſeveral places, to the ruin of the inhabitants. An *earthquake* is a violent ſhaking of ſome large part of the earth, ſuppoſed to be cauſed by a nitrous and ſulphureous vapour included in the bowels of the earth, which by ſome accident taking fire produces an exploſion, which occaſions theſe terrible motions and ſhocks of the earth at that time. The firing theſe ſulphureous vapours, may be owing to their fermentation, or to the falling of rocks and ſtones within the hollow places of the earth, and ſtriking fire againſt one another.

Thunder and lightening ariſes much from the ſame cauſe, for the nitro-ſulphureous vapours, exhaling

Fig. haling out of the earth, and floating about in the
11. air, at laſt by their fermentation take fire, and produces that dreadful exploſion called Thunder.

The violence of earthquakes is ſometimes ſo great as to ſplit and tear the earth, and cauſe it to open for many miles; gaping at the exit of the ignified vapour, and cloſing again after. And ſometimes tearing and disjoining the ground, ſo that after the ſhock, it immediately ſinks; and thus a hill or a mountain is ſometimes reduced to a great pool of water or a lake. After this manner ſeveral conſiderable tracts of land, with the cities and towns, plants and trees, have been ſwallowed up, and totally ſunk.

The ſtrength of this confined vapour is ſo great, that it forces up ſtones, water, earth, and all things that fall in its way, with a very great impetuoſity, caſting them ſometimes to immenſe diſtances. In this manner they are thrown out of the mouths of *Volcano's*, or any place where the inflamed vapour can find vent, or break open. And from the heat of the inflamed vapour, which forces out theſe bodies, they are all of them alſo heated and inflamed thereby; ſo that flames of fire are ſometimes ſeen to proceed out of the earth, with great quantities of melted metal or metalline ſubſtances. Great floods of this kind are generally thrown out of burning mountains.

When this happens in a place ſeated underneath the ſea, the motion of the earth below, puts the ſea into horrible tumult, making it rage and roar, with a hideous noiſe, raiſing its ſurface into prodigious waves, which are rolled and toſſed about in a furious manner, overſetting and ſinking ſhips, or throwing them on the land. By this, the ſea is raiſed far above its ordinary height, and depreſſed again as far below. And thus it continues in the moſt extreme agitation, till the hot vaporous matter be all ſpent and diſcharged; and then the earthquake ceaſes, till a freſh collection of this ſulphureous vapour produces another. And this ſulphureous vapour diſcharged into the atmoſphere may furniſh matter for thunder and lightening; as they are both generated from the ſame cauſes, and require the ſame matter for both to work upon. Fig. 11.

The ſhock of an earthquake is always felt at the ſame moment in all places, tho' it has ſometimes been known to reach many hundred miles; and altho' the ſeveral places be parted by the ſea lying between them.

Mountainous countries are the moſt ſubject to earthquakes; for mountains are commonly ſtony and hollow within, and therefore fit to receive theſe vapours; and particularly ſuch places as contain a great deal of ſulphur and nitre, are of all others the moſt ſubject to earthquakes, and generally ſuffer moſt thereby. Such combuſtible matter taking fire will burn for a long time, and ſend out vaſt flames with a horrid noiſe, as is frequent in mount *Etna* and *Viſuvius*, and all ſuch places. And theſe Volcano's are of uſe for diſcharging the matter, and preventing worſe conſequences.

Earthquakes are far more frequent than great floods; not one age paſſes without ſeveral inſtances of this ſort, with their direful effects, ſuch as ſinking of towns and mountains, as happened lately at *Liſbon*. The changing the land into ſea; and the ſea into land, by throwing up new iſlands where none appeared before. And by ſuch events as theſe, the face of the earth muſt be continually changing, and muſt be quite altered from what it was many ages ago.

Other ſmaller changes I paſs by, ſuch as the alteration made in the ſurface of the earth by cultivation, the cutting down woods, incloſing ground; building of towns and cities; as likewiſe the devaſtation

Fig. 11. vaſtation made by armies, by the invaſion of countries; where many beautiful places have been laid waſte, towns burnt and depopulated, and innumerable ſuch like accidents; all which help to give a different aſpect to almoſt all countries upon the earth.

Beſides the changes already mentioned, moſt places have changed their original names; ſo that our modern geographical deſcriptions or maps hardly give us any region under the ſame name, by which it was known in former times. And this has occaſioned great controverſy and diſpute about diverſe countries mentioned by ancient writers; and not only countries, but cities and towns have loſt their original names, and at preſent go under other names. And all this may be in great meaſure owing to the conqueſts made by one nation making war againſt another. For by this means the boundaries of countries have been quite changed, ſometimes by tearing a part from one country and throwing it to another, or throwing ſeveral of them into one, or dividing one country into ſeveral parts. And the conquerors, either not knowing or not liking the old names, of towns and countries, impoſed other of their own. Thus the ancient *Scythia* is now called *Tartary*. What was *Thrace* is now *Romania*, what was *Libya* is now *Tripoli*, or a part of *Barbary*. *Achaia* is now *Lividia*. *Cyrene* is *Barca*. As to towns, *Athens* is become *Settines*. *Berenice* is *Suez*. *Leopolis* is *Lemberg*. *Olympia* is *Longinico*. *Præneſte* is now *Paleſtrina*. *Suſa* is *Schonſter*. *Sparta* is *Miſitra*. *Delphos* is *Livadia*. *Peluſium* is *Damietta*, and numbers of others.

PROP.

Fig. 11.

PROP. XXII. *Prob.*

To ſhew the diviſion of the earth into its ſeveral kingdoms and countries.

The deſcription of the ſeveral countries of the world is the leaſt part of my buſineſs, as belonging more to an hiſtorian than a mathematical writer; yet I ſhall here give a ſhort account of each country, and how it is ſituated; without which it is hardly poſſible to underſtand any thing of hiſtory. And to have a right notion of any place, of its ſituation, and extent, the geographer ought to furniſh himſelf with maps of each country, which will ſhew him at once all theſe particulars, as alſo what cities, towns, caſtles, rivers, bridges, mountains, and other remarkable things, are contained in it; and without ſuch maps we have very faint ideas of any nation or country.

The earth is generally divided in four quarters,

1 *Europe.* 3 *Africa.*
2 *Aſia.* 4 *America.*

But this diviſion is very imperfect; for there are vaſt tracts of land under both the poles, not yet diſcovered, and which belong to none of theſe four diviſions. Therefore taking in theſe two polar regions, the earth is more properly divided into ſix general parts, which will take in the whole earth whether land or ſea.

1. EUROPE, is bounded on the north partly or wholly by the frozen ſea; on the ſouth, by the Mediterranean ſea, on the weſt by the Atlantic ocean, and on the eaſt by the rivers Tanais, Oby, and Volga, which laſt falls into the Euxine ſea. In deſcribing the ſeveral countries, I ſhall begin with the north parts, and proceed to the ſouth.

Sweden.

Fig. 11.

Sweden.

This country is contained between the n. latitudes 60 and 72, and longitudes 30 and 56 from *Ferro*. It is bounded on the eaſt by *Muſcovy*, on the weſt with *Norway*, on the north with the *Frozen ſea*, on the ſouth by *Denmark* and the *Baltic ſea*.

It is divided into the provinces *Finmark*, *Lapland*, *Sweedland*, *Finland*, *Gotbland*, *Livonia*, and the Swediſh iſlands.

Norway.

It is contained between the n. latitudes 60 and 68, and longitudes 30 and 38. It is bounded on the eaſt by *Sweden*, on the weſt by the *weſt ſea*, on the north by the *frozen ſea*, and on the ſouth by an arm of the *weſt ſea*.

Denmark.

Contained between 54 and 57 degrees of n. lat. and between 30 and 35 degrees of longitude. It contains *Jutland*, and the iſlands *Zealand*, *Funen*, &c. *Jutland* is bounded on the ſouth by *Germany*; on the other ſides by the ſea.

Muſcovy or Ruſſia.

This is a large country, extending from the n. lat. 45 to 71; and between the long. of 48 and 107. It is bounded on the eaſt by *Tartary*, on the weſt by *Sweden*, on the north by *Lapland* and the *Frozen ocean*, on the ſouth by *Poland* and the *Caſpian* and *Euxine* ſeas. It ſtands partly in Europe and partly in Aſia; it is divided into the provinces of *Trines*, *Kargapolia*, *Divina*, *Condora*, *Siberia*, *Obdora*, *Vologda*, *Caſan*, *Mordowitz*, *Kiſnovogrod*, *Volodimar*, *Moſcow*, *Aſtracan*, *Novogrod*, *Weliki*, *Pleſkow*, *Severia*, and many more.

Fig. 11.

Great Britain.

This is an iſland, and contains England and Scotland; it is contained between the n. lat. 50 and 59, and long. 14 and 22. It is divided into a great many counties or ſhires; and there are a great number of ſmall iſlands, lying to the north and north weſt thereof.

Ireland.

This is alſo an iſland on the weſt ſide of Great Britain, and is contained between the n. lat. 51 and 56; and long. 9 and 14; which is alſo divided into ſeveral counties.

Germany.

This lies between latitude 45 and 55, and long. 26 and 39. It is a large country, and conſiſts of many provinces, dukedoms, circles, &c. It is bounded on the eaſt with *Pruſſia*, *Poland*, and *Hungary*, on the weſt with *France*, on the north by *Denmark* and the *Baltic ſea*, on the ſouth by *Italy*. It contains on the north, *Holland*, *Flanders*, *Weſtphalia*, *Lower Saxony*, *Upper Saxony*. In the middle, *Lower Rhine*, *Upper Rhine*, *Franconia*. On the ſouth, *Suabia*, *Bavaria*, *Auſtria*.

By ſubdiviſion Holland contains *Zealand*, *Utrecht*, *Zutphen*, *Over-Iſſel*, *Frieſland*, *Groningen*. And Flanders contains *Gelderland*, *Brabant*, *Luxembourge*, *Limbourge*, *Arteſia*, *Hannonia*, *Namur*. Weſtphalia contains *Oldenbourg*, *Hoya*, *Diepholt*, *Schomberg*, *Embden*, *Lingen*, *Tecklenburg*, *Ravenſburg*, *Benthem*, *Munſter*, *Lip*, *Mark*, *Berge*, *Cleves*, *Juliers*. Lower Saxony contains *Holſtein*, *Lawenburg*, *Mecklenburg*, *Bremen*, *Verden*, *Lunenburg*, *Brunſwick*, *Halberſtat*, *Magdeburg*. Upper Saxony contains *Miſnia*, *Thuring*, *Brandenburg*, *Pomerania*, &c. but the ſubdiviſions of theſe countries are endleſs.

Pruſſia.

Fig. 11.

Pruſſia.

This extends from lat. 51 to 53, and long. 35 to 45; it is bounded on the eaſt by *Poland*, on the weſt by *Germany*, on the north by the *Baltic*, on the ſouth by part of *Poland*.

Saxony.

Bounded on the north by *Pruſſia*, on the ſouth by *Bohemia*, on the eaſt by *Poland*, and weſt by *Germany*.

Bohemia.

Bounded on the eaſt by *Poland*, on the weſt by *Germany*, on the north by *Saxony*, on the ſouth by *Hungary*.

Sileſia.

Bounded on the north by *Brandenburg*, on the ſouth by *Hungary*, on the eaſt by *Poland*, on the weſt by *Bohemia*.

Poland.

Bounded on the eaſt by *Ruſſia*, on the weſt by *Sileſia*, on the north by *Pruſſia*, on the ſouth by *Hungary*.

Hungary.

This country extends from 45 to 49 deg. n. lat. and from the long. 34 to 41. It is bounded on the eaſt by *Turkey*, on the weſt by *Germany*, on the north by *Poland*, on the ſouth by *Turkey*.

France.

This country is contained between the n. lat. 43 and 51, and long. 14 and 28. It is bounded on the eaſt by *Germany*, on the weſt by the *Atlantic Ocean*, on the north by the *Engliſh Channel*, on the ſouth by *Spain* and the *Mediterranean Sea*.

Fig. 11.

Spain.

It is contained between the n. lat. 36 and 45, and long. 10 and 23. It is bounded on the eaſt by the *Mediterranean*, on the weſt by *Portugal* and the *Atlantic Ocean*, on the north by *France* and the *Bay of Biſcay*, on the ſouth by the *Atlantic Ocean* and part of the *Mediterranean*.

Portugal.

Bounded on the eaſt by *France*, on the weſt by the *Atlantic Ocean*, on the north by *Spain*, on the ſouth by the *Atlantic Ocean*.

Italy.

This is contained between the n. lat. 38 and 47, and long. 27 and 41. It is bounded on the ſouth and eaſt by the *Mediterranean Sea*, on the north by *Germany* and *Switzerland*, on the weſt by *France* and the *Mediterranean*.

Switzerland.

Bounded on the north and eaſt by *Germany*, on the ſouth by *Italy*, on the weſt by *France*.

Turkey.

This is contained between 37 and 49 deg. of n. lat. and 38 and 55 deg. of longitude. It contains *Greece*, *Little Tartary*, and part of *Hungary*, and many provinces beſides. It is bounded on the eaſt by the *Euxine Sea*, on the weſt by the *Gulph of Venice*, on the north by *Hungary*, on the ſouth by the *Mediterranean*.

Iſlands in Europe.

The principal are the *Orkneys*, the *Shetland* iſles, and Weſtern iſlands near Scotland. The Iſle of *Man*, *Angleſey*, and iſle of *Wight*, near England, *Jerſey*

Fig. *Jerſey*, *Guernſey*, *Alderney*, near France. The
11. iſlands *Rugen*, *Bornholm*, *Oeland*, *Gothland*, *Oeſal*, *Dago*, *Aland*, *Ween*, *Zealand*, *Funen*, *Langland*, *Laland*, *Fulſtar*, *Mona*, *Femeren*, *Alſen*, near Denmark and Sweden. Alſo *Carmen*, *Hitteren*, *Sanien*, *Suray*, near Norway, and an infinite number of ſmaller ones.

The *Azores* or *weſtern iſlands*, lying weſt from Spain, being 9 in number; they are St. *Michael*, St. *Maria*, *Tercera*, *Gratioſa*, St. *George*, *Pico*, *Fyal*, *Flores*, *Cuervo*.

Iſlands in the Mediterranean, are *Yoica*, *Majorca*, *Minorca*, lying near Spain. *Corſica*, *Sardinia*, *Sicily* and *Malta*, near Italy. *Candia* near Greece, *Cyprus* near Syria in Aſia; and a vaſt number of ſmall ones near Greece in the Archipelago.

2. ASIA, is bounded on the eaſt by the *Eaſtern ſea*, on the weſt by *Europe* and the *Red ſea*, on the north partly by the *Frozen ſea*, on the ſouth by the *Indian Ocean*.

Siberia.

This country reaches from n. latitude 50 to 68, and from the long. 105 to 170. It is bounded on the eaſt by the *Japan Ocean*, on the weſt by *Ruſſia*, on the north by the *Frozen ſea*, on the ſouth by *Tartary*. This is a wild, barren, cold country, and thinly inhabited, and under the emperor of Ruſſia.

Tartary.

This was anciently called *Scythia*; it reaches from 40 to 60 deg. of n. latitude, and from 56 to 180 longitude. It is bounded on the north by *Siberia*, on the ſouth by *Perſia* and *China*, on the eaſt by the *Eaſtern Ocean*, on the weſt by *Ruſſia*.

Turkey.

Contained between the n. latitude 30 and 45, and long. 56 and 78. It is bounded on the eaſt by *Perſia*, on the weſt by the *Archipelago*, on the north, by the *Euxine ſea*, on the ſouth, by the *Mediterranean* and *Arabia*. *Natolia* and *Syria* are two provinces, and *Phœnicia*, *Judea*, *Meſopotamia*, *Armenia*, *Aſſyria*, *Chaldea*, *&c.* Fig.

Arabia.

This is contained between the n. lat. 12 and 31, and 55 and 76 of longitude. It is bounded on the eaſt by *Perſia* and the *Perſian Gulph*, on the weſt by the *Red Sea*, on the north by *Turkey*, on the ſouth by the *Indian Ocean*.

Perſia.

This country reaches from n. lat. 26 to 44, and long. 73 and 100. It is bounded on the eaſt by *India*, on the weſt by *Armenia*, on the north by *Ruſſia* and the *Caſpian Sea*, on the ſouth by the *Arabian Sea* and *Perſian Gulph*. This country is under *Kouli Kan*, or the *great Sophy*.

India.

This lies between n. lat. 8 and 40, and long. 92 and 130. It is bounded on the eaſt by *China* and the *Sea*, on the weſt by *Perſia* and the *Indian Sea*, on the north by *Tartary*, on the ſouth by the *Indian Sea*. This is under the *great Mogul*.

China.

It lies from n. lat. 20 to 41, and long. 118 and 140. It is bounded on the north by *Tartary*, on the ſouth and eaſt by the *Sea*, on the weſt by *India* and *Tartary*. The north part of the country is defended by a ſtrong wall 1500 miles long.

Iſlands in Aſia.

Yedſo or *Jeſſo*, and *Japan*, both lying eaſt of *Tartary*; theſe are two large iſlands. The *Phil-*

Fig. *lipine* iſlands, *Sunday iſles*, the *Molucca* iſlands, alſo *Borneo*, *Sumatra*, *Java*, *Ceylon*, in the Indian ſea, with an infinite number of ſmaller ones, as the *Ladrone* iſles, and a cluſter of iſlands lying far eaſt.

Alſo ſeveral parts of countries not yet fully diſcovered, as land of *Compania* near *Yedſo*, *New Guinea*, and *Carpentaria* near the *Molucca* iſlands, and *New Holland* more to the weſt.

3. AFRICA.

Is bounded on the eaſt by the *Red Sea* and the *Indian Ocean*, on the weſt by the *Ethiopic Ocean*, on the north by the *Mediterranean Sea*, and on the ſouth by the *Southern Ocean*.

Barbary.

This country extends from n. lat. 25 to 35, and from long. 4 to 52. It is bounded on the eaſt by *Egypt*, on the weſt by the *Atlantic Ocean*, on the north by the *Mediterranean Sea*, and on the ſouth by *Mount Atlas* and *Numidia*. It comprehends the kingdoms of *Morocco*, *Fez*, *Telenſin*, *Algiers*, *Tunis*, *Tripoli*, and *Barca* or *Cyrene*.

Egypt.

It is extended from n. lat. 21 to 30, and from long. 52 to 63. It is bounded on the eaſt by the *Red Sea*, on the weſt by *Numidia*, on the north by the *Mediterranean Sea*, on the ſouth by *Nubia* and *Abiſinia*.

Numidia or Bildulgerid.

This is comprehended between n. lat. 22 and 33, and long. 2 and 55. It is bounded on the eaſt by *Egypt*, on the weſt by the *Atlantic Ocean*, on the north by *Barbary*, on the ſouth by the deſart of *Zaara*. The eaſtermoſt part joining to Egypt was anciently called *Lybia*.

Zaara, or the Deſart.

It reaches from n. lat. 21 to 28, and from long. 2 to 50. It is bounded on the eaſt by *Egypt* and *Nubia*, on the weſt by the *Atlantic Ocean*, on the north by *Numidia*, on the ſouth by *Negroland*. It contains a part of ancient *Lybia*.

Negroland.

It is contained between the n. lat. 10 and 23, and long. 0 and 42. It is bounded on the eaſt by *Nubia*, on the weſt by the *Atlantic Ocean*, on the north by *Zaara*, on the ſouth by *Guinea*.

Nubia.

This is ſituated between the n. lat. 9 and 23, and longitude 42 and 57. It is bounded on the eaſt by *Ethiopia*, on the weſt by *Zaara* and *Negroland*, on the north by *Egypt* and *Numidia*, on the ſouth by *Ethiopia*, and is reckoned by ſome to be a part of it.

Abiſſinia.

This reaches from n. lat. 2 to 12, and long. 50 to 72. It is bounded on the eaſt by the *Red Sea* and *Indian Ocean*, on the weſt by *Nubia* and the *Deſarts*, on the north by *Egypt*, on the ſouth by *Ethiopia*. The *Nile* has its riſe from a lake in the middle of this country.

Guinea.

This is contained between the n. lat. 4 and 12, and long. 3 and 30. It is bounded on the eaſt by *Ethiopia*, on the weſt by the *Atlantic Ocean*, on the north by the *Great River*, which parts it from *Negroland*, on the ſouth by the *Ethiopic Ocean*. The inhabitants of this country are quite black.

Ethiopia.

This is a vaſt country, reaching from 12 deg. n. lat. to 33 degrees ſ. lat. and from long. 28 to 66. It is bounded on the eaſt, ſouth and weſt by the *Ethiopic Ocean*, on the north by *Nubia* and *Abiſſinia.* It contains a great many provinces and kingdoms. In this country are the *Mountains of the moon*, lying in 10 degrees of ſouth latitude. At the ſouth point of it, is the country of the *Hottentots.*

Iſlands in Africa.

Madeira lying weſt of *Morocco.* The *Canaries*, lying weſt of *Numidia*; they are 7 in number, the principal are *Tenerif*, for its high mountain; and *Ferro*, from which longitude is reckoned, being the weſtermoſt, and about the ſame longitude with *Cape Blanco*, or the weſtermoſt part of the continent. The *Cape de Verd Iſlands*, being 10 in number, lying near *Cape Verd*, in *Negroland. St. Mathes*, *Aſcenſion*, *St. Helena*, in the *Ethiopic Ocean.* And the large iſland *Madagaſcar* lying eaſt of *Ethiopia*, and the *Comore Iſlands* alſo eaſt of *Ethiopia* in the *Indian Sea.* And beſides, many ſmall ones.

4. AMERICA.

Is bounded on the eaſt by the *Atlantic Ocean*, on the weſt by the great *South Sea*, on the north by *lands unknown*, on the ſouth by the *Southern Ocean.*

New Wales.

A country lying on the weſt of *Hudſon*'s *Bay*, between the n. lat. 51 and 64, and long. 278 and 298. It joins to *Canada* on the ſouth, on the north and weſt upon unknown countries.

Labrador.

This country reaches from n. lat. 50 to 63, and long. 297 and 323. It is bounded on the eaſt by the *Atlantic Ocean*, on the north and weſt by *Hudſon's Bay*, on the ſouth by *Canada* and the *River St. Laurence*. This is alſo called *New Britain* and *Eſquimaux*. Fig.

Canada.

It reaches from n. lat. 45 to 53, and long. 287 and 306. It is bounded on the eaſt by the *River St. Laurence*, on the north by *Labrador*, on the weſt by unknown countries. This is alſo called *New France*.

Nova Scotia.

It is contained between n. lat. 44 and 50, and long. 309 and 315. It is bounded on the eaſt by the *Gulf of St. Laurence*, on the north by the *River St. Laurence*, on the weſt by *New England*, on the ſouth by the *Sea*.

New England.

It is contained between the n. lat. 41 and 46, and long. 305 and 311. It is bounded on the eaſt and ſouth by the *Sea*, on the north by *Canada*, on the weſt by *New York*. This contains *Maſſachuſets*, *New Hampſhire*, *Connecticut*, *Rhode Iſland*, and *Providence*.

New York.

It is bounded on the eaſt by the *Sea*, on the weſt by the *Iroquois* or 5 *nations*, on the north by *New England*, on the ſouth by *New Jerſey*.

New Jerſey.

This is bounded on the eaſt by the *Sea*, on the weſt by *Penſilvania*, on the north by *New York*, on the ſouth by the *Sea*.

Penſilvania.

It lies between 39 and 42 deg. of n. lat. and 295 and 303 of long. It is bounded on the eaſt by *De-*

Fig. *laware River*, on the weſt by ſome part of *Canada*, on the ſouth by *Maryland*.

Maryland.

It is bounded on the eaſt by the *Sea*, on the weſt by the *Apaluchian Mountains*, on the north by *Penſilvania*, on the ſouth by *Virginia*.

Virginia.

This is bounded on the eaſt by the *Sea*, on the weſt by *Louiſiana*, on the north by *Maryland*, on the ſouth by *Carolina*.

Carolina.

It lies between 31 and 36 n. lat. and between 292 and 303 of longitude. It is bounded on the eaſt by the *Sea*, on the weſt by *Louiſiana*, on the north by *Virginia*, on the ſouth by part of *Florida* and the *Sea*. On the ſouth part of this country is *Georgia*.

Louiſiana.

It is contained between n. lat. 30 and 42, and long. 278 and 288. It is bounded on the eaſt by the river *Miſſiſippi*, on the weſt by *New Mexico*, on the north by the *Inland countries*, on the ſouth by the *Gulph of Mexico*.

New Mexico or *Granada.*

It reaches from n. lat. 23 to 45, and from long. 200 to 245. It is bounded on the eaſt by *Louiſiana*, on the weſt by the *Sea*, on the north by countries unknown, on the ſouth by the *South Sea*. It includes *California* and *New Albion*.

Old Mexico.

This country reaches from n. lat. 7 to 26; and from long. 258 to 297. It is bounded on the eaſt by the *Gulph of Mexico*, on the weſt and ſouth by Fig. the *South Sea*, on the north by *New Mexico*. It contains a great many provinces. It is alſo called *New Spain*.

Florida.

The end of the peninſula, which lies ſouth of *Florida*; it is contained between the n. lat. 24 and 30, and long 293 and 296. It is all environed with the ſea, except on the north, where it joins upon *Georgia*. All theſe belong to *North America*; the following belong to *South America*.

Terra Firma.

It extends from the equator to 12 deg. n. lat. and from long. 298 to 330. It is bounded on the eaſt by the *Atlantic Ocean*, on the weſt by the *South Sea*, on the north by the *North Sea*, on the ſouth by *Peru* and *Amazonia*. It is divided into many provinces.

Peru.

It extends from 3 degrees ſ. lat. to 25, and from long. 261 to 318. It is bounded on the eaſt by *Amazonia*, on the weſt by the *South Sea*, on the north by *Terra firma*, on the ſouth by *Chili* and *Peragua*. On the eaſt ſide are the *Cordileer mountains*. It alſo conſiſts of ſeveral provinces.

Amazonia.

It extends from the equator to 16 deg. ſ. lat. and from long. 308 to 328. It is bounded on the eaſt by *Braſilia*, on the weſt by *Peru*, on the north by *Terra firma*, on the ſouth by *Peragua*. This country is wholly unknown.

Braſilia.

It extends from ſ. lat. 1 to 33, and from long. 324 to 342. It is bounded on the eaſt by the *Atlantic*

Fig. *lantic Ocean*, on the weſt by *Amazonia* and *Paragua*, on the north by *Terra firma* and the *Sea*, on the ſouth by the *Sea* and the river *Plate*. It conſiſts of many provinces.

Paragua.

This country reaches from ſ. lat. 12 to 37; and from long. 308 to 336. It is bounded on the eaſt by *Braſilia*, on the weſt by *Chili*, on the north by *Amazonia*, on the ſouth by *Patagonia*. It is alſo called *La Plata*. It contains ſeveral provinces.

Chili.

This country lies on the weſt coaſt of *America*. It extends from ſ. lat. 25 to 45, and from long. 295 to 305. It is bounded on the eaſt by *Paragua*, on the weſt by the *South Sea*, on the north by *Peru*, on the ſouth by *Patagonia*.

Patagonia.

This is the ſouthermoſt part of *America*. It extends from ſ. lat. 35 to 54, and long. 295 to 320. It is bounded on the eaſt by the *Atlantic Ocean*, on the weſt by *Chili*, on the north by *Paragua*, on the ſouth by the *Straits of Magellan*. It is alſo called *Terra Magellanica*. This country is very little known.

American Iſlands.

Newfoundland lying in the *Gulph of St. Laurence*, a large iſland, and *Cape Breton* more to the ſouth. *Bermudas* eaſt of *Carolina*. *Bahama iſlands*; *Antilles iſlands*, the greateſt being *Cuba*, *Hiſpaniola*, and *Jamaica*, and *Porto Rico*. The *Lucayas* iſles, the *Charibbe* iſles; all theſe are eaſt of the *Gulph of Mexico*. The *Sotovento* iſles along the n. coaſt of *Terra firma*. Among theſe there is an infinite number of very ſmall iſlands.

Terra del Fuego, on the ſouth end of *America*, a large a large iſland. The *Gallipagos iſlands*, lying weſt under the equinoctial. And the iſles of *Solomon* lying far weſt. There are alſo a great number of iſlands lying weſt from *Mexico*, diſperſed all over in the South Sea; and likewiſe ſeveral up and down in the Atlantic ocean, far from any continent. Fig.

5. REGIONS *about the North pole.* To this head may be referred *Greenland*, *Frieſland*, *Iceland*, *Spitzburg*, and whatever countries are near the north pole, not yet diſcovered.

6. REGIONS *about the South pole.* *Van Demen*'s *land*, *New Zeeland*, *Drake*'s *land*, *Cape Circumciſion*, *Bovet*'s *land*, *Terra auſtralis.* But not one place is diſcovered near the pole.

SCHOLIUM.

Having given a ſhort deſcription of all the known countries of the world; it cannot be amiſs to inſert a table of the latitude and longitude of places, which will be ſerviceable for the readily finding any place we want. The longitude is here reckoned from *Ferro*; and both latitude and longitude is given only in whole degrees; it being preſumed, that this will be near enough for finding out any place upon a globe or map.

A Table

A Table of the latitude and longitude of places.

A.	lat.	lon.
Aalberg, Alberg	57 n	29
Aarhuus	65 n	30
Aback, Weltenburg	49 n	32
Abakanſkoi	53 n	114
Abalak	57 n	84
Abazkaia	50 n	89
Abbeville	50 n	22
Abenrade, Axenrade	55 n	30
Aberdeen	57 n	18
Abo	60 n	43
Abydas	40 n	47
Acapulto	17 n	276
Acerra	41 n	36
Achen	6 n	114
Achyr	50 n	57
Acqs, Aux, Dax,	43 n	21
Acquapendente	45 n	32
Acra	5 n	340
Adon	47 n	29
Adrianople, Andrianople	42 n	46
Aerſchot	51 n	24
Agde	43 n	22
Agen	44 n	19
Agra	26 n	99
Agria, Eger,	48 n	41
Ajazzo	37 n	57
Aichdat	49 n	31
Aigneperes	45 n	22
Aileſbury	52 n	17
Aix	43 n	24
Aix la Chapelle	51 n	25
Alaix, Alais,	44 n	24
Albano	41 n	33
Albany	43 n	334
Albarrazin	40 n	22
Albemarle	50 n	22
Albuquerquy	39 n	27
Albaregalis	47 n	39
Albert	44 n	17
Alby	45 n	20
Alcazar, Alcaraz,	38 n	17
Alcudia	40 n	25
Aldborough	52 n	21
Aldenburg	60 n	32
Alenzon, Alencon,	48 n	19
Aleſſio	42 n	40
Alexandria	40 n	51
Algeri	41 n	28
Algiers	37 n	23
Alhama	37 n	24
Alicant	38 n	17
Allendorf	51 n	30
All ſaints bay	12 ſ	338
Almeria	37 n	4
Almiſſa	44 n	39
Almunecar	37 n	14
Altamont	40 n	38
Altena	54 n	30
Altenburg, Owar,	51 n	33
Altorf	49 n	33
Amadubat	24 n	92
Amadan, Hamadan	35 n	67
Amadia	37 n	63
Amalfa	41 n	35
Amapalla	12 n	285
Amaſia	42 n	56
Ambara, Ambamarjam	13 n	55
Amberg	49 n	32
Ambrun	44 n	26
Amelia	42 n	33
Amiens	50 n	21
Amphipolis	42 n	60
Amſterdam	52 n	24
Anclam	54 n	34
Ancona	43 n	35
Andernach	50 n	26
Andero	43 n	13
Andrea	43 n	35
St. Andrews	56 n	18
Andujar, Anduxar,	38 n	24
Angelo	41 n	36
Angelos	19 n	275
Angerſburg	54 n	43
Angermund	51 n	26
Angers	47 n	18
Angol	38 n	300
Angoleſme	46 n	18
Angoura, Anchira,	41 n	53

Angra

	lat.	lon.		lat.	lon.
Angra	39 n	350	Aſtracan	47 n	72
Anna	33 n	62	Ath, Aeth	51 n	23
Annan	55 n	18	Athens	38 n	44
Anapolis	39 n	300	Attock	53 n	92
Annecy	46 n	26	Avallon	47 n	23
Anſlo, Obſto,	59 n	29	Aubigne	47 n	22
Anſpach	49 n	30	Auch	44 n	40
Antiquera	37 n	13	Aveiro	40 n	29
Antibes	43 n	26	Augſburg, Auſburg	48 n	31
Antioch	36 n	57	St. Auguſtine.	30 n	297
Antivaxi	38 n	44	Auguſtow	53 n	43
Antwerp	51 n	23	Avignon	43 n	24
Apenzel	46 n	29	Avila	41 n	135
Aquilla	42 n	35	Aviles	43 n	14
Aracan	20 n	113	Autengabad	19 ſ	95
Arauco	37 n	300	Aurich	53 n	26
Arbela	35 n	64	Auſburgh	48 n	31
Arbon	48 n	29	Autun	47 n	23
Archangel	64 n	65	Auxerra	47 n	22
Arco	46 n	30	Axim	5 n	16
Ardenburg	51 n	23	Axonne	42 n	24
Ardra	5 n	24	Axuma	15 n	50
Arebon	5 n	25	Ayamonte	37 n	12
Aremburg	50 n	26	Awlen	49 n	30
Arequippa	17 n	305	Axel	51 n	23
Arezzo	43 n	33	Axipoli	45 n	51
Argenton	46 n	21	B.		
Argun	52 n	124	Babilon	32 n	64
Arhuſen	56 n	29	Baca	37 n	16
Arien, Aire,	50 n	21	Baccaſſerai	45 n	55
Arica	18 ſ	308	Bacherak	50 n	27
Aries	43 n	24	Bachu	40 n	69
Armagh	54 n	14	Badajox	39 n	10
Arlon	49 n	25	Baden (Swabia)	49 n	24
Armentiers	50 n	22	Baden (Swiſſerland)	47 n	28
Arnheim	52 n	25	Baeza	38 n	14
Arona	46 n	28	Bagdat	33 n	63
Aronches	39 n	14	Baguagar	16 n	97
Arras, Atrecht	50 n	21	Bagnialuck	44 n	39
Arundel	51 n	18	Bahus	58 n	31
Arzilla	36 n	15	Baja	47 n	40
Aſaph	53 n	17	Balbec	33 n	57
Aſchaffenburg	50 n	29	Balca	37 n	85
Aſcoli	42 n	36	Baliſore	21 n	105
Aſoph, Aſow	47 n	64	Bamberg	50 n	31
Aſti	45 n	28	Bancalis	2 n	119
Aſtorga	42 n	13	Bancock	13 n	121

Bandora

	lat.	lon.		lat.	lon.
Bandora	19n	92	Bertraud	43n	18
Bantam	6n	125	Berwick	56n	18
Baracoa	21n	302	Beſancon	47n	25
Baranco	11n	303	Beſiers	43n	23
Baranwahr	46n	41	Beſtertze, Beſtricia	48n	42
Barberino	44n	32	Bethlehem	31n	56
Barcelona	40n	20	Bethune	50n	22
Barcelonetta	44n	26	Betlis	37n	65
Barcelor	15n	94	Bibrach, Biberach	48n	29
Bardt	55n	34	Biel	47n	27
Bari	41n	34	Bielogrod	47n	54
Barleduc	48n	25	Bilboa	43n	14
Barletta	41n	38	Bimlipatan	18n	103
Baſartſchick	42n	44	Bir	35n	80
Baſil, Baſſe	48n	27	Birkenfield	50n	26
Baſſaim	19n	92	Biſchweller	49n	27
Baſtia	42n	30	Biſignano	40n	37
Baſtia	38n	43	Biſuagar	14n	98
Batavia	6ſ	125	Bitonli	41n	38
Bautzen, Pautzem	51n	34	Blankenburg	52n	31
Bayonne, Bajonne	43n	17	Blavet	47n	14
Beaufort	45n	26	Blois	48n	20
Beauvais	49n	21	Blouits	52n	41
Beja	38n	29	Bochara	40n	85
Beichlingen	51n	31	Bodon	45n	46
Belcaſtro	39n	37	Boglio	44n	27
Beleiſora	59n	61	Bogoto	4n	304
Belgrade	45n	42	Boiſleduc	52n	24
Bellents	46n	29	Bologna, Bologne	44n	31
Beller	46n	25	Bon	50n	27
Belluno	46n	33	Bona	36n	28
Beltz	50n	45	Bonifacio	41n	29
Belvedere	37n	42	Bonneville	46n	26
Bencoolen	4ſ	121	Boppart	50n	27
Benevento	41n	37	Borja	42n	16
Benguela	11ſ	34	Boſſa	39n	30
Benin	7n	25	Boſſora	30n	67
Benſheim	43n	29	Boſton	42n	306
Bentheim	52n	27	Bouillon, Buillon	46n	25
Bentivoglio	44n	32	Bovino	41n	36
Berenice, Fuez	30n	55	Boulogne	51n	21
Bergamo	45n	29	Bourdeaux	45n	18
Bergen	60n	26	Bourg	46n	25
Bergen op Zoom	51n	24	Bourges	47n	21
Bergerac	45n	19	Boxthude	54n	29
Berlin	52n	34	Bracciano	42n	34
Bern	47n	27	Braga	42n	8

Bra-

	lat.	lon.		lat.	lon.
Braganza	42 n	12	Calahorra	42 n	16
Brandenburg	52 n	33	Calais	51 n	21
Braſlan	56 n	47	Calcedon	41 n	59
Breda	52 n	24	Calenburg	52 n	30
Bremen	53 n	28	Calicut	11 n	95
Breſſcia	45 n	31	Calmar	57 n	36
Breſlaw	51 n	37	Cambaia	12 n	124
Breſt	48 n	14	Cambra	51 n	22
Breſte, Breſſini	52 n	45	Cambridge	52 n	19
Bridgetown	13 n	319	Campagna	41 n	35
Brieg	51 n	37	Campeachy	19 n	285
Bril	52 n	24	Candahor	35 n	87
Brin	49 n	36	Candia	35 n	45
Briſac	48 n	27	Candy	8 n	99
Briſtol	51 n	16	Canterbury	51 n	20
Brixen	47 n	31	Canton	23 n	132
Bruges, Brugge	51 n	22	Cape of good hope	34 ſ	40
Brunſwick	52 n	30	Cape blanco	21 n	0
Bruſſels	51 n	24	Cape verd	15 n	0
Bucaw	48 n	29	Capona	41 n	36
Buchorn	47 n	29	Cardonna	41 n	21
Buckereſt	44 n	47	Carelſcroon	56 n	35
Buckingham	52 n	17	Careſen, Laſſeen	16 n	72
Buda	48 n	40	Cargapol	63 n	56
Budoa	42 n	41	Carlſtadt	45 n	36
Budweis	49 n	35	Carmona	37 n	13
Buenos Ayres	36 ſ	318	Carpentras	44 n	25
Bugia	36 n	24	Cars	42 n	64
Bugie	22 n	56	Carthage	36 n	29
Bulac	30 n	52	Carthagena	37 n	17
Bulgar	54 n	71	Carthagena	10 n	301
Burgos	42 n	13	Carwar	15 n	93
Burgow	48 n	30	Caſal, Cazal	45 n	28
Buric	52 n	26	Caſbin	36 n	68
Byghof	53 n	50	Caſhan	34 n	70
C.			Caſſel	51 n	29
Cabo	45 n	35	Caſſimere	35 n	95
Cabul	33 n	89	Caſtanowitz	46 n	37
Cachao	23 n	125	Caſtel novo	43 n	41
Cadiz, Cales	36 n	10	Caſtello branco	39 n	28
Caen	49 n	18	Caſtiglione	45 n	31
Caffa	45 n	57	Caſtle town	54 n	14
Cagliari	39 n	30	Caſtro	43 n	296
Cahors	44 n	20	Catanea	38 n	35
Cajana	64 n	47	Cataro	42 n	39
Caifum	35 n	123	Catherlouge	53 n	12
Cairo grand	30 n	52	Caudebec	49 n	19

	lat.	lon.
Caxamarca	7 ſ	303
Cayenne	5 n	325
Cephalonia	38 n	41
Cervia	44 n	33
Ceſena	44 n	32
Ceuta	36 n	36
Chalons	48 n	24
Cham	49 n	25
Chamberry	45 n	25
Charlemont	54 n	14
Charleroy	50 n	24
Charles town, Charlton	32 n	299
Charolleis	46 n	24
Chartres	47 n	20
Chateau	50 n	23
Chatelet	50 n	24
Chatham	51 n	20
Chaves	42 n	13
Chelm	51 n	43
Chelmsford	52 n	19
Cheraſco	45 n	27
Cherburg	50 n	18
Chereſoul	36 n	65
Cheſter	53 n	16
Cheſter (new)	40 n	304
Chiapa	16 n	280
Chiarenza	38 n	41
Chicheſter	60 n	18
Chinca	13 ſ	302
Chiney	50 n	25
Chios, Scio, Kio	38 n	47
Chitor (India)	23 n	96
Chitor (Italy)	45 n	27
Chitro	40 n	43
Chinſi	43 n	33
Chriſtiana	60 n	30
Chriſtianople	27 n	35
Chriſtianſtadt	56 n	34
Cifalu	38 n	33
Cinoloa	26 n	265
Cittadella	40 n	23
Cividad real	39 n	14
Civita vechia	42 n	33
Clagenfurt	47 n	34
Clavenna	46 n	39
Clauſenburg	47 n	44

	lat.	lon.
Clermont, Champagn	49 n	25
Clermont, Avergne	46 n	22
Cleve	52 n	25
Coblentz	50 n	27
Coblon	13 n	100
Cobourg	50 n	31
Cochin	9 n	96
Cogniac	46 n	49
Coire	47 n	29
Coimbra	40 n	99
Colberg	54 n	36
Colima	19 n	269
Colmar	48 n	27
Cologne	51 n	26
Com	34 n	69
Comerci	48 n	25
Comorra	48 n	38
Compeigne	49 n	22
Compoſtella	43 n	8
Conception	10 n	295
Condom	44 n	18
Coni	44 n	27
Conquet	48 n	13
Conſenza, Coſenza	39 n	37
Conſtance	48 n	29
Conſtantina	36 n	27
Conſtantinople	42 n	48
Conteſſa	41 n	44
Copenhagen	56 n	32
Copiapo	25 ſ	303
Coquimbo	39 ſ	302
Cordoua	38 n	12
Corfu	40 n	40
Coria	40 n	26
Corinth	37 n	43
Cork	52 n	10
Coron	37 n	43
Cortona	43 n	23
Corunna	43 n	29
Coventry	52 n	17
Cowes	51 n	16
Cracow	50 n	40
Crema	45 n	30
Cremnitz	49 n	39
Cremona	45 n	39
Creſcentino	45 n	27
Creutznach	50 n	27

	lat.	lon.
Croia	42 n	42
Cronſtat	47 n	47
Croſſen	52 n	35
Cuenca, Cuenza	40 n	16
Culiacan	24 n	264
Cullembach	50 n	31
Culm	53 n	39
Cuſcó	13 ſ	308
Cuſtrin	52 n	35
Czernihow	52 n	55
D.		
Dabal	18 n	93
Daca	24 n	119
Dagno	41 n	41
Dalaborc	59 n	38
Damaſcus	33 n	57
Dambea	15 n	54
Damietta	31 n	52
Damvillars	49 n	25
Dancala	17 n	53
Daneburg	53 n	31
Dantzig	54 n	39
Darmſtadt	49 n	28
St. Davids	12 n	101
Debrecin, Dubitza	47 n	42
Delft	52 n	23
Delly	28 n	99
Delmonhurſt	53 n	28
Dendermond	51 n	24
Denia	39 n	18
Derbent	42 n	72
Derpt	58 n	49
Deſſaw	52 n	33
Deventer	52 n	25
Deux ponts	49 u	24
Diarbeck	47 n	62
Die	45 n	24
Dieppe	50 n	20
Diepholt	53 n	28
Digne	44 n	26
Dijon	47 n	24
Dillenburg	51 n	28
Dillengen	48 n	30
Dinkenſpoil	49 n	30
Diul	26 n	89
Dixmude.	51 n	23
Doblin, Dobetin	56 n	45

	lat.	lon.
Dol	48 n	16
Dolk, Dole,	47 n	25
Doltabad	20 n	95
St. Domingo	18 n	309
Donawert	49 n	30
Dort	51 n	24
Dovay	50 n	23
Dreſden	51 n	33
Dreux	49 n	20
Drontheim	64 n	30
Dublin	54 n	12
Duiſburg	51 n	26
Dumfries	55 n	15
Dunbarton	56 n	14
Dundee	56 n	15
Dunkirk	51 n	21
Durazzo, Brazzi,	42 n	43
Durham	55 n	15
Duſſeldorp	51 n	26
E.		
Ecija	37 n	12
Edam	52 n	25
Edinburg	56 n	16
Egra	50 n	37
Ehenheim	48 n	27
Ekrenford	55 n	29
Elbaſſen	41 n	43
Elbin	54 n	39
Elcatif	25 n	59
Elſimburg	56 n	33
Elſinore, Halſingore,	56 n	32
Elvas	39 n	10
Ely	52 n	18
Emboli	42 n	46
Emmeric	52 n	25
Emden	53 n	27
Euchuiſen	52 n	24
Engers	50 n	27
Ens	48 n	34
Enſiſheim	48 n	27
Epheſus	38 n	47
Eppingen	40 n	29
Erfurt	51 n	31
Erivan	40 n	45
Erkelens	50 n	25
Erpach	49 n	29
Erzerum	40 n	80

	lat.	lon
Eſlingen, Elſing,	49 n	29
Eſna	25 n	55
Eſſen	51 n	26
Eſtella	43 n	15
Eſtremots	38 n	10
Eu	50 n	20
Evora	39 n	9
Exeter	51 n	15
Eyndhoven	51 n	25
Eyſenach	51 n	30
F.		
Faenza	44 n	32
Falkenburg	54 n	36
Falmouth	50 n	14
Famaguſta	35 n	56
Fano	44 n	34
Farewel cape	60 n	334
Faro	36 n	11
Feſanta	36 n	269
Feſanta de Bagota	4 n	305
Ferden, Verden	53 n	29
Ferentino	42 n	34
Fermo	43 n	36
Ferrara	44 n	33
Ferrol	43 n	12
Felipour	27 n	98
Fez	33 n	15
Finale	44 n	29
Flenſburg	55 n	29
Florence	43 n	32
Flour	45 n	23
Fluſhing, Uliſſingen,	51 n	23
Fogaras	47 n	48
Foix	43 n	21
Foken	27 n	139
Fondi	41 n	36
Fontainbleau	48 n	22
Fonterabia	43 n	16
Fontenoi	51 n	23
Forbi	44 n	33
St. Foy	44 n	18
Fraga	41 n	18
Francfort (Saxony)	52 n	34
Francfort (Rhine)	50 n	27
Frankendal	49 n	28
Fraſcati	41 n	34
Frederica	31 n	297

	lat.	lon.
Frederickſtadt	56 n	32
Fredericſode	56 n	31
Freiburg	48 n	27
Freiſengen	49 n	32
Friburg	46 n	37
Fridburg	48 n	31
Friedburg	50 n	28
Friſach	47 n	35
Fritzler	51 n	30
Fuld	51 n	29
Fuligno	42 n	34
Furnes	51 n	22
Furſtenfelt	47 n	37
G.		
Gaeta	41 n	35
Gallipoli	41 n	47
Gandia	39 n	17
Gangea	41 n	67
Gani	16 n	102
Gap	44 n	26
Gaveren	51 n	23
Geertrudenburg	52 n	24
Geldres	52 n	26
Galenhauſen	50 n	29
Gemund	49 n	30
Geneva	46 n	26
Gengenbach	48 n	27
Genoa, Genoua,	44 n	29
Germenſheim	49 n	28
Gerumenha	39 n	29
Gepping	48 n	30
Gevalia	60 n	38
Ghent, Gant,	51 n	23
Giawle	60 n	29
Gibraltar	36 n	11
Gieſſen	51 n	29
Gingen	49 n	30
Ginge	12 n	99
Giovenazzo	41 n	37
Girace	39 n	36
Girge	26 n	52
Girona	42 n	22
Giſors	49 n	20
Giula	47 n	43
Giuſdendil	43 n	45
Glandeves	43 n	26
Glaſcow	56 n	14

Glatz

	lat.	lon.		lat.	lon.
Glatz	50 n	37	Guarda	40 n	12
Glouceſter	52 n	16	Guatimal	15 n	280
Glogaw	51 n	37	Guaxaca	18 n	277
Gluckſtadt	54 n	29	Guben	52 n	35
Gueſne, Guiſen	52 n	38	Gueret	47 n	21
Goa	16 n	94	Guiaquil	2 ſ	297
Goes	51 n	24	Guiara	11 n	312
Golconda	17 n	98	Gurk	47 n	35
Golnau	54 n	36	Guſtrow	54 n	33
Gombron	28 n	76	H.		
Gor	32 n	105	Hadderſleve	56 n	29
Gorcum	52 n	24	Hadramut	16 n	70
Goree	15 n	28	Haerlem	52 n	24
Goritia	46 n	34	Hagenan	48 n	27
Gorlitz	51 n	35	Hague	52 n	24
Goſlar	52 n	30	Hailbrun	49 n	29
Gotha	51 n	31	Halabas	27 n	102
Gottenburg	58 n	32	Halberſtat	52 n	31
Gottingen	52 n	30	Hall	49 n	30
Gottorp	55 n	30	Halmſtadt	57 n	13
Gowde	52 n	24	Ham	52 n	27
Grace	43 n	27	Hamadan	35 n	68
Gradiſca	45 n	39	Hamburg	54 n	30
Gran	48 n	39	Hamchen	30 n	140
Granada (Spain)	37 n	14	Hamelin	52 n	29
Granada (America)	11 n	289	Hanan	50 n	29
Grandents	53 n	40	Hanover	53 n	29
Gratz	47 n	37	Hapſal	59 n	45
Grave	52 n	26	Harburg	54 n	30
Gravelin	51 n	22	Harderwick	52 n	16
Gravenec	48 n	29	Havanna	23 n	297
Gravina	41 n	38	Havre de grace	49 n	37
Grenoble	45 n	25	Hean	22 n	126
Grimborb	50 n	26	Heidelburg	50 n	29
Gripſwald	54 n	34	Helmſtadt, Halmſtat	53 n	31
Grodno	54 n	45	Helvoetſluys	52 n	24
Groengen	53 n	26	Heraclea	41 n	48
Groll	52 n	26	Herat	34 n	80
Groſetto	42 n	32	Herborg	51 n	28
Grotſka	45 n	40	Hereford	51 n	52
Grupenhagen	51 n	30	Herentals	51 n	24
Guadaiajara	21 n	270	Hermanſtadt	47 n	46
Guadalaxara	40 n	14	Heſilin, Heſdin,	50 n	22
Guadix, Acci,	37 n	15	Hilderſheim	52 n	30
Gualeor	26 n	99	Hindown	27 n	98
Guamanca	12 n	305	Hirchfield	51 n	30

	lat.	lon.
Hochſtet	49 n	30
Hoenzotlern	48 n	29
Homburg	50 n	27
Honfleur	49 n	19
Horn	53 n	24
Hornburg	48 n	29
Hove	53 n	29
St. Hubert	50 n	25
Hegly	23 n	108
Hueſca	42 n	17
Hulſt	51 n	24
Hunninghen	47 n	28
Huſum	25 n	29
Huy	51 n	26
I.		
Jaca	43 n	17
Jaffa, Joppa	33 n	56
Jaen	38 n	14
Jagendorf	50 n	38
St. Jago	20 n	302
St. Jago	34 ſ	301
Jagodna	44 n	45
Jaitzo	45 n	39
Jamba	31 n	101
Jamby	2 n	97
Jamets	49 n	25
Janna	39 n	42
Japara	6 ſ	130
Jariſlaw	58 n	63
Jaſques	25 n	78
Jaſſy	47 n	49
Jawer	51 n	36
Jazy	42 n	50
St. Jean de Maurienne	45 n	26
Jeddo	36 n	161
Jena	51 n	32
Jenkopin	58 n	34
Jeruſalem	32 n	52
Jeſſelmery	27 n	94
Jever	54 n	27
Iglaw	49 n	35
Ihor	3 n	124
Illock	46 n	40
Imola	45 n	32
Imperial	40 ſ	294
Ingolſtadt	48 n	31
Inſpruck	47 n	32

	lat.	lon.
Joinville	49 n	26
Iſenach, Eyſenach	51 n	31
Iſernia	42 n	36
Iſney	48 n	30
Iſpahan	33 n	70
Judenburg	47 n	35
Judoign	50 n	25
St. Julian	49 ſ	313
Juliers	51 n	26
Jurea	45 n	28
K.		
Kaffa	47 n	61
Kakenhauſen	57 n	47
Kaliſh	52 n	38
Kallo	48 n	42
Kaminec	48 n	47
Kaniſca, Caniſa	47 n	38
Kargapol	62 n	60
Kauſburen	48 n	31
Keiſerberg	48 n	27
Keiſerſwert	51 n	28
Kempten	47 n	30
Kerman	30 n	76
Kexholm	61 n	53
Kiel	55 n	31
Kilia	47 n	54
Kimi	66 n	44
Kingſton (Jamaica)	18 n	302
Kiow	51 n	52
Kiattan	46 n	33
Knyſſen	53 n	45
Kola	69 n	54
Komorra	49 n	39
Kongel	58 n	32
Koningſburg, Regimont,	54 n	42
Koningſgratz	50 n	36
Kowno	55 n	45
Krainſlaw	51 n	43
Krempen	54 n	29
Krems	48 n	36
Krim	48 n	61
Kufstain	48 n	32
Kuttenburg	50 n	35
L.		
Labiau	55 n	42
Ladenburg	50 n	29
Ladogna	41 n	36

Lagos

	lat.	lon
Lagos	37 n	8
Laholm	57 n	32
Lahor	34 n	95
Lambego	41 n	9
Lambeſco	43 n	24
Lampſacus	40 n	48
Lancaſter	54 n	15
Lanchang	21 n	121
Landaf	51 n	15
Landan	50 n	28
Landaw	49 n	28
Landen	51 n	24
Landrecy	50 n	23
Landſcroon	56 n	34
Landſhut	48 n	32
Landſcroon	56 n	33
Landſperg (Brand)	53 n	36
Landſperg (Bavaria)	48 n	31
Langeac	45 n	23
Langrez	48 n	25
Laon	50 n	23
Lar	28 n	74
Laredo	44 n	14
Lariſſa	39 n	45
Larta	39 n	42
Lavamund	47 n	35
Laubach	46 n	35
Lauda	49 n	31
Lauffenburg	47 n	28
Laugingen	48 n	30
Lauſanne	46 n	26
Lawenburg	54 n	31
Laxemburg	48 n	37
Lebus	52 n	35
Leccie	40 n	40
Lectour	44 n	19
Leerdam	52 n	25
Leghorn	43 n	31
Leipſick	51 n	33
Leith	56 n	16
Lemburg, Leopolis	49 n	44
Leucicia	52 n	39
St. Leo	44 n	34
Leon (Spain)	43 n	12
Leon (America)	12 n	286
Leontini	37 n	36
Leopolſtadt	49 n	39
Lepanto	38 n	41

	lat.	lon.
Leria, Reiria	40 n	29
Lerida	41 n	19
Leſcar	43 n	18
Leſſines	50 n	23
Lettere	40 n	35
Leucate	43 n	22
Leutkirk	47 n	30
Leutmeritz, Letomeritz	50 n	34
Lewarden	53 n	25
Lewiſburg	45 n	318
Leyden	52 n	24
Lezina	41 n	36
Libaw	56 n	42
Lichtenberg	50 n	32
Lida	53 n	47
Lidcopin	59 n	33
Liege, Luyck	51 n	25
Liere	51 n	24
Lignitz	51 n	36
Lima	12 ſ	303
Limburg	51 n	26
Limeric	52 n	10
Limoges	46 n	20
Limpurg	50 n	28
Lincoln	53 n	18
Lincopen	58 n	36
Lindow, Lindaw	47 n	30
Lingen	53 n	28
Lintz	48 n	34
Lippe	52 n	28
Liſbon	39 n	9
Liſieu x Liſeux	49 n	19
Liſle, Lille	51 n	22
Liſſa	52 n	36
Lita	41 n	47
Litchfield	53 n	16
Livadia	38 n	42
Loango	3 ſ	31
Lochem	52 n	26
Loches	47 n	21
Lodeve	43 n	23
Lodi	45 n	30
Logronto	42 n	16
Lojowogrod	42 n	15
Lombes	43 n	19
London	51 n	18
Londonderry	55 n	11

	lat.	lon.
Longueville	50 n	19
Longwy, Longovy	50 n	25
Loo	52 n	26
Lorca	38 n	16
Loredo	45 n	33
Loretto, Lauritana	43 n	35
Loudun	47 n	26
Lovitz	52 n	40
Loyrebrander	25 n	88
Lourde	43 n	18
Louvain, Lovain	51 n	24
Louvo	15 n	121
Loxa	5 ſ	301
Loya, Loja, Loxai	37 n	14
Lubec	54 n	31
Luben (Bohemia)	51 n	36
Luben (Saxony)	32 n	34
Lublin	51 n	43
Lubow, Lublaw	49 n	41
Luc	45 n	25
Lucca, Luca	44 n	31
Lucern	44 n	27
Lucinganno	43 n	33
Lugano	46 n	28
Lugo	43 n	9
Lunden	56 n	34
Lunenburg	54 n	30
Luneville	49 n	26
Lure	48 n	36
Luſſon	46 n	17
Luſne, Lucko	51 n	45
Lutzen	51 n	32
Luxemburg	50 n	26
Luzara, Luzzara	45 n	31
Lyons	40 n	24
M.		
Macerata	43 n	35
Mackeran	26 n	66
Madenburg, Manheim	49 n	28
Madre de popa	11 n	302
Madrid	40 n	14
Madrigal	1 n	303
Maeſeyck	51 n	25
Maeſtricht	51 n	25
Magadoxa	2 n	61
Magdeburg	52 n	32
Magliano	49 n	34

	lat.	lon.
Majorca	39 n	20
Malaga	37 n	13
Malda	25 n	108
Malmoe	55 n	33
St. Malo	48 n	16
Malta, Medina	35 n	34
Malvaſia	36 n	49
Manreſa	41 n	20
Mans	48 n	18
Mansfield	52 n	31
Mante	49 n	21
Mantua, Mantoua	45 n	31
Maracaibo	11 n	308
Marana	41 n	46
Marans	46 n	17
St. Marco	39 n	39
Mardike	51 n	22
Margentheim	49 n	32
Margoreſt	47 n	49
Marienburg (Germ.)	50 n	24
Marienburg (Poland)	54 n	40
Marienburg, Sweden	59 n	34
Marmande	44 n	23
Marognia	42 n	51
Marpurg	51 n	29
Marſala	37 n	34
Marſalquieur	36 n	18
Marſeils	43 n	25
Marſico	40 n	38
Martaban	17 n	99
Martha	12 n	303
Martocano	39 n	38
Maſcon, Mazon	46 n	24
Maſſa	44 n	31
Maſſerano	45 n	28
Maſulipatan	16 n	101
Mætera	41 n	38
Materan	7 ſ	130
Maubeuge	50 n	23
Maulcon	43 n	17
St. Maupa	38 n	44
Mayenne	48 n	18
Mazagan	33 n	8
Mazara	35 n	34
Meaco	36 n	156
Meaux	49 n	23
Mecca	21 n	64
Mechlin	51 n	24

	lat.	lon.
Medenblic	53 n	25
Medina celi	41 n	15
Medina talmari	25 n	61
Medina ſidonia	36 n	11
Mednick	56 n	44
Meiſſen	51 n	33
Meldorp	54 n	29
Meliapour	13 n	100
Melinda	3 ſ	39
Melito	28 n	38
Melle	52 n	28
Melros, Meurs	56 n	16
Melum, Melun	48 n	22
Memel	56 n	43
Memmingen	48 n	30
Menchou	49 n	24
Menen, Menin	51 n	23
Mentz	49 n	28
Meppen	53 n	27
Mequinez	34 n	8
Meran, Moran	47 n	31
Mergentheim	49 n	32
Merida	39 n	11
Mero	17 n	116
Merſburg	51 n	132
Merſperg, Merſpurg	47 n	29
Meſched	36 n	77
Meſeen	66 n	66
Meſember	44 n	53
Meſſina	38 n	37
Metling	46 n	36
Metz	49 n	26
Meurs	51 n	26
Middleburg	51 n	23
Milan	45 n	29
Milland	44 n	22
Mindelheim	48 n	30
Minden	52 n	28
Minſk, Minſki	54 n	49
Miranda de douro	42 n	11
Miranda de ebro	43 n	14
Mirandola	45 n	32
Mirepoix	43 n	20
Mittau	56 n	44
Moco, Mocho	13 n	65
Modena	45 n	31
Modica	36 n	36
Modon	35 n	46

	lat.	lon.
Modruſch, Madruſch	45 n	37
Mohilow, Mogilof	54 n	54
Moſette	41 n	39
Mola	41 n	37
Molina	41 n	16
Moliſe	42 n	36
Molſheim	48 n	21
Monaco	44 n	27
Moncaller, Moncale	44 n	27
Moncon	42 n	24
Mondonnedo	43 n	10
Mondovi	44 n	27
Monluſſon	46 n	22
Monopoli	41 n	38
Mons, Bergen	50 n	23
Montallo	43 n	35
Montaugis	48 u	20
Montauban	44 n	19
Montbelliard	47 n	26
Monthriſon	45 n	23
Monte verde	41 n	36
Montpelier, Mompelier	43 n	24
Montreal (Sicily)	38 n	35
Montreal (Spain)	43 n	16
Morlaix	48 n	14
Morocco	32 n	9
Mortaigne	49 n	19
Mortara, Montara	45 n	29
Moſambique	15 ſ	42
Moſbach, Moſpach	49 n	28
Moſcow	55 n	65
Moſpurb	48 n	32
Moulins	46 n	23
Mouſul, Moſul	36 n	43
Mucyſlaw	54 n	53
Mulhauſen	48 n	27
Multon	30 n	92
Mungats, Munkats	49 n	42
Munick, Munchen	48 n	32
Munſter	52 n	27
Munſterburg	51 n	36
Murcia	38 n	17
Muro	41 n	37
Muſtagar	36 n	26
Muxara, Muxacra	37 n	16
Muyden	52 n	25

N.	lat.	lon.
Naerden	52 n	24
Najara	42 n	15
Nacſivan	39 n	65
Namur	50 n	25
Nancy	49 n	26
Nangaſaqui	32 n	151
Nankin	32 n	129
Nants, Condivineum	47 n	17
Naples	41 n	38
Napoli	36 n	49
Narbonne	43 n	22
Nardo	41 n	39
Narenza, Narenta	43 n	39
Narni	42 n	34
Narſinga	15 n	98
Narva	59 n	48
Naſſaw	50 n	27
Nata	9 n	296
Navereins	43 n	17
Naverino	35 n	46
Naumburg	51 n	32
Negapatun	11 n	99
Negrais	17 n	112
Negropont	38 n	48
Nemours	48 n	21
Nepi	42 n	34
Neuenburg, Nuberg	48 n	31
Nevers	41 n	22
Neufchattel	47 n	26
Neuſtadt	47 n	33
Newburg	49 n	31
Newcaſtle	55 n	16
Newmark	47 n	45
Newport	51 n	21
Nice, Nizza	43 n	27
Nicomedia	41 n	51
Nicopin	59 n	37
Nicopolis, Nigepoli	45 n	47
Nimeguen, Nimmegen	52 n	25
Ninove	57 n	23
Niort	46 n	18
Niſmes, Nimes	43 n	24
Niſna	56 n	65
Niſſa	43 n	43
Nitracht, Neytra	49 n	39
Niville	51 n	24

	lat.	lon
Nixabour, Naſabor	36 n	38
Nocera	43 n	34
Nola, Nole	41 n	36
Nombre de dios	10 n	300
Nona	45 n	37
Norcia, Nurſia	43 n	35
Norden	54 n	26
Norkepin	58 n	36
Northeim	51 n	30
Northanſen	52 n	30
Norwich	53 n	20
Noto	36 n	37
Novara	45 n	28
Novigrad	45 n	45
Novogrod, Weliki	58 n	55
Noyon	49 n	22
Nurenburg	49 n	31
Nuys	51 n	26
Nyburg	55 n	30
Nyenburg	53 n	29
Nyſlot	63 n	50
O.		
Oberweſel	50 n	27
Ochrida	41 n	43
Ochenfurt	49 n	31
Oczacow	47 n	55
Odenſee	56 n	30
Odor	50 n	38
Odenburg (Weſtph.)	53 n	27
Oldenburg (Denmark)	54 n	31
Oleron	43 n	17
Olika	51 n	47
Olinda	8 ſ	343
Olite	42 n	16
Olivenza	38 n	10
Olmutz	49 n	37
Olympea, Longinico	38 n	41
St. Omers	51 n	22
Ommenburg	51 n	28
Oneglia	44 n	28
Oppelen	50 n	38
Oppido	38 n	38
Oran	37 n	18
Orange	44 n	24
Oratavia	28 n	1
Orbitello	42 n	32

	lat.	lon
Ordunna	43 n	14
Orenſe	42 n	9
Origuella	38 n	16
Oriſtagni	39 n	29
Orixa	20 n	105
Orleans	48 n	20
Orleans new	30 n	288
Oropeza	20 ſ	312
Orta, Horta	42 n	34
Orvietto	43 n	33
Oſnabrug	52 n	27
Oſtend	51 n	22
Oſtuni	41 n	40
Oſwego	44 n	300
Otranto	40 n	40
Oudenard	51 n	23
Oviedo	43 n	12
Oxford	52 n	17
P.		
Padang	1 ſ	119
Paderborn	52 n	28
Padua, Padoua	45 n	32
Paita	5 n	298
Palamboang	8 ſ	134
Palamos	41 n	21
Palanca	48 n	40
Palazzuoli	30 n	36
Palencia	42 n	13
Palermo	37 n	35
Paleſtrina, Præneſte,	42 n	34
Palma	38 n	8
Palmyra	33 n	59
Pamiers	43 n	20
Pampeluna	43 n	16
Panama	10 n	296
Panay	11 n	129
Pancale	44 n	27
Papa	47 n	37
Papenheim	49 n	32
Paraiba	7 n	323
Parenzo	45 n	35
Paris	49 n	22
Parma	44 n	31
Parnaw	58 n	46
Paſſan	48 n	34
Paſto	2 n	301
Patan	27 n	109
Patchuca, Patioque,	21 n	275

	lat.	lon
Patna	26 n	105
Patras	38 n	47
Patti	38 n	36
Pan	43 n	18
Pavia	45 n	29
Paul, Pol de leon	49 n	15
St. Paul	24 n	328
Paz	18 n	312
Pazzi	41 n	52
Pedir	5 n	114
Pegu	17 n	117
Pekin	40 n	121
Pella	41 n	43
Peluſium	51 n	51
Peniche	39 n	9
Pennon de velez	35 n	14
Percaſlaw	50 n	53
Perga	39 n	41
Pergamus	39 n	47
Perigux	45 n	19
Permavelec	62 n	81
Peronne	50 n	24
Perpignan	43 n	22
Perſapolis	31 n	74
Perthamboy	41 n	300
Perugia	43 n	34
Peſaro	44 n	34
Peſcara	42 n	36
Peſt	48 n	40
Peterborough	52 n	18
Peterſburg	60 n	56
Peterwaradin	46 n	42
Petricow	51 n	40
Pettaw, Petaw	47 n	37
Pettipoly	17 n	100
Pfortſheim	49 n	28
Pfulendorf	46 n	28
Phaliſburg	49 n	27
Pharnacia	41 n	58
Pharſalus, Pharſa,	39 n	43
Philadelphia, (Aſia)	38 n	49
Philadelphia (America)	41 n	304
Philippi	41 n	45
Philippopoli	42 n	45
Philipſburg	49 n	28
Placenza	45 n	30
Pienza	43 n	33

	lat.	lon.
Pignerol	44 n	27
Pilau	55 n	41
Pilſen	50 n	33
Pilſno, Pilzow	50 n	40
Pinſk	52 n	47
Piombino	42 n	32
Pipley	21 n	106
Piſa	43 n	31
Piſca	14 ſ	302
Piſcataway	44 n	308
Placentia	43 n	15
Plata	23 ſ	311
Platea	38 n	46
Plawen	50 n	32
Pleſcow	57 n	50
Pluviers	48 n	21
Poggio imperial	43 n	32
Poictiers	46 n	19
Pola	45 n	35
Policaſtro	40 n	37
Polignano	41 n	39
Polockz	55 n	51
Pondicherry	12 n	100
Pont de eſprit	44 n	24
Ponta mouſon	49 n	26
Popa madre	10 n	301
Popayan	3 n	302
Porentru	47 n	27
Portalegre	39 n	10
Portrentru	47 n	27
Port l'orient	48 n	16
Porto	41 n	9
Porto bello	9 n	296
Porto hercole	42 n	32
Port St. Mary	37 n	11
Porto rico	18 u	313
Port royal	17 n	301
Poſega	45 n	39
Poſna	52 n	37
Potenza	40 n	37
Potoſi	22 ſ	311
Pourſelui	28 n	120
Prabat	16 n	121
Prague	50 n	34
Precopia, Orciup	44 n	44
Premiſlaw	49 n	42
Preſburg	48 n	38

	lat.	lon.
Preveſa	38 n	44
Priaman	1 ſ	118
Priſſeran	43 n	42
Priſtina	43 n	41
Procupia	44 n	41
Prom	19 n	114
Ptolomais	32 n	56
Puy envelay	45 n	24
Puzzuolo, Pozzuolo	41 n	36
Q.		
Quakenburg	52 n	27
Quebeck	47 n	304
Queda, Keda	7 n	118
Quedlinburn	52 n	31
Queſlin, Queviling	26 n	129
Queinoy	50 n	23
Queyang	27 n	126
Quillebeuf	49 n	23
Quiloa	10 ſ	59
Quimper	48 n	14
Quimperlay	47 n	15
Quinque eccleſiæ	46 n	40
St. Quintin	50 n	23
Quito	13 ſ	300
R.		
Raab	48 n	38
Rachelſburg	47 n	37
Raguſa	43 n	40
Rajamahal	24 n	106
Rajapour	22 n	97
Rain	49 n	31
Rakaw	36 n	59
Rakonic	50 n	33
Rambervillers	48 n	26
Ramillies	51 n	25
Ranchira	12 n	306
Ranetz	55 n	43
Rantzow	54 n	30
Raorconda	17 n	99
Raperſwill	47 n	28
Razeburg	60 n	44
Ratipor	50 n	38
Ratiſbon	49 n	32
Ratzeburg	54 n	31
Rava, Rawa	52 n	40
Ravello	41 n	36
Ravenna	44 n	33

	lat.	lon.
Ravenſberg (Weſtp.)	52 n	27
Ravenſberg (Swab.)	47 n	29
Real, Chiapa,	17 n	281
Realejo	12 n	307
Rebuick	45 n	47
Reccandi	43 n	36
Rees	52 n	25
Reggio	44 n	31
Regio	38 n	37
Rennes	48 n	17
Reſcht	38 n	72
Rethel	49 n	24
Rethigen	48 n	29
Revel	59 n	46
Reutlingen	48 n	29
Rezan	55 n	64
Rheines	49 n	24
Rhineberg	51 n	26
Rhinfelden	47 n	27
Rhodes	36 n	40
Richelieu	47 n	18
Ries	43 n	25
Rieux	43 n	19
Riga	57 n	45
Rimini	44 n	34
Rio, Riom	45 n	23
Ripatranſone	43 n	35
Ripen	56 n	29
Riſano, Riſne	43 n	42
Riva	46 n	31
Rivadec	43 n	10
Rivoli	45 n	27
Roanne	46 n	23
Rochelle	46 n	17
Rocheſter	51 n	19
Rochfort	46 n	17
Rocroix	50 n	24
Rodez, Rodes	44 n	22
Roermond	51 n	25
Röleduc	51 n	26
Romans	45 n	25
Rome	42 n	34
Ronciglione	41 n	35
Ronda	36 n	11
Roſchid, Roſchild	56 n	32
Roſes	42 n	22
Roſetto	31 n	61

	lat.	lon.
Roſienne	55 n	44
Roſſanno	40 n	38
Roſtock	54 n	33
Roſtow, Roſtof	57 n	60
Rotenburg	49 n	31
Rottenburg	53 n	29
Rotterdam	52 n	24
Rotweil	48 n	26
Roven, Roan	49 n	21
Rovigo	45 n	32
Roye, Roya	49 n	23
Rudelſwert	46 n	36
Rudiſto	42 n	53
Ruffacu	48 n	27
Ruvo	41 n	38
Rzeczyca	53 n	51
S.		
Sabionetta	45 n	31
Saccai	36 n	155
Saderaſapaton	12 n	100
Sagan	52 n	35
Said	27 n	52
Saintes	46 n	17
Salamanca (Spain)	41 n	11
Salamanca, America,	17 n	285
Salamis	47 n	44
Salenkamen	45 n	42
Salem	42 n	308
Salerno	41 n	37
Salins	47 n	26
Saliſbury	51 n	16
Salle	34 n	11
Salm	48 n	27
Salona	44 n	39
Salonichi	41 n	46
Salſes	43 n	20
Salſona	42 n	19
Saltſburg	48 n	33
Saluzzo	45 n	27
Samarcand	40 n	86
Samaria	33 n	58
Sandec, Sandecz	49 n	41
Sandomir	50 n	42
Sangueſſa	47 n	17
Santa cruz	23 n	293
Santa maria	8 n	298
Santafe	36 n	269

	lat.	lon.
Santa fe de bagota	4 n	304
Santarem	39 n	10
Santen	52 n	26
Santillana	43 n	13
Santuliet	51 n	23
Santorini	36 n	45
Saquem	19 n	59
Saragoſa	41 n	7
Saraio, Seraio	45 n	40
Sarbruc	29 n	27
Sardis	37 n	48
Sarlat	45 n	21
Sarno	41 n	36
Sarſina	44 n	33
Sarzana	44 n	31
Saſſari	40 n	29
Savanna	32 n	297
Savigliano	44 n	27
Samur	47 n	30
Savona	44 n	29
Sayarook	41 n	306
Scala	41 n	36
Scanderoon	36 n	57
Scaros, Saros,	49 n	41
Schafhauſen	47 n	28
Schamachia	41 n	70
Scharding	46 n	33
Schleſtat	48 n	27
Schemnitz	49 n	39
Schiras, Chiras	30 n	73
Schomberg	50 n	26
Schonſter, Suſa	31 n	68
Schweidnitz	51 n	37
Schweinfurt	50 n	31
Scio	39 n	54
Scrivan	10 n	276
Scutare	42 n	42
Sebenico	44 n	38
Secandra	27 n	99
Sedan	49 n	25
Sees	49 n	19
Segedin, Seged	46 n	42
Segeſwar	47 n	45
Segewold, Sevold	57 n	46
Segni	42 n	35
Segorbe	40 n	17
Segovia (Aſia)	18 n	139

	lat.	lon.
Segovia (Spain)	41 n	13
Selivrea	43 n	53
Semendria	45 n	43
Senez, Senes	44 n	26
Sens	48 n	23
Seregippe	11 ſ	339
Seſſa	41 n	36
Setines, Athens	38 n	44
Severino	39 n	41
Seviero	42 n	37
Seville	37 n	11
Sibet	15 n	65
Sichem	32 n	56
Sidon, Zidon	33 n	57
Siegen	51 n	28
Sienna	43 n	32
Sigan	34 n	128
Sigeth, Zyget	46 n	39
Sigiſtan	31 n	82
Siguenza	41 n	15
Siliſtria, Doreſtero	44 n	47
Sillebar	4 ſ	121
Simmeren	50 n	27
Sincapora	1 n	124
Sinope	42 n	56
Sion	46 n	27
Sirad	52 n	38
Siranager	31 n	100
Sirmium	45 n	41
Siſeg	46 n	38
Siſteron	44 n	25
Slawkaw	49 n	37
Sleſwick	55 n	30
Sluezk	53 n	50
Sluys	51 n	23
Smalkaden	51 n	31
Smolenſko	55 n	54
Symrna	38 n	47
Snakenburg	53 n	30
Sneek	53 n	25
Soconſco	15 n	280
Suczow	47 n	48
Soeſt	52 n	28
Sofala	20 ſ	55
Sofia	43 n	47
Soignes	50 n	23
Soiſſons	49 n	23

	lat.	lon.
Solms	50 n	28
Solothurn	47 n	27
Sondro	36 n	30
Sora	42 n	35
Soraw	52 n	35
Sorrento	41 n	36
Sottotitza	63 n	60
Sovanna	42 n	32
Spaa, Spaw	50 n	26
Spaletto, Spalatro	44 n	39
Spandaw	52 n	34
Spires	49 n	28
Spoleto	42 n	34
Sprotaw	52 n	36
Squillace	39 n	38
Stade	54 n	29
Stagno	43 n	41
Stanz	47 n	28
Staria	58 n	54
Stavenger	59 n	26
Stavern	53 n	25
Steenburg	52 n	24
Stenay	49 n	25
Stendal	53 n	32
Stereburg	53 n	36
Stetin	53 n	35
Steyre	49 n	34
Stockholm	59 n	39
Stolp	54 n	38
Stralſund	54 n	34
Straſburg	48 n	27
Straubing	49 n	33
Strongolo	39 n	38
Stugart	48 n	29
Suez	30 n	54
Sulmona	42 n	36
Sunneberg	53 n	35
Surinam	6 n	322
Suſa	45 n	27
Suſdal	57 n	66
Swallen	21 n	92
Swerin	54 n	32
Switz	47 n	29
Syracuſe	37 n	36
T.		
Tabaſco	18 n	283
Tacungo	14 ſ	300

	lat.	lon.
Tafala	43 n	17
Tafilet	30 n	13
Taitchen	29 n	141
Talamone	42 n	32
Tanaſſerim	12 n	118
Tancrowal	14 n	4
Tangier	36 n	11
Tanjour	11 n	99
Taormina	38 n	36
Taracon	52 n	20
Taragona	41 n	19
Tarento	40 n	39
Taraſcon	43 n	24
Tarbe	43 n	18
Targarod	47 n	47
Targoviſco	48 n	46
Tariffa	36 n	12
Tarku	42 n	71
Tarodant	30 n	8
Tarracon	42 n	16
Tarragon	40 n	19
Tarſus, Teraſſo	37 n	55
Tatta	26 n	88
Tavaſtus	61 n	45
Tavira, Tavir	37 n	9
Tauris	38 n	66
Tayiren	39 n	128
Tecklenburg	52 n	27
Tegapatan	8 n	96
Tellickerry	12 n	95
Temeſwaer	46 n	42
Tergowiſk	46 n	46
Termini, Termola	42 n	37
Terni	42 n	34
Ternova	43 n	46
Tervel	40 n	17
Teſchin	50 n	39
Thebes	38 n	48
Thionville	40 n	36
St. Thomas (Aſia)	13 n	100
St. Thomas, America	7 n	316
Thonon	46 n	26
Thorn	53 n	39
Thoulon, Toulon	47 n	26
Tholouſe, Toulouſe	44 n	19
Titul	45 n	41
Tivoli, Tibar	42 n	34

Tibolſki

	lat.	lon.
Tobolſki	65 n	93
Tocat	41 n	59
Tockay	48 n	42
Todi	43 n	33
Toledo	39 n	13
Toloſa	43 n	15
Tombut	14 n	7
Tomebamba	2 ſ	301
Torcello	43 n	33
Tordecillis	41 n	13
Torgaw	51 n	33
Torna	48 n	41
Torne	66 n	43
Tortona	44 n	28
Tortoſa	40 n	18
Toul	48 n	25
Tournay	50 n	23
Tournon	45 n	24
Tours	47 n	19
Traerbach	50 n	27
Tranchin, Trenchin	49 n	38
Trani, Trane	41 n	38
Trapano	38 n	33
Trapezond	42 n	62
Traw	44 n	38
Trent	46 n	31
Trevigio	45 n	32
Trevoux	46 n	24
Triers, Trevers	50 n	26
Trieſte	46 n	34
Trinidada	13 n	284
Trinquebar	12 n	101
Tripoli	33 n	13
Triquier	48 n	14
Trois rivieres	47 n	303
Troki	54 n	46
Tropea	39 n	37
Troppaw	50 n	38
Troy	39 n	46
Troyes	48 n	24
Truxillo (Europe)	39 n	11
Truxillo (America)	8 n	301
Tubingen	48 n	29
Tucuyo	7 n	310
Tunja	4 n	306
Tunis	36 n	30
Turin	44 n	27

	lat.	lon.
Tuy	42 n	8
Twer, Twere	57 n	60
Tyre	32 n	58
Tzorlic	43 n	53

V.

	lat.	lon.
Vado	44 n	29
Vaiſon	44 n	26
Valence	45 n	25
Valencia	39 n	17
Valenchiennes	50 n	23
Valenza	45 n	29
Valkenburg	51 n	25
Valladolid	42 n	13
Vallona	41 n	41
Valparaiſo	33 ſ	301
Van	38 n	64
Vannes, Vennes	47 n	15
Varna	45 n	53
Vaudemont	48 n	26
Ubeda	38 n	14
St. Ubes Setuval	38 n	8
Udenſkoi	53 n	116
Udino	46 n	33
Velletri	41 n	34
Vence	43 n	27
Vendoſme	48 n	18
Venice	45 n	33
Venlo	51 n	26
Venta de cruz	9 n	297
Ventimiglia	42 n	27
Vera cruz	18 n	276
Vera pax, Coban	15 n	285
Vercelli	45 n	28
Verdun	49 n	25
Vernon	49 n	21
Verona	45 n	31
Verſailles	48 n	22
Verua	45 n	28
Veſprin, Weiſbrun	47 n	39
Vezelai	47 n	23
Viana	42 n	16
Vianden	50 n	26
Vicenza	45 n	32
Vich	42 n	20
Vienna	48 n	37
Vienne	45 n	24
Vigevano	45 n	29

Vigo

	lat.	lon.
Vigo	42 n	8
Villarica	20 n	278
Villena	38 n	16
Vire	48 n	17
Viſet	51 n	26
Viſiapour	17 n	96
Viterbo	42 n	33
Vitoria, Victoria	43 n	15
Viviers	44 n	24
Vize	43 n	52
Uladiſlaw	53 n	39
Ulm	48 n	30
Ulmen	50 n	26
Uma	64 n	38
Umbriatica	40 n	41
Voidenar	40 n	45
Volona	42 n	40
Volterra	43 n	31
Upſal	60 n	38
Urbine	43 n	34
Urgel	42 n	19
Uſcopia	43 n	46
Utica	37 n	29
Utrecht	52 n	25
Uzez	44 n	24
W.		
Wagin ingen	52 n	25
Walcourt	50 n	24
Waldſhut	47 n	28
Wangen	47 n	30
Waradin	47 n	42
Waraſden	47 n	38
Warſaw	52 n	41
Warta	52 n	43
Wartenburg	51 n	37
Waterford	52 n	11
Weil, Weilſtat	49 n	28
Weilburg	50 n	28
Weimar	51 n	31
Weiſenburg Tranſil.	46 n	45
Wells	48 n	34
Werden	51 n	26
Werdenburg	47 n	30
Werl	51 n	27
Wertheim	50 n	29
Weſel	51 n	26
Weſterwick	58 n	37

	lat.	lon.
Wetzler	50 n	25
Wexio	57 n	34
Whidah, Fida	6 n	23
Wiburg	56 n	29
Wiburn	61 n	50
Widen, Vidden	44 n	46
Wihitz	45 n	37
Wilkomers	55 n	46
Williamſburg	37 n	302
Williamsfort	23 n	107
Williamſtat	51 n	24
Wilna	55 n	46
Wimpen	49 n	29
Wincheſter	51 n	17
Windaw	57 n	43
Winnicza	49 n	50
Winoxbergen	51 n	21
Wiſby	57 n	38
Wiſmar	54 n	31
Witepſki	55 n	54
Wittenburg	51 n	33
Woerden	52 n	24
Wolaw	51 n	37
Wolfenbuttle	52 n	30
Wolgaſt	54 n	34
Wolkowſka	55 n	44
Wolodimar	50 n	45
Wolodomer	57 n	50
Wologda	59 n	62
Worcom	51 n	24
Worms	49 n	28
Woronitz, Worotin	54 n	65
Wurtzburb	50 n	31
Wynendalf	51 n	23
Wynoxberg	51 n	21
X.		
Xaca, Sacca	37 n	34
Xaliſco	22 n	268
Xativa	39 n	17
St. Xavier	24 ſ	328
Xeres de badajos	38 n	10
Xeres de la frontera	37 n	11
Xeres de guadiana	37 n	9
Xiceu	27 n	122
Xilopolis	42 n	47
Xinian	31 n	140
Xixona	38 n	17

Xuanogrod

	lat.	lon.
Xuanogrod	45 n	38
Y.		
Yenne	45 n	25
Yeſd	33 n	74
Ylſt	53 n	25
Yoangfu	31 n	134
York	54 n	17
New York	41 n	304
Ypres	51 n	23
Yſendic	51 n	22
Yſondun	47 n	22
Yvica	39 n	19
Yvoix	50 n	25
Z.		
Zabern, Saburn	49 n	28
Zabern elſas	48 n	27
Zabes	47 n	45
Zabola	47 n	47
Zagrab	46 n	37
Zamora	42 n	12
Zamoſki	50 n	44
Zamora	41 n	26
Zara	44 n	37
Zargona	45 n	46
Zarnaw	51 n	40
Zaſlow	50 n	50
Zator	50 n	46
Zawolocze	57 n	51
Zeby	57 n	30
Zeigenheim	51 n	29
Zeitz	51 n	32
Zell	53 n	30
Zemlin	48 n	42
Zeng, Segua	45 n	37
Zenobiz	45 n	44
Zenonis	48 n	63
Zerpſt	52 n	32
Zers	45 n	48
Zeverinum	46 n	45
Ziericze	51 n	25
Ziget	46 n	52
Ziltz	44 n	4
Zintzheim	49 n	29
Ziſſera	57 n	54
Zittaw	51 n	34
Znaim	48 n	36
Zolaritzo	64 n	60
Zollern	48 n	28
Zolnock	47 n	41
Zornajam	56 n	60
Zug	46 n	28
Zurick	47 n	29
Zutphen	52 n	26
Zweybruck	49 n	27
Zwickaw	50 n	32
Zwifalen	48 n	29
Zwingenburg	49 n	28
Zwol, Swol,	53 n	25
Zwonik	44 n	42
Zytomierz	51 n	52

SECT.

SECT. II.

The uſe of the Terreſtrial Globe, in the Solution of Geographical Problems.

THE parts of the terreſtrial Globe are much the ſame as in the celeſtial; theſe are the brazen meridian, the wooden horizon, the quadrant of altitude, the horary circle, the ſemicircle of poſition, and the mariners compaſs. The eaſtern edge of the brazen meridian is divided into degrees, and repreſents the true meridian. The upper ſide of the wooded horizon is divided into degrees, and repreſents the true horizon. The quadrant of altitude is a thin braſs plate, fixt to the meridian at any point of it required, by a nut and a ſcrew. That edge of it which is divided into degrees is called the fiducial edge, and properly repreſents the correſponding great circle. The horary circle is divided into 24 hours, making twice 12; and 12 at noon is at the upper part of the meridian, and 12 at night at the oppoſite ſide; the morning hours to the eaſt, the evening ones to the weſt; the end of the axis repreſenting the pole, carries round the hand, which ſhews the hour. The ſemicircle of poſition has its extremities fixt to the north and ſouth points; about which it moves freely, and may be ſet to any poſition. The mariners compaſs is placed on the foot of the globe, and ſerves for ſetting the globe in the ſame poſition as the heavens.

The circles drawn on the terreſtrial globe are, the equinoctial, and the parallels of latitude to every 10 degrees, the two tropics, and the two polar circles; alſo the meridians at every 10 degrees diſtance upon the equinoctial, beginning at the

Fig. the firſt meridian, which is or ought to be, that paſſing thro' the iſland of *Ferro*, one of the Canaries. The ecliptic is alſo drawn on the terreſtrial Globe, tho' it belongs properly to the celeſtial; and is put upon the terreſtrial for the ſake of reſolving ſome problems of the ſphere. Alſo rumb lines are drawn upon ſome vacant place of the terreſtrial globe, ſhewing the 32 winds or points of the compaſs.

The ſurface of the terreſtrial Globe differs from that of the celeſtial, in this particular; that upon the celeſtial are deſcribed the ſeveral conſtellations of the heavens, with the true places of all the ſtars. But upon the terreſtrial, the ſeveral countries of the world are deſcribed; all lands and ſeas adjoining to them; all iſlands, capes, bays, mountains, rivers, towns, and other things upon the earth are delineated in their proper ſituation, juſt as the places themſelves, which they repreſent, are upon the earth. So that this globe or little earth will be exactly ſimilar to the great globe of the earth; and all their correſpondent parts will be in a ſimilar poſition. And if this terreſtrial globe be turned about its axis, it will likewiſe have a ſimilar motion with the earth itſelf. And ſo there is an exact agreement between the natural poſition of the ſeveral parts of the earth and ſea, and their artificial repreſentation upon the terreſtrial globe; by which means a great many problems of the ſphere may be very eaſily reſolved. I ſhall here lay down ſuch as belong properly to the terreſtrial globe, referring the reſt to the celeſtial globe, delivered in Sect. III. of the Aſtronomy.

PROB. I.

To find the latitude and longitude of a given place upon the globe.

Turn the globe round its axis, till you bring the place under the braſs meridian, there let it reſt. Then the arch of the meridian, comprehended between the given place and the equator, is the latitude; which is north, if it lie towards the north pole; or ſouth, if towards the ſouth pole. And the arch of the equator contained between the firſt meridian and the braſs meridian, is the longitude.

If you would have this longitude in hours, bring the firſt meridian under the braſs meridian, and ſet the hour index to 12, then turn the globe towards the weſt, till your place come under the braſs meridian; and then the index will ſhew the hours correſponding to the longitude, or the longitude in time.

PROB. II.

To find upon the globe, any place whoſe latitude and longitude is given.

Reckon the longitude along the equator, and bring that point of it to the brazen meridian; then count the latitude along the braſs meridian northwards or ſouthwards, and there you will find the place required, for either north or ſouth latitude.

PROB. III.

The latitude of any place being given; to find all thoſe places that have the ſame latitude.

Bring the place to the brazen meridian, make a mark there; then turn the globe about, and all

Fig. thoſe places that paſs under that mark, have the ſame latitude as the place given.

Likewiſe all thoſe places may be found that have the ſame longitude as a given place; for if that place be brought to the meridian, all thoſe places then lying under the meridian have the ſame longitude.

PROB. IV.

To find the difference of latitude, and difference of longitude between any two given places.

For the difference of latitude. Bring each of the places to the braſs meridian, and mark the points of interſection; then count the degrees of the meridian contained between theſe marks; and you have the difference of latitude.

For the difference of longitude. Bring each of the places to the braſs meridian, and mark the two points of the equinoctial cut by the meridian; count the degrees of the equinoctial between the marks, and that is the diff. longitude. Or for the diff. longitude in time; bring one place to the meridian, and ſet the hour circle to 12; then bring the other place to the meridian; and the number of hours between 12 and where the hand ſtands, is the diff. longitude in time.

PROB. V.

To find the diſtance of two places upon the globe.

Apply the quadrant of altitude, or braſs ſemicircle to the globe, to paſs thro' the two places. Then the number of degrees between them, counted upon the quadrant or ſemicircle is the diſtance.

Or take the diſtance between the two places with a pair of compaſſes, and apply it to the equator, and it will ſhew how many degrees are contained Fig. in that diſtance.

If you would know the diſtance in miles, multiply the degrees by 60, gives the minutes or geographical miles.

PROB. VI.

To rectify the globe.

Set the globe upon a horizontal plane, ſo that the needle may point towards the north, allowing for the variation.

Raiſe your elevated pole ſo many degrees above the horizon as is equal to the latitude; moving the meridian up and down, till that number of degrees touches the horizon, there let it reſt. Then the globe is rectified for the latitude.

Count the degrees of latitude from the equator towards the elevated pole, and where it ends, is the zenith. To this point fix the quadrant of altitude, ſo as the graduated edge of it may touch that point.

Bring the ſun's place in the ecliptic to the meridian, and then ſet the hour index to 12 at noon; ſo the globe is rectified for the ſun's place. And if your place be brought to the braſs meridian, then the meridian of the globe will coincide with the meridian of the place, and the globe will be in the ſame ſituation as the earth at noon.

PROB. VII.

To find how any given place bears from your place, upon a great circle.

Having rectified the globe for your latitude, and fixed the quadrant of altitude in the zenith, by the laſt Prob. Turn the globe about till your place come to the braſs meridian, there ſtop it; then turn the

Fig. quadrant of altitude about, till it paſs thro' the other place, let it reſt there; then count the number of degrees upon the horizon intercepted between the braſs meridian and the quadrant of altitude, and that will be the angle of poſition.

By this means you may know how all the countries round about are ſituated in reſpect of your dwelling. And alſo what places bear upon any point of the compaſs you deſire.

PROB. VIII.

To find the antæci, periæci, and antipodes to any given place.

Bring the given place to the meridian, and find its latitude; reckon the ſame number of degrees from the equator to the contrary pole, and where the reckoning ends is the antæci.

The given place being ſtill at the meridian, ſet the hour index to 12 at noon, then turn the globe about till the index points to 12 at night; then the place in the ſame parallel which is under the meridian, is the place of the periæci.

The globe having this laſt ſituation, that place which is in the oppoſite parallel, and under the meridian, is the place of the antipodes.

PROB. IX.

To find what a clock it is at any given place; when it is noon, or any other hour at your place.

Rectify the globe, and bring the place of your habitation to the brazen meridian, and ſet the index to 12 at noon, if it be noon at your place, or elſe to the given hour; then turn the globe about, till the given place comes to the braſs meridian;

and then the index will point to the given hour at that place. Fig.

PROB. X.

The hour of the day being given at your place; to find the place or places where it is noon, or any other hour propoſed.

Rectify the globe, and bring your place to the meridian, and then letting it reſt, ſet the index to the hour given; then turn the globe round, till the index points to 12 at noon, if it is noon at the other place, or elſe to the hour given at that place; and then all the places lying under the braſs meridian are theſe required; or where they reckon that time of the day.

And thus all thoſe places are found, where the ſun is in their meridian at a given hour, for it is then 12 a clock there.

PROB. XI.

The day and hour being given; to find the place of the earth where the ſun is vertical at that time.

Having rectified the globe for your place; get the ſun's place for that time, and find it in the ecliptic, and bring it to the brazen meridian, and mark the place of the meridian with chalk. Then turn the globe about till your place comes under the meridian, there ſtop it, and ſet the index to the given hour. Then turn the globe about till the index point to 12 at noon, there ſtop it, and obſerve what place is under the mark on the brazen meridian, for that is the place where the ſun is vertical.

You may obſerve, as the globe turns round, that the ſun will be vertical ſucceſſively to all theſe places that lie in that parallel.

PROB. XII.

To find all the places where the ſun is in the horizon riſing or ſetting, at a given hour.

The globe being rectified, bring the ſun's place in the ecliptic to the brazen meridian, and mark the place on the meridian. Turn the globe about till your place comes under the meridian, then ſet the index to the given hour; then turning the globe till the index points to 12 at noon; obſerve what place is under the mark. Bring this place into the zenith, then all places at the horizon will be the places where the ſun is riſing or ſetting, the riſing being to the weſt ſide, and the ſetting to to the eaſt.

PROB. XIII.

To find what a clock it is by the globe, when the ſun ſhines.

Rectify the globe to the latitude of your place, and ſet it in a true poſition by the compaſs; then letting the ſun's light fall upon it, obſerve on the dark ſide, that point of the equinoctial that ſeparates the light from the dark part; reckon how many times 15 degrees is contained between that point and the horizon, and ſo many hours it wants of 12 a clock, if the dark ſide is on the weſt; or it is ſo many hours paſt 12, if the dark ſide be eaſtward.

PROB. XIV.

To find in what places it is day, and in what places night, at a given time.

This is done by Prob. XII. For all thoſe places that are above the horizon have day, and thoſe under it have night. And all thoſe places under the meridian above the horizon have their noon, and all places under the meridian below the horizon have their midnight.

Otherwiſe.

If the ſun ſhines it may be done thus; rectify the globe according to the latitude, and ſet it due north and ſouth by the compaſs. Then bring your place to the meridian, and in this poſition let the ſun ſhine upon it, and it will be day to all the illuminated part, and night time to the dark part of the globe; and ſun riſe or ſun ſet, at the circle ſeparating the light from the dark part.

For the globe is now in a like poſition as the earth, and therefore it is illuminated in the very ſame manner that the earth itſelf is illuminated.

PROB. XV.

To find the time of ſun riſing and ſetting.

Rectify the globe according to the latitude, and bring the ſun's place to the braſs meridian, and then ſet the hour index to 12 at noon. Then turn the globe about till the ſun's place comes to the eaſtern ſide of the horizon; then the index will ſhew the time of ſun riſing; and if the ſun's place, by turning the globe, be brought to the weſtern part of the horizon, the index will ſhew the time of ſun ſet.

Hence if the time of ſun ſet be doubled it gives the length of the day. And if the time of ſun riſe be doubled it gives the length of the night.

PROB. XVI.

To find the time of day break, or the end of twilight, on a given day.

Rectify the globe to the latitude, and bring the ſun's place to the braſs meridian; and ſet the hour index to 12 a clock at noon. Then turn the globe about towards the eaſt, till the ſun's place be 18 deg. below the horizon, or the oppoſite point of the ecliptic 18 deg. above the horizon, and then it is day break, and the index ſhews the time. This diſtance of 18 deg. may be meaſured with a pair of compaſſes opened to that diſtance, or by the diviſions upon the quadrant of altitude. Again, turn the globe weſtward till the ſun's place in the ecliptic be 18 deg. below the horizon, and then the hour index will ſhew the time when twilight ends.

PROB. XVII.

To find what countries have no dark night at any time propoſed.

Bring the ſun's place at the given time to the braſs meridian, and count his declination upon the meridian, which is always towards the elevated pole. Then elevate the globe ſo, that the height of the equinoctial above the horizon may be equal to the declination + 18 degrees. Then reckon what height the pole is above the horizon, which will be the latitude, where twilight begins to be perpetual. Therefore in all greater latitudes, ſuch people as dwell there, will have no dark night, but twilight. Or the morning twilight will begin before the evening twilight ends.

PROB.

PROB. XVIII.

To find in what latitude the ſun begins to ſhine conſtantly without ſetting, at a given time of the year. And alſo at what latitude in the oppoſite hemiſphere, he begins to be totally abſent.

Bring the ſun's place in the ecliptic to the braſs meridian, and count his declination upon the meridian, and that will be the complement of the latitude. Therefore reckon the ſame number of degrees from the pole towards the equator, upon the meridian, and mark the place. Then turn the globe round, and all the places paſſing under this mark, are thoſe in which the ſun begins to ſhine conſtantly without ſetting, for that time. Mark the oppoſite point of the brazen meridian, and turning the globe round, all the places in that parallel under the mark are thoſe where total darkneſs begins; for there the ſun begins on that day to be quite abſent, or totally to diſappear.

PROB. XIX.

To find the length of the longeſt day or night in any given place.

Rectify the globe to the latitude of the place, and mark any point in the tropic of cancer for north latitude, or in the tropic of capricorn for ſouth latitude. Then bring that mark to the eaſt ſide of the horizon, and ſet the index of the horary circle to 12 a clock at noon. Turn the globe about till the mark in the tropic comes to the weſtern ſide of the horizon. Then obſerve upon the hour circle, how many hours the hand or index has gone thro' from 12 at noon; and that is the length of the longeſt day, as alſo of the longeſt night. And

Fig. And this ſubtracted from 24, leaves the length of the ſhorteſt night or ſhorteſt day, at the given place.

PROB. XX.

The latitude of the place in the frigid zone being given; to find how many days the ſun ſhines conſtantly upon it.

Rectify the globe according to the latitude; then turn the globe about, and obſerve what point of the firſt quadrant of the ecliptic falls upon the interſection of the horizon and meridian to the north, ſuppoſing the place lies north latitude. Find that point of the ecliptic upon the wooden horizon; and againſt it ſtands the day of the month when the ſun is in it. And then the longeſt day begins at that place.

Again, turn the globe about till ſome point of the ſecond quadrant of the ecliptic falls upon the north interſection of the meridian and horizon as before; obſerve what point of the ecliptic that is, and find it on the wooden horizon, and againſt it ſtands the day of the month when the ſun is there. And at that time the longeſt day at that place ends.

Then count the number of natural days contained between the beginning and ending of this longeſt day; and you will have the length of it. And the day firſt found is the day of his appearance; and the latter, the day of his departure. And the number of natural days in the longeſt night is the ſame; and begins and ends at the ſame diſtance from the winter ſolſtice, as the day does from the ſummer one.

Otherwiſe thus.

The globe being elevated according to the latitude, bring the ſolſticial point to the north part of the meridian. Then reckon the complement of the latitude from each ſide of the equator upon the meridian, and mark the places where the reckoning ends. Then turn the globe carefully about, and obſerve what two degrees of the ecliptic comes under the northern mark; then the intercepted arch of the ecliptic, reduced to time, gives the number of days that the ſun conſtantly ſhines, or remains above the horizon of the place; which is the length of the longeſt day there. And the oppoſite arch equal to it, gives the number of days he is abſent, or the length of the night. Or obſerving the points of the ecliptic paſſing under the ſouthern mark, the contained arch reduced to time, gives the length of the night. Fig.

The elevation of the pole continuing the ſame, obſerve the two degrees of the ecliptic that come under the north mark, and find them on the wooden horizon, and you will have the days that the ſun is in theſe points, which days are thoſe of the firſt and laſt appearance of the ſun. And if the points of the ecliptic, that come under the ſouth mark, be obſerved, you will find on the wooden horizon the correſpondent days when the long nights begins and ends in that place.

SECT. III.

The Theory of Navigation, Spherical, and Spheroidical. With the Solution of ſome uncommon Caſes. The actual Solution in Numbers, of the common Caſes of Sailing, by the Spheroid; and likewiſe of theſe in parallel Sailing. Rules for correcting a Reckoning. Of taking Altitudes.

HAVING formerly writ a practical book of Navigation, calculated entirely for the uſe of ſuch as frequent the ſea, I ſhall here only extract from it, ſo much of the theory as is proper for a courſe of mathematics. To which I ſhall add ſome things that were omitted there, or but ſlightly touched, as being leſs neceſſary for ſailors; but ſerve for compleating the theory. But as to the practical part, which is the moſt neceſſary thing for ſeamen, I ſhall meddle very little with it here. Yet if any perſon wants farther ſatisfaction in that point, he may conſult the other book at his leiſure.

DEFINITIONS.

DEF. I.

Navigation is the art of computing a ſhip's way, and guiding her thro' the ſea, in ſailing from one place to another.

DEF.

DEF. II.

Difference of latitude of two places, is an arch of the meridian contained between two parallels paſſing thro' theſe places.

DEF. III.

Difference of longitude, is an arch of the equator contained between the meridians of any two places.

DEF. IV.

Meridional diſtance, is the diſtance in the parallel of the place you are in, from the meridian of the place you came from.

DEF. V.

Rumb, is the line or way a ſhip deſcribes while ſhe ſails upon any one point of the compaſs, and cuts all the meridians at the ſame angle.

DEF. VI.

Courſe, is the angle which the rumb or ſhip's way makes with the meridian.

DEF. VII.

Departure, is the whole eaſting or weſting a ſhip continually makes, in any ſingle courſe.

DEF. VIII.

Plain ſailing, is the art of computing a ſhip's way by plain trigonometry, in regard to her difference of latitude and departure.

DEF. IX.

Middle latitude ſailing, is a method of computing the place of a ſhip as to longitude, by help of the middle latitude, which is the latitude lying in the

Fig. the middle way between the two places of the ſhip.

DEF. X.

Mercator's or Wright's ſailing, is the method of computing a ſhip's place as to longitude, by help of a table of meridional parts.

Inſtruments uſed in Navigation.

1. *Charts.* Theſe are maps of the ſea and ſea coaſts, and are generally conſtructed by Mercator's projection, where the meridians are all parallel, and the degrees of longitude equal in all latitudes, and the degrees of the meridian increaſe towards the poles, in the ratio of the coſ. latitude to the radius. Theſe ſorts of charts are fitteſt for the ſailors uſe.

2. *The Compaſs.* This is divided into 32 points or rumbs; by this the ſhip is ſteered, upon any courſe. For the needle always points to the north, except ſo far as it deviates by the *variation*, which is the difference between the true and the magnetical meridian, eaſt or weſt. An *azimuth compaſs* has beſides, an index with a thread and two ſights, and is moveable about the center. Its uſe is, to take the azimuth of the ſun or a ſtar.

3. The *Log line*; which is a cord divided into ſeveral parts called knots, and the end is faſtened to the log. Its uſe is to meaſure the ſhip's motion, and is heaved into the ſea every two hours; and the length run off, is known by a half minute glaſs. For ſo many knots as are run off, ſo many miles the ſhip ſails an hour.

4. The *Sea quadrant*, commonly called *Davis's* quadrant. This is an inſtrument to take the ſun's altitude.

5. *Hadley's quadrant*, another inſtrument for taking

king the altitude of the ſun or a ſtar; and is the beſt Fig.
inſtrument for that purpoſe.

6. The *Croſs ſtaff* or *fore ſtaff*, this is alſo uſed for taking altitudes.

7. The *Lead*, this is faſtened to a line divided into fathoms, and is uſed for ſounding the depth of the ſea.

PROP. I.

In plain ſailing, that is, where no longitude is concerned; as radius : to the diſtance run : : ſo the ſine of the courſe : to the departure.

Let P be the pole, HL the equinoctial, A the 13.
place of a ſhip, PAH a meridian paſſing thro it, ADFQ the rumb or way of the ſhip. Divide the meridian AP into an infinite number of equal parts, at k, y, x, &c. and thro' theſe points draw the parallels of latitude kB, yC, xD, &c. to cut the rumb in B, C, D, &c. thro' which points draw the meridians PB, PC, PD, &c. cutting the equinoctial in t, $ſ$, r, &c. By the ſeveral interſections of the parallels and meridians, and the rumb, there will be made an infinite number of ſmall elementary triangles ABk, BCp, CDg, &c. all equal and ſimilar. For all the ſides Ak, Bp, Cg, &c. are equal by conſtruction, for $Bp = ky$, and $Cg = yx$, &c. and the angles kAB, pBC, gCD, &c. are equal by Def. V. therefore (Geom. II. 7.), all the parts of the rumb, AB, BC, CD, &c. are equal, and alſo the ſides kB, pC, gD, &c. Now in any one of theſe triangles ABk, it will be, by plain trigonometry, as rad : ſ. kAB : : AB : kB : : BC : pC : : CD : gD, &c. and by compoſition, rad : ſ. kAB : : AB + BC + CD, &c. : $kB + pC + gD$, &c. But when the ſhip arrives at F, AB + BC + CD, &c. = AF the diſtance run; and $kB + pC + gD$, &c. is the departure by Def.

Fig. 13. Def. VII. Therefore it is, rad : ſ. *k*AB : : diſtance run : departure. And alternately rad : diſtance run : : ſ. courſe (*k*AB by Def. VI.) : departure.

Cor. 1. *As radius* : *diſtance run* : : *coſ. courſe* : *diff. latitude.*

For rad : ſ. *k*BA : : AB : A*k* : : BC : B*p* : : CD : C*g*, &c. : : (by compoſition) AB + BC + CD, &c. : A*k* + B*p* + C*g*, &c. or A*k* + *ky* + *yx*, &c. But when the ſhip comes to F, AB + BC + CD, &c. = AF the diſtance run, and A*k* + *ky* + *yx*, &c. = AR the difference of latitude. Therefore rad : ſ. *k*BA or coſ. courſe : : diſtance run AF : dif. lat. AR. Or alternately, rad : diſtance run AF : : coſ. courſe : dif. lat. AF.

Cor. 2. *As rad* : *diff. latitude* : : *tan. courſe* : *departure.*

For by this Prop. rad : diſtance : : ſ. courſe : departure : : (Cor. 1.) coſ. courſe : dif. latitude. And alternately, dif. latitude : departure : : coſ. courſe : ſ. courſe : : (trigonometry) rad : tan. courſe.

Cor. 3. *Hence, if a ſhip moves uniformly along its rumb line, it approaches the pole uniformly.*

For if all the lines AB, BC, CD, are equal; then all the lines A*k*, *ky*, *yx* are equal, and deſcribed in equal times. And hence appears the abſurdity of what ſome people aſſert, that the rumb can never reach the pole. For if this was true, then a ſhip keeping always in the rumb, could never reach the pole, tho' ſhe approaches it uniformly. Which is a direct abſurdity.

Cor. 4. *If two ſhips ſail from* A *at once, one along the meridian* AP, *with velocity* AK; *the other in the rumb* AF, *with velocity* AB; *they will both meet in the pole at once.*

It cannot be denied, that the firſt ſhip will get

Fig. 13.

to the pole in a finite time, which is eaſily determined from the velocity along AP. And in every point of time, they will be both in the ſame parallel of latitude. Thus they will be at K and B; at *y* and C, at *x* and D, at R and F, at once. And therefore when the firſt ſhip is at P, the ſecond cannot be ſuppoſed to be ſhort of it without an abſurdity. And it is the ſame thing if you take two points inſtead of the two ſhips.

Cor. 5. *As coſ. courſe : radius : : ſo length of the meridian* AP : *to the whole length of the ſpiral* AQP.

Cor. 6. *Hence, the diſtance run, the difference of latitude, and the departure being laid out in right lines; they will form a true right angled plain triangle.*

14. For make RA = RA (fig. 13.) and angle RAF = RAF. Then ſuppoſe the rumb AF to be unbent, or drawn back into a ſtraight line, and laid upon AF (fig. 14). The points B, C, D, will fall upon B, C, D, and the lines *k*B, *p*C, *g*D, upon *k*B, *p*C, *g*D, in the two figures; becauſe the angles at B, C, D, being equal; the lines B*k*, C*p*, D*g*, are all parallel to FR. And therefore the perpendicular RF being equal to all the lines *k*B, *p*C, *g*D, &c. will be equal to the departure. And ſo theſe three lines make the right angled triangle ARF.

Cor. 7. *Hence, the ſquare of the diſtance run, is equal to the ſum of the ſquares of the departure and difference of latitude.*

Cor. 8. *This Prop. and Corollaries will reſolve all the caſes of plane ſailing, being thoſe where no longitude is concerned.*

There are ſix caſes of plane ſailing, and they are all ſolved by working theſe proportions forward or backward, according to the different data.

Fig. 14.

SCHOLIUM.

Having made ſome remarks, in my book of Navigation, on the nature of plane ſailing; I ſhall here conſider it more particularly. And I ſay,

13. 1. That meridional diſtance, departure, and difference of longitude, are all eſſentially different. Let a ſhip ſail from A to F; when ſhe comes at F, ſhe will have made the meridional diſtance RF, and departure $kB + pC + gD$, &c. and difference of longitude HL. Now the meridional diſtance RF or $Rm + mn + no$, &c. is leſs than the departure $kP + pC + gD$, &c. and this departure is leſs than the longitude HL or $Ht + ts + fr$, &c. All this is plain from the converging of the meridians towards the pole. Again, if a ſhip ſails from F to A; ſhe makes the meridional diſtance Az, which is greater than the former RF; and is likewiſe greater than the departure $kB + pC + gD$, &c. but leſs ſtill than the difference of longitude. Here then in the ſame courſe back and forward, the departure and difference of longitude continue the ſame, but the meridional diſtance is different, being leſs towards the pole, and greater towards the equinoctial.

2. If a plain chart be conſtructed, ſo that the meridians be parallel to one another, and the degrees of latitude every where equal, and alſo equal to the degrees of longitude at the equator; then if the way of a ſhip be laid down upon ſuch a chart, according to the courſe and diſtance, the departure and difference of latitude will be truly found, as to their magnitude; but the place of the ſhip will have a wrong ſituation in reſpect of the meridians, by reaſon of the paralleliſm of the meridians, that ought to converge to the pole. And hence any traverſe or compound courſe laid down upon ſuch a chart will give all the places of the ſhip

in

Fig. 14. in a wrong ſituation, in reſpect of the meridians; and conſequently in reſpect to one another. For that reaſon the plane chart is erroneous, except in places near the equator, where the meridians are nearly parallel. Yet in a ſmall compaſs, as in a day's run, the error will not be ſenſible in any latitude. For in ſo ſmall a compaſs, the meridians continue parallel as to ſenſe; and therefore a traverſe may be wrought exact enough for the length of 24 hours, as is the common practice. But the error of the plane chart does not affect the preſent propoſition, for this will give the departure truly, for any courſe tho' never ſo long.

14. 3. Hence it follows, that the triangle ARF in this chart will be ſuch, that if AR be the true difference of latitude, AF will be the true diſtance (ADF) upon the globe, RAF the true courſe (*k*AB,) and RF the true departure; and therefore this conſtruction is true, and gives us a true way of ſailing. It is no objection to ſay, that it does not give the longitude truly, for that is inconſiſtent with its nature. A man may as well ſay that Mercator's projection is falſe, becauſe it does not give the diſtance or difference of latitude truly. For by its nature and conſtruction it is only to give the longitude truly. In ſhort, both conſtructions are true, according to their ſeveral natures. Plain ſailing is true ſo far as courſe, diſtance, difference of latitude and departure are concerned; and Mercator's is true ſo far as the longitude is concerned; but one of them cannot anſwer for the other.

13. 4. What has been ſaid relates to a ſingle courſe as ADF, whoſe angle of the courſe is *k*AB. But inſtead of ſailing directly from A to F, if a ſhip ſhould ſail to R and then to F; the departure along AR would be nothing; and along RF, would be equal to RF; for in any parallel the diſtance and departure are the ſame. And conſequently

Fig. the departure in theſe two courſes, is leſs than in
13. the ſingle courſe AF. Likewiſe a ſhip ſailing back from F to A, will make the ſame departure as from A to F; and from F to R and A, the ſame as from A to R and F. But if ſhe ſails from F to z, and then to A, ſhe makes no departure along Fz; and from z to A, ſhe makes the departure zA, which is greater than from F to A. Therefore a ſhip running upon ſeveral courſes, makes a leſs departure when nearer the pole, and a greater when nearer the equinoctial, than in a direct courſe to the ſame place. But in ſmall diſtances, as a day's run, the difference will be inſenſible.

5. Since the departure in the courſe AF, is greater than the meridional diſtance RF, and leſſer than AL; it will be very near a mean between them; and therefore the departure will be very near the meridional diſtance xw, lying in the middle latitude between A and F.

6. The departure in ſome of the caſes, is the moſt proper and natural medium, for finding the longitude. And therefore I cannot but be ſurpriſed at thoſe authors that exclude it from the practice of navigation, at the ſame time they are forced to make uſe of it, or what is equivalent to it, for the ſame purpoſe. For inſtead of RF, they
14. do not ſcruple to take the $\sqrt{AF^2-AR^2}$, as if that was not the ſame thing; when it might eaſily be had by a ſimple proportion in this propoſition. And they are ſometimes forced to uſe the enlarged diſtance upon Mercator's chart; a line which there is no manner of occaſion for, but for ſuch a ſhift; by which means, they give us the moſt clumſy ſolutions imaginable. Such is the unaccountable prejudice of ſome people. But further,

13. 7. I ſay, this Prop. is true, not only in the ſphere, but in the ſpheroid, or on the ſurface of any ſolid, generated by revolving round an axis, and even up-

on a plane or cylindric ſurface. For ſince all the angles kAB, pBC, gCD, &c. are equal, by the nature of the rumb; the elementary triangles are all ſimilar and equal; and therefore the ſame concluſions will follow as before, viz. that rad : diſtance run : : ſ. courſe : departure : : coſ. courſe : diff. latitude. And rad : diff. latitude : : tan. courſe : departure, whatever ſort of a ſolid it is. And the diſtance, diff. latitude, and departure will make a right angled triangle as before. So far is this Prop. from being falſe, that it is univerſally true, whatever ſort of a line the meridian is; and therefore it can never lead into any error. I ſhall only add, that in a ſingle day's run, where the latitude is little varied, the (ſum or difference) of all the departures made by different courſes, in that time, will be the ſame as the ſingle departure, made between the firſt and laſt place of the ſhip; which will come to the ſame as the meridional diſtance, for that latitude. And this is the very practice of all ſeamen; to find the eaſtings and weſtings for every ſeparate courſe, and then take the total amount (that is the ſum or difference) for the departure. A method ſo eaſy, natural, and expeditious, that it is impoſſible to invent any thing that can in this reſpect exceed it; and therefore the Navigator need make no ſcruple of continuing his practice. And ſo much for plane ſailing. Fig. 13.

PROP. II.

In parallel or eaſt and weſt ſailing; coſ. latitude : radius : : diſtance of two places in any parallel : difference of longitude.

Let RF be the diſtance in that parallel, then HL will be the difference of longitude. The coſine of latitude is the radius of the parallel, and 13.

Fig. 13. the radius of the equinoctial is the radius of the ſphere; and the radii of all circles are as any ſimilar parts of the circumferences, as a degree, &c. Therefore *coſ. latitude at* R : *radius* : : *diſtance* RF : *diſtance* HL *on the equinoctial*, which is the difference of longitude. This has likewiſe been demonſtrated in Prop. 15, Sect. I.

Cor. 1. *The length of a degree in any parallel, is as the coſine of latitude.*

Cor. 2. *This Prop. will ſolve all the caſes of parallel ſailing, which are 3; by working the proportion backwards or forwards, as occaſion requires.*

PROP. III.

By middle latitude ſailing, as coſ. middle latitude : radius : : departure : difference of longitude.

Suppoſe a ſhip ſails from A to F, making the difference of longitude HL. Then ſince the departure $kB + pC + gD$, &c. is greater than the meridional diſtance RF, and leſs than the meridional diſtance Az; and if x be taken in the middle between A and R, then xw, which is a mean between RF and Az, is nearly equal to the departure, or the ſum of the lines $kB + pC + gD$, &c. therefore if xw be taken for the departure; then by the laſt Prop it is, as coſ. of the lat. of x : radius : : ſo is xw : HL. That is, as coſ. middle latitude : radius : : departure : difference of longitude.

Cor. 1. *As coſ. middle latitude : ſ. courſe : : diſtance run : difference of longitude.*

For coſ. middle lat. : rad : : departure : dif. long. and (Pr. I.) rad : diſt. : : ſ. courſe : departure. Therefore coſ. mid. lat. : diſt. : : ſ. courſe : dif. long.

Cor. 2. *Coſine of middle latitude : tan. courſe : : difference of latitude : difference of longitude.* Fig. 13.

For coſ. mid. lat. : rad : : departure : dif. longitude, and (Cor. 2. I.) rad : dif. latitude : : tan. courſe : departure.

Therefore,

Coſ. mid. lat. : dif. lat. : : tan. courſe : dif. long.

Cor. 3. *This Prop. and Corollaries will reſolve all the caſes of ſailing, where the longitude is concerned, and very near the truth, if the courſe be not too long.*

SCHOLIUM.

This method of ſailing is very ſhort and eaſy, and alſo very exact in ſhort courſes; and does not differ much in long courſes, and therefore is a very uſeful approximation. And I believe I ſhall never live to ſee a ſhip ſail to that exactneſs, and this Prop. can determine it. And as this method is ſo eaſy, and performed without any other tables than the common ones; it is ſtrange that any perſon ſhould condemn or depreciate it. Nothing will go down with ſuch people, but rules formed to the utmoſt accuracy, as if any ſhip could ſail by ſuch rules. Sailing or guiding a ſhip at ſea, is but a mechanical art, and cannot be executed to any great degree of exactneſs. And therefore it is needleſs to inſiſt upon rules made to the utmoſt geometrical rigour; and the difficulty that attends the computation of a ſhip's way by ſuch rules, is ſufficient to cauſe them to be laid aſide, when eaſier will do as well. But when navigation is ſo far improved, as that a ſhip can ſail to any degree of exactneſs, it will be time then to ſeek out ſuch rules as are perfect and accurate, and adequate to that end.

PROP. IV.

The length of a minute on the ſphere : is to the length of a minute on the meridian of Mercator's chart, for any latitude : : as the coſ. of latitude : to radius.

15. For ſince the rumb cuts all the meridians at the ſame angle, in order to have the rumb a right line, all the meridians muſt be parallel; whence every coſine will become equal to the radius. Therefore every particle on the ſphere, muſt be increaſed proportionally, that is, as the coſine of the latitude of that point, is to the radius. Therefore ſince the radius of the parallel is made equal to the radius of the ſphere, a minute on the globe muſt alſo be increaſed in that proportion. And therefore as coſ. lat : rad : : ſo a minute on the globe, to a minute in the meridian line.

Cor. 1. *From hence, by the method of fluxions, may be calculated the length of the meridian line in the chart, or the meridional parts for any latitude.*

Let AC be the radius of the equinoctial, AB the latitude, draw BD, CG perp. to AC, and BE parallel to it, and let Bb be a minute; draw bn parallel to BD. And put CA $= r$, arch AB $= v$, DB $= y$, BE $= x$, and $z =$ length of the meridian line for the lat. AB. The triangles BDC and Bnb are ſimilar; whence CD (x) : CB (r) : : bn : Bb $= \frac{r}{x} \times bn$. And by this Prop. BE (x) : AC (r) : : Bb $\left(\frac{r}{x} \times bn\right)$: a minute of the meridian line in the chart $= \frac{rr}{xx} \times bn$. Now, if for theſe ſmall lines we put the fluxions, we ſhall have

$\dot{z} = \frac{rr}{xx}\dot{y} = \frac{rr\dot{y}}{rr - yy}$; and the fluent is $z =$ Fig. 15.
$\frac{2.302585r}{2} \times \text{Log.} : \frac{r+y}{r-y}$.

Cor. 2. *Therefore* $z = 2.302585r \times$ *into log.: cotangent of half the complement of the latitude — log.: radius* (10).

For $z = \frac{2.302}{2} r \times \log. : \frac{r+y}{r-y} = 2.302r \times \log. : \sqrt{\frac{r+y}{r-y}}$. But (by Schol. Prop. II. Trigon.) $r\sqrt{\frac{r+y}{r-y}} =$ cot. ½arch, that y is the coſine of $=$ cot. ½BG $=$ cotang. half the complement of the latitude. Whence $z = 2.302r \times \log. \frac{\text{cotan. } \frac{1}{2}\text{com. lat.}}{r}$

Cor. 3. *Since the meridional parts are expreſſed in minutes, we ſhall have* $z = 7915.7 \times$ *log.* $\frac{\text{rad.}}{\text{tan. } \frac{1}{2}\text{co. lat.}}$

For $\frac{2.30258 \times 180 \times 60}{3.14159} = 7915.705$, and $\frac{\text{cot. } \frac{1}{2} \text{ comp. lat.}}{r} = \frac{r}{\text{tan. } \frac{1}{2} \text{ com. lat.}}$, by trigonometry.

PROP. V.

In Mercator's ſailing, as proper difference of latitude : to meridional difference of latitude : : ſo the departure : to the difference of longitude.

Let A, F be two places on the globe, ADF the rumb leading from one to the other. And if the difference of latitude AR be divided into an infinite number of equal parts, and the parallels of latitude drawn thro' the points of diviſion; there 13.
will

Fig. 13, will be formed an infinite number of triangles *k*AB, *p*BC, *g*CD, &c. which have been proved (Prop. I.) to be ſimilar and equal. And upon Mercator's chart, theſe triangles will be projected into others ſimilar to them. For by the laſt Prop. A*k* will be to its length on the chart, as coſ. HA to radius. And ſince the radius of the parallel at A, is increaſed to the radius of the ſphere; any part of the circumference will be increaſed in the ſame proportion; therefore *k*B, will be to its length on the chart, as coſ. HA or H*k* to radius. And therefore the triangle *k*AB will be ſimilar to its projection on the chart; and all the triangles on the globe will be ſimilar to thoſe on the chart. And therefore it will be as A*k* to *k*B, ſo the ſum of all the ſides A*k*, B*p*, C*g*, &c. to the ſum of all the ſides *k*B, *p*C, *g*D, &c. upon the globe; and ſo is the ſum of all the ſides A*k*, B*p*, C*g*, &c. to the ſum of all the ſides *k*B, *p*C, *g*D, &c. on the chart. But the ſum of A*k*, B*p*, C*g*, &c. on the globe is the proper difference of latitude. And the ſum of *k*B, *p*C, *g*D, &c. is the departure. Alſo the ſum of A*k*, B*p*, C*g*, &c. on the chart, is the meridional difference of latitude; laſtly, the ſum of *k*B, *p*C, *g*D, &c. on the chart is the difference of longitude, ſince any one *k*B is projected into its dif. longitude. Whence this proportion follows, as proper difference of latitude : departure : : ſo meridional difference of latitude : to difference of longitude, of the places A and F.

Cor. 1. *As radius : to tangent of the courſe : : ſo meridional difference of latitude : to difference of longitude.*

For (Cor. 2. Prop. I.) rad : tan. courſe : : diff. latitude : : to the departure : : (by this) meridional diff. latitude : diff. longitude.

Cor. 2. *As the number, .00012633 : to the tangent of the courſe : : ſo the diff. of log : tangents of half the complements of the latitudes : to the difference of longitude.* Fig. 13.

For (Prop. IV. Cor. 3.) the mer. parts for one lat. L is 7915 × log : $\frac{\text{rad.}}{\text{tan. } \frac{1}{2} \text{ co. lat. L}}$; and the mer. parts for the other lat. *l* is 7915 × log. : $\frac{\text{rad.}}{\text{tan. } \frac{1}{2} \text{ co. lat. } l}$. Therefore the meridional dif. latitude = 7915 × diff. log. tangents of half the complements of the latitudes L and *l*; put this = 7915 × D. Then we have (by Cor. 1.) rad : tan. courſe : : 7915 D : diff. longitude. And dividing by 7915, $\frac{\text{rad.}}{7915}$: tan. courſe : : D : diff. longitude, but $\frac{\text{rad.} = 1}{7915.7}$ = .00012633.

Cor. 3. *As in plain ſailing, the rumb, the difference of latitude, and departure make a right angled triangle; ſo in the chart, the rumb, the meridional difference of latitude, and difference of longitude, alſo make a right angled triangle, ſimilar to the former. Thus, in the triangle* ADE, *if* AB *is the diff. lat.* BC *the departure,* AC *the diſtance. Then* AD *is the mer. dif. lat.* DE *the diff. longitude.* 17.

Cor. 4. *This Prop. and Cor. 1. will reſolve all the caſes of Mercator's ſailing, or where the longitude is concerned, with the help of a table of meridional parts.*

For by the table of meridional parts, the meridional difference of latitude is had, which is only the difference of the meridional parts belonging to the two latitudes. And that table is conſtructed by Cor. 3. Prop. IV.

Fig. Cor. 5. *By Cor. 3. and Prop. I. all the caſes of ſailing may alſo be reſolved.*

SCHOLIUM.

13. Altho' all the caſes of ſailing may be ſolved by help of Mercator's chart, yet this chart is not a true repreſentation of the earth. For though every ſingle particle, or elementary triangle on the chart is ſimilar to its correſpondent part on the ſphere, yet the whole is not ſimilar to the whole; for the parts of the chart being bigger than thoſe in the ſphere, yet they do not increaſe in the ſame proportion; being nearly equal at the equator, and infinitely greater at the poles; and at the intermediate places, in all imaginable proportions. So that the whole cannot be ſimilar to the whole, but very much diſtorted. Nor is it poſſible that any projection upon a plain can be ſimilar to a curve ſurface, ſuch as a ſphere or a ſpheroid.

The actual ſolution of the ſeveral caſes of ſailing are eaſily drawn from the foregoing propoſitions, and I ſhall not trouble the reader with them here, but refer him to the book of Navigation. But the three following problems, not being common, I ſhall here give their ſolutions.

PROP. VI. *Prob.*

One latitude, departure, and difference of longitude being given; to find the other latitude.

By Prop. III. ſay,

As diff. longitude :
to departure : :
So radius :
to coſine of middle latitude.

The difference between the middle latitude and the given latitude, added to, or ſubtracted from, the middle latitude, as the caſe requires, gives the other latitude. Fig. 13.

If this latitude found, be not thought exact enough, it may be corrected by Mercator. For the latitude found muſt be ſuch, that the proper diff. latitude, may be to the meridional diff. latitude; in the given ratio of the departure to the difference of longitude; and is eaſily found in a few trials. Or it may be found after two trials by the rule of falſe.

PROP. VII. *Prob.*

One latitude, diſtance, and difference of longitude being given; to find the other latitude.

1. Aſſume any angle for the courſe, as near as you can gueſs, by the circumſtances of the queſtion. From this, find the difference of latitude, by Cor. 1. Prop. I. making,

As radius :
to the diſtance : :
So the coſine of the courſe :
to the diff. latitude.

2. Both latitudes being had, find the longitude by Cor. 1. Prop. V. thus,

As radius :
to the tangent of the courſe : :
So the mer. diff. latitude :
to difference of longitude.

3. If the longitude found, differs from the longitude given, as it generally will; correct the courſe, by making it greater or leſs as there is occaſion; then find the latitude, and then the longitude again. Repeat the operation till the longitude agrees with that given. But when you come near the matter, you may find the courſe truly from two trials, by the rule of falſe.

Or

Fig. 13.

Or thus,

1. Say as rad : coſ. given latitude : : diff. longitude : to the departure.
2. Find the ſum and difference of the departure and diſtance ; take the logarithms of the ſum and difference, and add them together ; half the ſum of theſe logarithms is the log : of the diff. of latitude. Then find the other latitude. Again (by Prop. V.)
3. Say, as meridional diff. latitude : proper diff. latitude : : diff. longitude : departure, more exact.

Repeat the 2d and 3d articles over and over, always with the laſt departure and diff. latitude ; till at laſt you get the ſame diff. latitude twice ; and this is the true diff. of latitude, whence you will get the other latitude exact.

PROP. VIII. *Prob.*

The courſe, departure, and difference of longitude, being given ; to find both latitudes.

Find the diff. latitude by Cor. 2. Prop. I. and the meridional diff. latitude by Cor. 1. Prop. V. thus,

As tan. courſe :
to radius : :
So the departure :
to the difference of latitude.
And ſo the diff. longitude :
to the meridional diff. latitude.

The difference of latitude being had in degrees and minutes, ſeek in the table of meridional parts, for two latitudes, ſuch as may exceed one another by that difference ; and that ſubtracting the meridional parts of one from the other, the remainder may be equal to the meridional diff. latitude before found: Fig. which is eaſily done in a few trials. And theſe are the two latitudes.

If inſtead of the departure, the diſtance or diff. latitude be given; the ſolution is the ſame, after the diff. latitude is known.

PROP. IX.

Let ABP be a quadrant of the ſphere, AHF a 16. quadrant of an ellipſis repreſenting the ſpheroidical figure of the earth, AC the equinoctial; P, F, the poles; BE, HI, parallels of latitude of the places B, H; BD, HL perpendiculars upon AC. And put AC $= r$, FC $= a$, AB $= v$, LH $= y$, CL or HI $= x$, $rr - aa = dd$, s, c, t, $ſ =$ ſine, coſine, tangent, ſecant of the latitude of any place B or H, radius $= 1$. Then,

The radius of a parallel in the ſphere BE : *is to the radius of the parallel, having the ſame latitude, in the ſpheroid,* HI : : *as* $\sqrt{1 - \frac{dd}{rr}ss}$: *to* 1.

If two places upon the ſphere and ſpheroid B and H, have the ſame latitude; then the tangents at B and H will be parallel; for they muſt point to the ſame ſtar. And the ordinate LH in the ſpheroid, will be nearer the equinoctial A, than the ordinate BD in the ſphere. For by the conic ſections, if both tangents be drawn from points in the ſame line BD or LH, they will meet in ſome point of the axis CA. And conſequently, that they may be parallel, the point H muſt be nearer A.

Suppoſe the tangents from H and B to meet the axis CA, in T and S; then the triangles CBD, DBS, LHT will be ſimilar. And by nature of the ellipſis LH or $y = \frac{a}{r}\sqrt{rr - xx}$, and DB $= sr$,

Fig. 16. BE $= cr$. Alſo CT $= \frac{rr}{x}$, and LT $= \frac{rr - xx}{x}$. Whence by ſimilar triangles, BE or CD (cr) : DB (sr) : : DB : DS : : LH $\left(\frac{a}{r}\sqrt{rr - xx}\right)$: LT $\left(\frac{rr - xx}{x}\right)$: : $ax : r\sqrt{rr - xx}$. Whence $sax = cr\sqrt{rr - xx}$, and $ssaaxx = ccr^4 - ccrrxx$. Therefore $x = \frac{crr}{\sqrt{ccrr + aass}} =$ HI. Therefore BE : HI : : $cr : \frac{crr}{\sqrt{ccrr + aass}}$: : $\sqrt{ccrr + qass} : r$: : $\sqrt{rr - rrss + aass} : r$: : $\sqrt{rr - ddss} : r$: : $\sqrt{1 - \frac{dd}{rr}ss} : 1$.

Cor. 1. *Hence* HI $=$ BE $+ 1 + \frac{1}{2}qss$, *putting* $= q = \frac{dd}{rr}$. *And therefore the radii of the parallels for the ſpheroid are eaſily calculated.*

For HI $= \frac{\text{BE}}{\sqrt{1 - qss}} = \frac{\text{BE}}{1 - \frac{1}{2}qss} = \text{BE} \times \overline{1 + \frac{1}{2}qss}$, nearly.

Cor. 2. HI or $x = \frac{crr}{\sqrt{aa + ddcc}} = \frac{rr}{\sqrt{rr + aatt}}$.

For $x = \frac{crr}{\sqrt{ccrr + aass}} = \frac{crx}{\sqrt{ccrr + aa - aacc}}$, becauſe $ss = 1 - cc$. Alſo $t = \frac{s}{c}$, whence $x = \frac{crr}{\sqrt{ccrr + aass}} = \frac{rr}{\sqrt{rr + \frac{aass}{cc}}} = \frac{rr}{\sqrt{rr + aatt}}$.

Fig. 16.

Cor. 3. LH or $y = \frac{aat}{\sqrt{rr + aatt}}$.

For $yy = aa - \frac{aa}{rr} \times xx = aa - \frac{aa}{rr} \times \frac{r^4}{rr + aatt}$ $= aa - \frac{aarr}{rr + aatt} = \frac{a^4tt}{rr + aatt}$.

Cor. 4. HI *or* $x = \frac{rc}{1 - \frac{1}{2}qss} = rc \times \overline{1 + \frac{1}{2}qss}$, *nearly. Where* $q = \frac{rr - aa}{rr} = \frac{dd}{rr}$.

For $x = \frac{rrc}{\sqrt{rrcc + aass}} = \frac{rrc}{\sqrt{rr - rrss + aass}}$ $= \frac{rc}{\sqrt{1 - qss}} = \frac{rc}{1 - \frac{1}{2}qss} = rc \times \overline{1 + \frac{1}{2}qss}$, taking the two firſt terms of the ſeries.

Cor. 5. *In the ſame latitude* B *and* H, DB : *is to* LH : : *as* $r\sqrt{rr - ddss}$: *to* aa.

For DB $= rs$, and LH $= \frac{a}{r}\sqrt{rr - xx}$, where $xx = \frac{ccr^4}{ccrr + aass}$.

PROP. X.

The ſame things ſuppoſed as in the laſt Prop. The length of a minute on the ſpheroid at H : *is to the length of a minute on the chart for that latitude* : : *as the radius of the parallel* HI : *to the radius of the equinoctial* AC.

For ſince the meridians in the chart, are all parallel, the radii of all the parallels become equal to the radius of the equator; and every particle on the ſphere being projected into a ſimilar figure in

Fig. 16. the chart, it is increaſed every way in that proportion. Therefore HI : AC : : as a minute at H : to a minute on the chart.

Cor. 1. *Hence a minute on the ſpheroid : is to its projection on the chart* : : 1 : $\sqrt{1 + \frac{aa}{rr}tt}$.

For it will be as x or $\frac{rr}{\sqrt{rr + aatt}}$ to r, that is, as r to $\sqrt{rr + aatt}$, or as 1 to $\sqrt{1 + \frac{aa}{rr}tt}$.

Cor. 2. *A minute on the ſpheroid : is to a minute on the chart* : : $rc : a\sqrt{1 + \frac{dd}{aa}cc}$.

For it is as x to r, or as $\frac{crr}{\sqrt{aa + ddcc}}$ to r, that is, as cr to $\sqrt{aa + ddcc}$.

PROP. XI.

The ſame things ſuppoſed as in Prop. IX; *the length of a minute on the ſphere; is to the length of a minute on the ſpheroid at* H : : *as* rra : to $\overline{aa + \frac{dd}{aa}yy}\big|^{\frac{3}{2}}$.

Theſe lengths are as the radii of curvature; and this radius in the ſphere is r, and in the ſpheroid is $\frac{\overline{aabb + 4a^2y^2 - 4aby^2}\big|^{\frac{3}{2}}}{2bba^3}$ (by Prob. 5. ex. 2. Sect. II. Fluxions), and putting $2r$ for a, and $\frac{2aa}{r}$ for b the latus rectum, it is reduced to $\frac{\overline{aa + \frac{rr - aa}{aa}yy}\big|^{\frac{3}{2}}}{ar}$.

Therefore a minute in the circle : a minute in the ellipſis : : $r : \frac{\overline{aa + \frac{dd}{aa}yy}\big|^{\frac{3}{2}}}{ar} : : arr : \overline{aa + \frac{dd}{aa}yy}\big|^{\frac{3}{2}}$. Fig. 16.

Cor. 1. *A minute on the ſphere : is to a minute on the ſpheroid at* H : : *as* $a \times \overline{1 + \frac{dd}{aa}cc}\big|^{\frac{3}{2}}$: *to* r.

For $yy = aa - \frac{aa}{rr}xx = aa - \frac{aa}{rr} \times \frac{r^4cc}{aa + ddcc} = aa - \frac{rraacc}{aa + ddcc} = \frac{a^4 + aaddcc - rraacc}{aa + ddcc} = \frac{aa - \overline{rr - dd}.cc}{1 + \frac{dd}{aa}cc} = \frac{aa - aacc}{1 + \frac{dd}{aa}cc} = \frac{aass}{1 + \frac{dd}{aa}cc}$. And $\frac{dd}{aa}yy = \frac{ddss}{1 + \frac{dd}{aa}cc}$. Alſo $aa + \frac{dd}{aa}yy = aa + \frac{ddss}{1 + \frac{dd}{aa}cc} = \frac{aa + ddcc + ddss}{1 + \frac{dd}{aa}cc} =$ (becauſe $cc + ss = 1$) $\frac{aa + dd}{1 + \frac{dd}{aa}cc} = \frac{rr}{1 + \frac{dd}{aa}cc}$. And $arr : \overline{aa + \frac{dd}{aa}yy}\big|^{\frac{3}{2}} : :$ $arr : \frac{r^3}{\overline{1 + \frac{dd}{aa}cc}\big|^{2}} : : a \times \overline{1 + \frac{dd}{aa}cc}\big|^{\frac{3}{2}} : r$.

Cor. 2. *If* $K = \sqrt[3]{\frac{aa}{rr}}$, *then* $c = \sqrt{\frac{aa}{dd} \times \overline{K - 1}}$, *the coſine of the latitude, where a minute of the meridian of the ſpheroid, is equal to a minute of the equinoctial.*

For then $\overline{1 + \frac{dd}{aa}cc}\big|^{\frac{3}{2}} = \frac{r}{a}$, and $\overline{1 + \frac{dd}{aa}cc}^{3} = \frac{rr}{aa}$, and $1 + \frac{dd}{aa}cc = \sqrt[3]{\frac{rr}{aa}} = K$, and $\frac{dd}{aa}cc =$ K—

Fig. 16. $\overline{K-1}$, whence $cc = \frac{aa}{dd} \times \overline{K-1}$.

Cor. 3. *If* v *and* V *be the length of the meridian of the ſphere and ſpheroid, at the ſame latitude,* $q = \frac{dd}{aa}$. *Then* $V = \frac{r - \frac{3}{4}rq}{a}v - \frac{\frac{3}{8}rqS}{a}$; *where* S *is the ſine of twice the latitude.*

For let $p = \frac{dd}{aa}$, then $a \times \overline{1 + pcc}^{\frac{3}{2}} : r :: \dot{v} : \dot{V}$ $= \frac{r\dot{v}}{a \times \overline{1+pcc}|^{\frac{3}{2}}} = \frac{r\dot{v}}{a \times \overline{1 + \frac{3}{2}pcc}} = \frac{r\dot{v}}{a} \times \overline{1 - \frac{3}{2}pcc + \&c.}$ = (becauſe q is ſomething leſs than p) $\frac{r\dot{v}}{a} - \frac{3rqcc}{2a}\dot{v}$. Whence $V = \frac{rv}{a} - \frac{3rq}{2a}$ × fluent of $cc\dot{v}$, or ſum of all the cc in the arch.

But ſuppoſing the radius = 1 (fig. 15) by ſimilar triangles, $\dot{v}$ (Bb) : $\dot{s}$ (nb) :: CB : BE :: 1 : c, and $c\dot{v} = \dot{s}$, and $cc\dot{v} = c\dot{s}$. And the Flu : $c\dot{s}$ (or the ſum of all the BE × nb) is = area ABEC = Fl : $cc\dot{v}$, and the area ABEC $= \frac{v}{2} + \frac{cs}{2}$. Therefore

$$V = \frac{rv}{a} - \frac{3rq}{2a} \times \overline{\frac{v}{2} + \frac{cs}{2}} = \frac{r - \frac{3}{4}rq}{a}v - \frac{3rq}{4a}cs$$

$$= \frac{r - \frac{3}{4}rq}{a}v - \frac{\frac{3}{8}rq}{a}S;$$ becauſe $cs = \frac{1}{2}S$.

Cor. 4. *If* v *and* V *be the meridians for the ſame latitude, in the ſphere and ſpheroid, in minutes of the equator; then* $V = \frac{r - \frac{3}{4}rq}{a}v - \frac{1289rq}{a}S$.

For if the lengths V and v be reduced to minutes, they muſt be multiplied by $\frac{180 \times 60}{3.1416}$ or 3438, which is the minutes in the radius. But V and

Fig. 16. and v, now denoting minutes, are already reduced; and reducing $\frac{3rq}{8a}$ S, we ſhall have $V = \frac{r - \frac{3}{4}rq}{a}v - \frac{\frac{3}{8}rq \times 3438}{a} S = \frac{r - \frac{3}{4}rq}{a}v - \frac{rq}{a} \times 1289\frac{1}{4} S$.

And hence it will be eaſy to make a table of the arches of the ſpheroid in minutes, exact enough for the earth. For if the ſpecies of the earth's ſpheroid be known, q is known; and then $\frac{r - \frac{3}{4}rq}{a}$ and $\frac{1289rq}{a}$ will be given quantities; whoſe logarithms, added to the logarithms of v and S, will give two numbers, whoſe difference is the ſpheroidal meridian. Thus if $q = .022$ according to *Maupertuis*, then $V = .9945v - 28.7\ S$; or $V = v - .0055v - 28.7\ S$, where $.0055v + 28.7\ S$ is the reduction, or diminution of the ſpherical arch.

PROP. XII.

The ſame things being ſuppoſed, as in Prop. IX. *a minute on the ſpherical chart : is to a minute on the ſpheroidal chart, at the ſame latitude : : as* $1 + \frac{dd}{aa}cc$ *: to* 1.

For by Prop. IV.

$$\left.\begin{array}{l}\text{a minute on the}\\ \text{ſpherical chart}\end{array}\right\} : \left.\begin{array}{c}\text{a minute on}\\ \text{the ſphere}\end{array}\right\} :: 1 : C.$$

And by Cor. 1. Prop. XI.

$$\left.\begin{array}{c}\text{a minute on}\\ \text{the ſphere}\end{array}\right\} : \left.\begin{array}{c}\text{a minute on}\\ \text{the ſpheroid}\end{array}\right\} :: a \times \overline{1 + \frac{dd}{aa}cc}\Big|^{\frac{3}{2}} : r.$$

Fig. 16. Alſo by Cor. 2. Prop. X.

$$\left.\begin{array}{l}\text{a minute on}\\ \text{the ſpheroid}\end{array}\right\} : \left.\begin{array}{l}\text{a minute on the}\\ \text{ſpheroidal chart}\end{array}\right\} :: rc : a\sqrt{1+\frac{dd}{aa}cc}.$$

Therefore ex equò,

$$\left.\begin{array}{l}\text{a minute on the}\\ \text{ſpherical chart}\end{array}\right\} : \left.\begin{array}{l}\text{a minute on the}\\ \text{ſpheroidal chart}\end{array}\right\} :: arc \times \overline{1+\frac{dd}{aa}cc}\Big|^{\frac{3}{2}} : arc\sqrt{1+\frac{dd}{aa}cc} :: 1+\frac{dd}{aa}cc : 1.$$

Cor. 1. *If z and Z be the length of the meridians on the chart, for the ſphere and ſpheroid,* $p = \frac{dd}{aa}$. *Then* $Z = z - qs$.

For (Prop. IV.) $c : 1 :: \dot{v} : \dot{z} = \frac{\dot{v}}{c}$. And (fig. 15) by ſimilar triangles, $c : 1 :: BE : BC :: nb : Bb = \frac{nb}{c}$, or $\dot{v} = \frac{\dot{s}}{c}$; therefore $\dot{z} = \frac{\dot{s}}{cc}$. But by this Prop. (putting $p = \frac{dd}{aa}$) $1 + pcc : 1 :: \dot{z} : \dot{Z} = \frac{\dot{z}}{1+pcc} = \dot{z} \times \overline{1 - pcc + \&c.} = \dot{z} - qcc\dot{z} = \dot{z} - qcc \times \frac{\dot{s}}{cc} = \dot{z} - q\dot{s}$. Whence $Z = z - qs$.

Cor. 2. *If z and Z be the meridional parts for the ſphere and ſpheroid, at the ſame latitude. Then* $Z = z - 3438qs$.

For to reduce the length z to minutes of the equator, it muſt be multiplied by $\frac{180 \times 60}{3.1416}$, or 3438. But Z and z are ſuppoſed to be already ſo reduced, and reducing qs, we have $Z = z - 3438qs$; and therefore it will be extremely eaſy to make a table of meridional parts for the ſpheroid, exact enough for the earth. For if the ſpecies of the Fig. ſpheroid be once determined, q is known; and 16. $3438q$ will be a given quantity; to the log. of which adding the log. ſine of the latitude (rejecting radius), and you have the log. of a number, which is the reduction or decreaſe of the meridional parts of the ſphere. Thus if $q = .022$, according to *Maupertuis*, then log : $3438q = 1.8787286$.

SCHOLIUM.

When three tables are made for the ſpheroid; one for the radii of the parallels, by Prop. IX. or Cor. 1. A table of the arches of the meridian, by Cor. 4. Prop. XI, and a table of meridional parts, by Cor. 3. Prop. XII. then all the caſes of ſailing may be ſolved, with little more trouble, by the ſame rules, as for the ſphere; that is, by Prop. II. and V. and their Corrollaries. But for ſuch as have not theſe tables, I ſhall lay down the two following propoſitions, by which the common caſes may be ſolved by the common tables only; if any body will be at the pains to work them, as they require ſomething more labour.

PROP. XIII.

Let v = *any latitude in minutes*, S = *nat. ſine of* 17. *twice this latitude. And put* $F = \frac{r - \frac{3}{4}rq}{a}v$, $G = \frac{1289rq}{a}S$; *and* f *and* g *the ſame for any other latitude.*

Alſo let M = *meridional parts of the ſphere, for that latitude, and* $N = 3438q \times$ *nat. ſine of the lat. and* m *and* n *the ſame for any other lat. Where* r = *radius of the equinoctial,* $a = \frac{1}{2}$ *axis, and* $q = \frac{rr - aa}{rr}$.

Fig. *Then on the ſpheroid, it will be,*

17. *As proper dif. latitude.* $\overline{F-G}-\overline{f-g}$:
mer. diff. latitude, $\overline{M-N}-\overline{m-n}$: :
departure :
diff. longitude,

on the ſame ſide of the equinoctial; but on different ſides, take $\overline{F-G}+\overline{f-g}$, *and* $\overline{M-N}+\overline{m-n}$.

For (by Prop. XI. Cor. 3.) the proper lat^s. on the ſpheroid are $= F - G$, and $f - g$. And (by Cor. 2. Prop. XII.) mer. parts on the ſpheroid is $z - 3438qs$ or M—N. And the dif. of the meridional parts for the two latitudes, or mer. dif. latitude, will be $\overline{M-N}-\overline{m-n}$. And on the ſpheroid it is, as proper dif. lat : mer. dif. lat : : departure : diff. longitude; or as AB : AD : : BC : DE.

Cor. 1. *Radius :*
tan. courſe : :
meridian dif. latitude, $\overline{M-N}-\overline{m-n}$:
dif. longitude.

For rad : tan. courſe : : proper dif. latitude : departure : : mer. dif. lat : diff. longitude.

Cor. 2. *The reduction for the lat.* v *is* $\frac{5\frac{1}{2}}{1000}v + 28.7S$. *And the reduction for the meridional parts of* v, *will be* N.

This follows from Cor. 4. Prop. XI. and Cor. 2. Prop. XII.

PROP. XIV.

The ſame things ſuppoſed, and put $s = S$. *latitude, Then,*

As $\overline{1 + \frac{1}{2}qss} \times$ *coſ. latitude* : Fig. 17.
Radius : :
diſtance in any parallel on the ſpheroid :
diff. longitude.

For (Cor. 1. Prop. IX.) HI = BE × $\overline{1 + \frac{1}{2}qss}$. And the degrees in the parallel being inlarged in the ratio of HI to AC, when they become equal to the degrees of longitude; therefore HI : AC : : diſtance in the parallel : to the difference of longitude.

Cor. *The reduction for the log : coſ. latitude is log* : $\overline{1 + \frac{1}{2}qss}$.

PROP. XV. *Prob.*

To ſolve the common caſes of ſailing, on the ſpheroid, by the common tables for the ſphere.

Before this can be done, the ſpecies of the ſpheroid muſt be determined, and the quantities r, a, q will then become known; by help of theſe, the arches of the ſphere, are reduced to thoſe of the ſpheroid and the contrary. Therefore if we ſuppoſe with *Maupertuis*, that $r = 1$; $aa = ,978$; $a = .98894$; $q = ,022$; we muſt then get, F, G, f, g, and M, N, m, n. And then Prop. XIII. and Cor. 1. will ſolve all the common caſes; working the proportions forward or backward, as there is occaſion. And all the caſes of parallel ſailing are ſolved in the ſame manner, by Prop. XIV. To facilitate the work we ſhall have,

$F = .9945v$; $G = 28.7 \times S$.
$f = .9945\text{v}$; $g = 28.7 \times f$.
M = mer. parts of v; N = 75.634 × ſine of v.
m = mer. parts of v; n = 75.634 × ſine of v.

Fig. 17. or Log: $9945 = -1.997605$; log: $28.7 = 1.457882$.
Log: $75.634 = 1.878728$.

reduction of $v = \frac{5\frac{1}{2}}{1000} v + 28.7$ S.

reduction of M is = N.

Caſe 1.

The latitude and longitude of two places being given; to find the courſe and diſtance.

Example.

A ſhip from A *in n. lat.* 5°, *ſails between the north and eaſt, into the n. lat.* 38 *at* E; *making her dif. of longitude* DE 43° = 2580′. *Required the courſe and diſtance,* BAC *and* AC.

Find the mer. dif. latitude for the ſpheroid, thus,

		M. P.	
$v = 38 =$	2280	$2468 = M$	$s.\ v = .615$
$V = 5 =$	300	$300 = m$	$S.\ v = .087$
diff.	1980	$2168 = M - m.$	dif. $= .528$

and $75.63 \times 528 = 40$

$M - m - N - n = 2128$. Then (by Cor. 1. Pr. XIII.)

mer. d. lat.	2128	- -	3.327971
d. long.	2580	- -	3.411619
Rad.			10.
Tan. courſe	50 29		10.083648, BAC.

Then find the proper dif. latitude for the ſpheroid.

.9945	-1.997605	$S.2v$	$= .970$
$198v$	3.296665	$S.2v$	$= .173$
1969	3.294270	dif.	.797

red. -22.7 $28.7 \times .797 = 22.7$

$1946.3 = F - f - G - g.$

Then coſ. courſe 50 29 — 9.803664 Fig.
d. lat. 1946.3 — 3.289210 17.
Rad. — — 10.

Diſt. 3058.7 — 3.485546, AC.

If the departure had been required, it would be found by the proportion in Prop. XIII.

Caſe 2.

The latitude of two places, and the courſe, being given; to find the dif. longitude, and diſtance.

Examp.

A ſhip ſails from A *in ſouth lat.* 25°, *to n. lat.* 30° 17.
at E; *upon a courſe of* 43°, *the dif. longitude and diſtance are required,* DE *and* AC.

	M. P.	
S. lat. 25° = 1500	1550 = M	s. v = .423
N. lat. 30 = 1800	1888 = m	S. v = .500
Sum 3300	3438	ſum .923

$.923 \times 75.63 = -69.8$

$3368.2 = \overline{M+m} - \overline{N+n}$.

Then, .9945	1.997605	S = .766
3300	3.518514	ſ = .866
3282	3.516119	ſum, 1.632

28.7	1.457882	3282
1.632	0.212720	—46.8
red. 46.8	1.670602	$3235.2 = \overline{F+f} - \overline{G+g}$.

Then (by Cor. 1. Prop. XIII.)
Rad. — 10.
Tan. courſe 43° 9.969656
m. d. lat. 3368 3.527372

d. long. 3141 3.497028, DE.

And

Fig. 17.

And coſ.	43	9.864127
d. lat.	3235	3.509874
	Rad.	10.
diſt.	4423	3.645747, AC.

Caſe 3.

Given the latitude of two places and the diſtance; to find the courſe and diff. longitude.

Exam.

A ſhip ſails from A *in S. lat.* 25, *to E. in N. lat.* 30°; *a diſtance of* 3235 *miles; required the courſe and diff. longitude,* BAC *and* DE.

Find the proper dif. latit. 3235 as in the laſt caſe,

Then	Diſt. 4423	3.645717
	Rad. —	10.
	d. lat. 3235	3.509874
	Coſ. courſe 43°	9.864157, BAC.

Then find the mer. dif. latitude 3368 as in the laſt caſe, and then (by Cor. 1. Prop. XIII.)

Rad. —	10.
Tan. courſe 43	9.969656
m. d. lat. 3368	3.527372
dif. longitude	3.497028
3141	= 52° 21′ = DE.

If the departure had been given, the longitude would be found by Prop. XIII.

Caſe 4.

One latitude, courſe, and dif. longitude being given; to find the other latitude, and the diſtance.

Examp.

A ſhip ſails upon a courſe of 37°, *between the ſouth*

and weſt, from E, *lat.* 54 *n. till her dif. longitude* Fig.
DE *was* 28° = 1680°. *To find the dif. latitude and* 17.
diſtance, AB *and* AC.

$$\begin{array}{rl} \text{merid. parts for } 54^\circ = & 3864.5 \\ 75.63 \times \text{S.}54\,(.809) = & -61.2 \\ \hline \text{M} - \text{N} = & 3803.3 \end{array}$$

Then (by Cor. 1. Prop. XIII.

Tan. courſe. 37°	9.877114
Rad. —	10.
d. long. 1680	3.225309
mer. d. lat. 2229	3.348195, AD.

Then as 2229 is leſs than 3803, the ſhip is ſtill in north latitude. Subtract the firſt from the laſt, and there remains

$$\begin{array}{r} 3803.3 \\ \underline{2229.} \end{array}$$

the mer. parts for 2d lat. 1574.3; if this belonged to the ſphere, it would anſwer to 25° 22′. But the meridian of the ſphere being greater than that of the ſpheroid, it muſt be reduced. And the reduction is n or 75.63 × ſine of v.

$$\begin{array}{rl} \text{Whence} \quad\text{———} & 1574.3 = m - n \\ 75{,}633 \times .428\,(\text{S. } 25^\circ 22') = & \underline{+\ 32.4} = n \\ m = & 1606.7 = \text{the} \end{array}$$

mer. parts for 25 52. If this had not been exact enough, you muſt have repeated it, with s for 25 52, which would rectify it. Then to find the proper d. latitude; 54° = 3240′. and 25° 52′ = 1552′. And $28.7 \times \overline{\text{S} - ʃ} = 28.7 \times .166 = 4.76$ the reduc.

$$\begin{array}{lr|r} v = 3240 & .9945 & -1.997605 \\ \text{v} = \underline{1552} & 1688 & \underline{3.227372} \\ v - \text{V} = \underline{1688} & 1679 & 3.224977 \\ & -\ 4. & 7 \\ \hline \end{array}$$

$$1674.3 = \text{pr. dif. lat.}$$

Then

Fig. Then cof. courſe 37° 9.902348
17. d. lat. 1674 3.223755
Rad. — 10.

diſtance 2096, 3.321407, CA.

Caſe 5.

One latitude, courſe and diſtance being given; to find the other latitude and difference of longitude.

Examp.

A ſhip ſails upon a courſe of 23°, from E. in n. lat. 45, for the diſtance 3700 miles; to find the lat. come to at A, and difference of longitude, DE.

45° = 2700, then S = 1. .9945 | −1.997605
2700 | 3.431364
2685 | 3.428969

and 28.7 × 1 is the reduction — 28.7

2656.3 = F — G.

Then rad. 10.
Diſt. 3700 3.568202
Coſ. courſe 23° 9.964026

d. lat. 3406 - 3.532228, BA.

and this being greater than 2656, ſhews that the ſhip has run into ſouth latitude.

3406
2656

750 = *f* — *g*.

And 750 would be 12 30 on the ſphere; but the ſpherical arch being greater than the elliptical, it muſt be reduced; here S = .422, and $\frac{5\frac{1}{2}}{1000}$ v + 28.7 × ſ = 4.1 + 12.1 = 16.2,

Fig. 17.

750
red. + 16.2

v = 766.2 = 12° 46′.

	M.P		
Lat. 45 —	3030	S. v = .707	1.878728
12 46	772	S. v = .221	— 1.967548
	3802	.928	1.846276
reduc.	70		
		N + n 70.2	

M—N+m—n, 3732

Then	Rad.	10.
	Tan. C. 23°	9.627852
	mer. d. lat. 3732,	3.571941
	dif. long. 1584,	3.199793, DE.

Thus all the caſes of Mercator's ſailing are ſolved, by the common rules, with the additional trouble of converting the arches of the ſpheroid into thoſe of the ſphere, and the contrary; and without any tables made on purpoſe. And in like manner, all the caſes of parallel ſailing are ſolved, by converting the radii of the parallels in the ſpheroid into thoſe of the ſphere, and the contrary, and with the ſame tables. And all the three caſes are ſolved by Prop. XIV. Where note, that in *Mauperluis* earth, $1 + \frac{1}{2} qss = 1 + .011ss$, s being the nat. ſine of the latitude.

Parallel Sailing.

Caſe 1.

Given the latitude and difference of longitude of two places in one parallel; to find their diſtance.

Example.

Example.

Suppoſe a ſhip ſails eaſt in the parallel of 52° 12′, till her difference of longitude be 857′; to find the diſtance ſailed.

.011	— 2.041393
s	9.897712
s	9.897712
.00687	— 3.836817

$1.0068 = 1 + \frac{1}{2}qss.$

Rad.	—	10
1.0068		0.002943
coſ. lat. 52 12		9.787394
d. long. 857		2.932981
diſtance 528.8		2.723318

Caſe 2.

Given the latitude, and diſtance of two places in one parallel; to find their difference of longitude.

Examp.

A ſhip ſails eaſt 528.8 miles in the latitude of 52° 12′; required her dif. longitude.

Find $1 + \frac{1}{2}qss = 1.0068$ as in caſe 1, then,

	1.0068	0.002943
Coſ. lat.	52 12	9.787394
	Rad.	10.
Diſt.	528.8	2.723291
		9.790337
d. long.	856.9	2.932954

Caſe

Caſe 3.

Having given the diſtance of two places in one parallel, and their difference of longitude; to find the latitude.

Examp.

A ſhip ſails eaſt 528.8 miles, and makes her dif. longitude $857'$; *required the latitude.*

Dif. long.	857	—	2.932981
Diſt.	528.8	—	2.723291
Rad.		—	10.
Coſ. lat.		—	9.790310

This anſwers to 51 54 if it was for the ſphere, but as the parallel is leſs in the ſphere, it muſt be reduced. Therefore find the reduction, viz. log. : $1 + .011SS$, putting $s = S.\ 51^\circ\ 54'$.

.011		—	2.041393
S. 51 54	—		9.895939
S. 51 54	—		9.895939
.0068	—	—	3.833271
Coſ. 51 54			9.790310
reduc. 1.0068			.002943
Coſ. lat. 52 12			9.787367 true.

PROP. XVI. *Prob.*

To correct a Reckoning at Sea.

The motion of a ſhip is ſubject to ſeveral inequalities, which muſt be corrected before the place of the ſhip can be truly known. And therefore the true latitude of the ſhip muſt be got, by obſerving the ſun's meridian altitude; by which and the

Fig. the ſun's declination being known, the ſhip's latitude becomes known, or inſtead of the ſun, any known ſtar may be uſed. Then if the obſerved latitude be the ſame as the computed latitude, there is no room for any correction. But when they differ, the error will be owing to ſome of theſe cauſes, the variation, the motion of a current, error in the courſe, or in the diſtance. And which it is, muſt be determined by judging of the ſeveral circumſtances that attend the reckoning.

1. *If the error lies in the variation*; which will be known, when an amplitude is had; take the difference of latitude and departure, all the time you have been wrong, and ſay,

As Rad : S. *error of variation* : :
So departure : *to correction in latitude* : :
And ſo diff. latitude : *to correction in departure.*

Then if the true courſe is greater, the departure muſt be increaſed, and the dif. lat. diminiſhed, by theſe corrections. And the contrary when the true courſe is leſs.

For let AC be the ſhip's way, AR the ſuppoſed meridian, A*m* the magnetic meridian, AH the true meridian, AR and CR the dif. lat. and departure as computed. Let fall CS perp. to the true meridian AH. Then inſtead of AR and CR, AS and CS will be the dif. lat. and departure. Therefore AR muſt be diminiſhed by the quantity R*n* or *d*S, to get the true dif. lat. AS; and CR muſt be increaſed by R*d* or *n*S, to get the true departure CS. And in the right angled triangles AS*n*, CR*n*, the angles at R and S are right, and the angles at *n* are equal; therefore RC*n* = SA*n* the error of variation. Therefore in the triangle CR*n*, rad : C*n* or CR : : S.RC*n* : R*n*, the correction in latitude. And in the triangle AR*d*, rad : A*d* or AR : : S.RA*d* : R*d* or S*n*, the correction in departure.

But

Fig. 18. But if the obſerved courſe RAC be diminiſhed by the error of variation, the perp. CS would fall on the other ſide of CR, then AR muſt be increaſed by *n*R, and CR decreaſed by R*d* or *n*S. For when one adds, the other ſubtracts.

19. 2. *If the error happens from the motion of a current.* If its direction be known, to find the quantity of the error; ſay, as *rad* : *error in latitude* : : *tan. current's courſe* : *correction in departure.*

This correction muſt be added to the departure, when the true dif. latitude is greater than the obſerved; but ſubtracted when leſs.

For if AR is the meridian, AQ the ſhip's way by the current, Q the ſhip's place by the reckoning, in the parallel PQ; and C her true place by obſervation, in the parallel RC; then QC is the error. Draw QD parallel to AR, then, in the triangle QDC, rad : DQ (PR) : : Tan. DQC (RAC) : DC, the correction in departure; which adds, if R is beyond P, otherwiſe it ſubtracts.

20. 3. *When the error is in the diſtance*; as is very likely when the courſe is near the meridian. To find the error, take the difference of latitude and departure from the time of the laſt obſervation, and ſay,

As diff. latitude : *departure* : : *ſo the error in latitude* : *to the correction in departure.*

Then the correction is to be added to the departure, when the computed dif. latitude is leſs than the true, that is, when the ſhip is before the reckoning; otherwiſe ſubtracted.

For if the ſhip's way be near the meridian, as in AB, and if BC be her parallel by the reckoning, and DE her parallel by obſervation. From A, the place of the laſt obſervation, deſcribe the arch BF to cut the parallel DE in F. Then if the error was in the courſe, it will be no leſs than the angle BAF, which is improbable; for ſuch a great miſ-

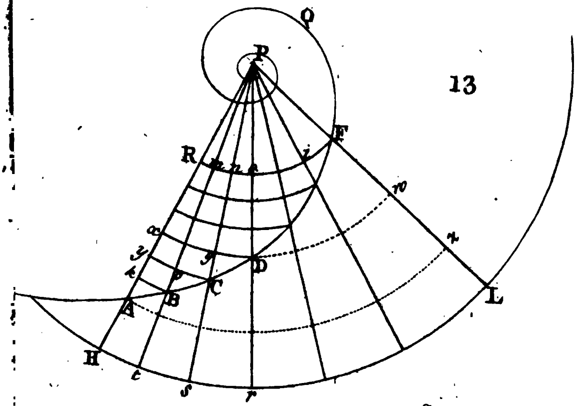
Fig. 20. take can hardly be made, if any care has been taken; therefore, it is more rational, to ſuppoſe it in the diſtance, and then it will be the ſmall quantity BD.

Produce AF to H, and draw HG parallel to the meridian AD. Then by ſimilar triangles AB (dif. lat.) : BH (departure) : : HG : FG. Then if BH be her parallel by the reckoning, HG muſt be taken from BH. But if DE be the parallel by the reckoning, or the ſhip foremoſt, then FG muſt be added to DF to get her true departure. If the courſe AH is very near the meridian AB, the correction of the departure FG is ſo ſmall, that it may be omitted.

4. *When the error is in the courſe*, as is very likely when it lies near the parallel. To find the error, ſay,

21. As *the departure* : *dif. latitude* : : *error in latitude* : *correction in departure*, from the time of the laſt obſervation. Then the correction muſt be added to the departure, when the reckoning is before the ſhip; otherwiſe ſubtracted. But when the ſhip's way is very near eaſt or weſt, this correction is ſo ſmall that it may be omitted.

For let AB be the meridian, BC the ſhip's parallel by the reckoning, DE her parallel by obſervation, AF her way by the reckoning, produce AF to cut BC in C. Then if the error was in the diſtance, it would be no leſs than the line FC, which is quite improbable; therefore it is rather the ſmall angle HAF in the courſe, which may very well happen. Draw HG parallel to the meridian AB, and from A deſcribe the arch HF. Then the angle AHF being right, the angle HFG = AHG, = HAD, and AHD = FHG, therefore the triangles ABH, FGH are ſimilar, whence BH (departure) : BA (dif. latitude) : : HG : GF, the correction in departure. And as F is her true place, 13



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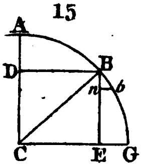

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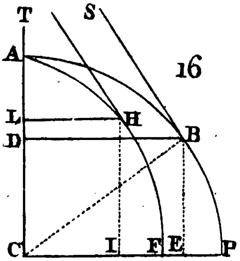

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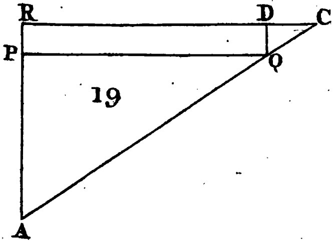

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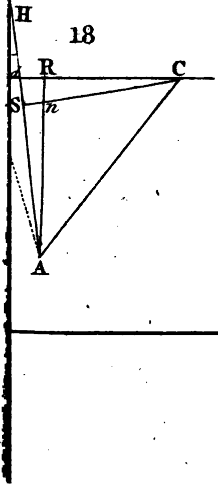

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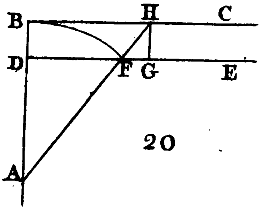

Pl. III. pa. 164.

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place, GF muſt be added to DG or BH to have her true departure DF. But if H had been her true place it muſt have been ſubtracted. If the courſe was in or near the parallel AP, GF would be nothing. Fig. 21.

5. *If a ſhip ſails along* AH *near four points*, and you know the error to be in the diſtance AH, then her true place will be at E, and the error of departure EG muſt be computed by Art. 3. But if you know the error to be in the courſe, then making AF equal to AH, her true place will be at F; and the error of departure FG muſt be computed by Art. 4. Therefore if both be wrong, ſhe will be in the middle of EF at G; that is, there will be no error in departure. Hence if it be not known, whether the courſe or diſtance is wrong, no error muſt be reckoned in departure, but it muſt remain as it was, for then BH is equal to DG. 22.

In general, ſailing near the meridian, or near a parallel, or near four points, the departure cannot be corrected. But it may be corrected when her courſe is within a point or two of the meridian, or any parallel.

5. The error in departure being found, the error in longitude lies the ſame way, being only greater in quantity; which is found thus, *as error in latitude : correction in departure : : ſo the meridional diff. lat. (between the computed and obſerved lat.) : correction in longitude.*

The correcting a reckoning, depends upon finding the latitude, and that is had by obſerving the ſun's meridian altitude, or zenith diſtance. And as the buſineſs of finding altitudes is uſeful upon many occaſions at ſea, I ſhall conclude with the two following propoſitions, for that purpoſe.

Fig. 22.

PROP. XVII. *Prob.*

The times of two equal altitudes of the ſun, taken in the forenoon and afternoon of the ſame day, being obſerved by a watch; to find by the ſame watch, the time of his being in the meridian.

I.

If the ſun's declination continues nearly the ſame, during the interval of the two obſervations, then the middle time between the obſervations, taken by the watch, is the time of his being in the meridian. But if he makes a ſenſible difference of declination in that time, then this mean time muſt be corrected as follows.

II.

23. Let HO be the horizon, Z the zenith, P the pole, GDC a parallel of altitude, IAC a parallel of declination; and let C, D be the two places of the ſun, ſuppoſed on different ſides of the meridian PZH. Then AD will be the difference of declination, between the times of obſervation. Then in the triangle ZPC there are given ZP the complement of the latitude; ZC the co-altitude, and angle ZPC anſwering the time, both by obſervation; therefore S.ZC : S.ZPC : : S.ZP : S.ZCP. Then having the angle C, let AD (the dif. declination) be taken in minutes of a degree. Then ſay, as coſ. ſun's declination : cotan. ZCP : : ſo 2AD : to the correction in ſeconds of time.

Then the correction muſt be added to the mean time, to find the true time of his ſouthing, when the ſun is going from the elevated pole; but ſubtracted, when coming towards it.

For, produce the meridians PAD, and PC to cut the equinoctial in B and E. Then the angles PCA and ZCD being right, by ſubtracting the common angle ZCA from both, DCA will be equal to ZCP. And in the ſmall rectilineal triangle ACD right angled at A, rad : AD :: cotan. DCA or ZCP : AC = $\frac{AD \times \text{cotan. } ZCP}{\text{rad.}}$. And by the ſpherical ſectors PAC and PBE, S.PC : rad :: AC $\left(\frac{AD \times \text{cot. } ZCP}{\text{rad}}\right)$: BE or angle APC = $\frac{AD \times \text{cot. } ZCP}{S.PC}$. But BE and AD being reckoned in minutes, and 1 minute of a degree being equal to 4 ſeconds of time, therefore BE reduced to ſeconds will be = $\frac{4AD \times \text{cot. } ZCP}{S.PC}$, and half the time in BE = $\frac{2AD \times \text{cot. } ZCP.}{S.PC}$. And if the ſun be going from A to D (from the pole), the morning obſervation will be at C, and $<$ CPZ being greater than (the half ſum) $\frac{CPZ + DPZ}{2}$, by half of CPD; that half muſt be added to the half ſum, to find CPZ, or the true time of noon. And the contrary, when the ſun comes towards the pole; for then D would be the morning obſervation.

Fig. 23.

III.

The foregoing correction is but ſmall, and in many caſes may be omitted, eſpecially at ſea. But there is another correction more material, owing to the change of latitude. For if this prob. is performed on ſhip board, and the ſhip is under ſail; if ſhe ſails uniformly upon one courſe during the interval of the obſervations, and alters her latitude much, then the following correction is neceſſary.

Let PZF be the meridian, IDC the ſun's parallel of declination; P the pole, D and C two

Fig. obſerved places of the ſun, ſuppoſed on different 24. ſides of the meridian PZF, Z the zenith of the place where D is obſerved, Q the zenith of the place where C is obſerved, PD and PC two hour circles. Then in the triangle ZPD, there will be given ZP, PD, and alſo ZD by obſervation, as alſo the angle DPZ from the obſerved mean time.

Then ſay, as S.ZD : S.ZPD : : S.PD : S.PZD. Therefore having the angle Z, and the arch ZQ in minutes, ſay, as coſ. latitude (ZP) : cotan. PZD : : 2ZQ : to the correction in ſeconds of time.

Then this correction muſt be ſubtracted from the mean time, if the ſhip goes towards the ſun; but added, if going from it; and it gives the true time of his ſouthing, by the watch.

For, produce the meridians PD and PC to cut the equinoctial in B and E, and draw ZD, QC the complements of the equal altitudes. Alſo draw Q*r*, D*n*, perpendicular to ZD, QC; then as ZQ, DC are extremely ſmall, D*r* will be equal to Q*n*, and ſince DZ is equal to QC, therefore $Zr = Cn$. But by ſimilar ſectors, we ſhall have, $\text{rad} : \text{S.DP} :: \text{BE (or angle DPC)} : \text{DC} = \frac{\text{BE}\times\text{S.DP}}{\text{rad}}$. And in the triangle ZDP, S.DP : S.Z : : S.ZP : $\text{S.PDZ} = \frac{\text{S.ZP}\times\text{S.Z}}{\text{S.DP}}$. And in the ſmall right-angled plain triangles Q Z*r* and CD*n*, we ſhall have, $\text{rad} : \text{ZQ} :: \text{S.ZQ}r \text{ or coſ. Z} : Zr = \frac{\text{ZQ}\times\text{coſ. Z}}{\text{rad}}$. And rad : DC : : S.CD*n* or PDZ : C*n*, that is, rad : $\frac{\text{BE}\times\text{S.DP}}{\text{rad}} :: \frac{\text{S.ZP}\times\text{S.Z}}{\text{S.DP}} : Cn = \frac{\text{BE}\times\text{S. ZP}\times\text{S.Z}}{\text{rad}^2}$ $= Zr \text{ or } \frac{\text{ZQ}\times\text{coſ. Z}}{\text{rad}}$, whence $\text{BE} = \frac{\text{ZQ}\times\text{coſ.Z}\times\text{rad}}{\text{S ZP}\times\text{S.Z}}$ $= \frac{\text{ZQ}\times\text{cotan. Z}}{\text{S.ZP}}$. Now if ZQ be expreſſed in

minutes,

Fig. 24.

minutes, BE will be expreſſed in minutes, and as 1 minute = 4 ſeconds of time, the time in BE will be $\frac{4ZQ \times \text{cotan. } Z}{S.ZP}$, and half the time in BE $= \frac{2ZQ \times \text{cotan. } PZD}{S.ZP}$. And if the ſhip goes from Z to Q, the morning obſervation will be at D, and the evening one at C; and the angle DPZ being leſs than half the ſum of DPZ and ZPC (which denotes the mean time), by half BE or half the angle DPC; therefore $\frac{1}{2}$DPC muſt be ſubtracted from this mean, to find the true time. And the contrary will happen, when the ſhip ſails from the ſun; for then C would be the ſun's place at the morning obſervation; and D at the evening one.

Cor. 1. *This Prob. may be equally uſeful for finding the time of the moon's ſouthing by the watch.*

Cor. 2. *If inſtead of the ſun, any ſtar be made uſe of; the correction, in the* II. *article, muſt be omitted; as the ſtar does not alter its declination.*

PROP. XVIII. *Prob.*

To take the altitude of the ſun or a ſtar, when the horizon is not viſible.

It frequently happens, that when an obſervation is wanted and the ſun or a ſtar is viſible, that the horizon is obſcured, by thick hazy weather; and often in calm weather, it is ſo ill defined, that there is no certainty in taking an obſervation from it; and yet in the common way, it is abſolutely neceſſary to have a clear horizon. Therefore in order to ſupply this defect, one method is this. Fix a ſpirit level to one ſide of a common quadrant. The ſpirit muſt be contained in a glaſs tube turned up

Fig. 24. up at the ends. This tube muſt be as long as the ſide of the quadrant, and very ſtrait in the middle, that the ſpirit may flow very ſlowly through it, and hinder its ſpilling, when held obliquely. The ends that turn up muſt be wider, and when the ſpirit is in, they muſt be cemented cloſe up, all but a very ſmall hole in each, in which two pins are to be put when the inſtrument is not uſed. But when the inſtrument is placed ready for uſe, theſe pins muſt be taken out, to admit the free ingreſs and egreſs of the air. To adjuſt the tube, let it be well fixt to the ſide of the quadrant; than place that ſide exactly horizontal; and taking out the pins, obſerve how high the ſpirit riſes in each end of the tube; mark theſe places exactly. And therefore when ever the quadrant is ſo placed, that the ſpirit riſes to theſe marks, you may be ſure, that ſide of the quadrant lies truly horizontal. This quadrant muſt have an index moveable on the center; and this index may either have plain ſights, or a ſhort teleſcope fixt to it. The whole inſtrument thus fitted up, may ſtand upon a ſtaff or pedeſtal. To obſerve with it, place it ſo that the ſpirit may riſe to the two marks; there fix it, and directing the index towards the ſun or ſtar, the graduated edge will ſhew the altitude. It would be convenient to have an aſſiſtant to keep the inſtrument level, whilſt you are taking the altitude.

Another method is, to add a mercurial level to *Davis's* quadrant, thus. Inſtead of having the ſight vane to move upon the 30 degrees arch, it is fixt to an index which moves about the center of the inſtrument. To this index is fixt the mercurial level; which, like the laſt, conſiſts of a glaſs tube turned up at the ends; the middle part of the tube muſt be extremely narrow, and the ends that turn up, muſt be wider, being twice or thrice the diameter of the middle or more; this is to prevent the vibratory motion of the mercury. The two ends have their tops ſtopped with pieces of wood cemented in them, and thro' theſe pieces of wood are made two very ſmall holes, in which two pins are put, when the inſtrument is at reſt; but taken out when uſed, to let the air enter, that the mercury may reduce itſelf to a level. The horizontal part of the tube muſt be as long as the index.

To adjuſt the inſtrument, let the index be ſo placed, that the ſlit in the ſight vane, and that in the horizontal vane may be exactly upon a level. Then pour mercury into the tube till it riſe to the ſame level in both ends of the tube, and no higher; then is your inſtrument correct. Therefore when at any time the index is ſo placed, that the mercury in both legs comes juſt to the line of viſion drawn between the two ſlits in the ſight vane and horizontal vane; then that line lies horizontal. And a viſible line ought to be drawn on the index to ſhew this.

To uſe the inſtrument, the obſerver muſt chuſe a convenient place, where there is the leaſt motion or wind to diſturb him. Then ſitting down, and taking hold of the inſtrument as uſual, and looking thro' the ſlits in the two vanes, move the index till the top of the mercury in the tube be in the ſame line, there keep it, whilſt you move the inſtrument up or down, till the ſun through the ſlit of the ſhadow vane fall on the ſlit of the horizontal vane; when all theſe things agree, obſerve the degrees cut, and you will have the zenith diſtance, as uſual. Here, in looking thro' the two ſlits, you obſerve the top of the mercury in the tube, which ſerves for the horizon; ſo that the end of the index is moved in the ſame manner, as the ſight vane uſually is.

To obſerve a ſtar, ſince it can caſt no ſhadow; to have it in a line with the horizon vane and ſhadow vane; another perſon muſt look for the ſtar

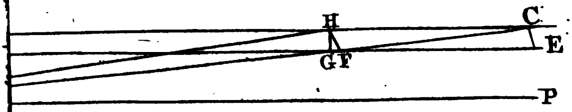
thro'

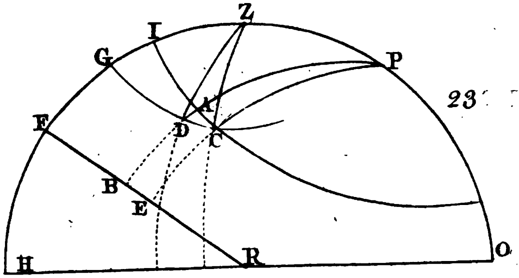
Fig. thro' the two ſlits; the index being kept in its proper poſition.

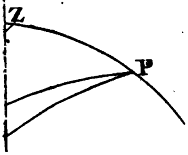
By this improvement, the want of a horizon is ſupplied; ſo that an obſervation may be made from any headland, or any harbour, or any place on ſhore, where the ſun can be ſeen, without any regard to the horizon; for the true level will always be preſerved, whether on the top of a mountain, or at the ſurface of the ſea.

To compleat the art of Navigation, theſe three things are abſolutely neceſſary; the variation of the compaſs, the latitude of the ſhip, and the longitude of it. The firſt may be found by an amplitude or azimuth; the ſecond is known from the ſun's meridian altitude, by the help of this Prop. the third is ſtill a ſecret, and likely to continue ſo. For, tho' many thouſand pounds have been paid for the pretended diſcovery thereof; I doubt we ſhall ſtill remain juſt as wiſe as we were before the diſcovery; except the ill ſucceſs of it happens to teach us ſo much wit, as to take better care of our money for the future. And indeed all unlikely ways and means for this purpoſe, have been propoſed and proſecuted; whilſt the only probable method is never thought of, or quite neglected.

FINIS.







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DIALLING.

OR THE

Art of drawing DIALS,

ON

All Sorts of PLANES whatſoever.

In THREE PARTS.

Sect. I. The fundamental Principles of DIALLING.

Sect. II. The Practice of DIALLING, illuſtrated on all kinds of Planes.

Sect. III. Of deſcribing the common Furniture of DIALS; and the Conſtruction of ſome uſeful DIALS of other kinds.

Tempora labuntur, tacitiſque ſeneſcimus annis;
Et fugiunt fræno non remorante dies.

OVID Faſt. L. VI.

THE

PREFACE.

AS the meaſuring of time exactly is a matter of great conſequence in all human affairs; without which we could not rightly know when to go about any particular buſineſs; therefore Dialling becomes a neceſſary art. For as certain times and ſeaſons are ſet apart as moſt proper for performing ſuch and ſuch actions, in which we are conſtantly imployed; not only in the ſeveral parts of the year, but at different times of the day; therefore we have need of ſome ſuch inſtrument as a dial, to direct us to theſe actions, and to inform us when theſe ſeveral periods of time are come. And tho' we be furniſhed with ſome ſorts of moving machines, which will do this, as clocks and watches; yet theſe are often out of order, apt to ſtop and go wrong, and therefore require frequently to be regulated and ſet right, by ſome unerring inſtrument as a dial; which being rightly conſtructed, will always (when the ſun ſhines) tell us truth. And therefore whether we have any clocks or not, we ſhould never be without a dial.

The original of Dials ſeems to be this. When men attentively obſerved the ſun's circular motion daily round the earth; that in the morning he roſe in the eaſt, moved about to the ſouth, and from thence to the weſt, in the evening, where he ſet. They would at the ſame time, obſerve the alteration of the ſhadows; that they were projected firſt one way and then another; moving round as the ſun did, but towards oppoſite points; and that they grew longer and ſhorter as the ſun aſcended higher or ſunk lower. This would give the moſt obvious hint of meaſuring time by ſhadows, or for making of dials: tho' they could not be exactly executed without the rules of art, which therefore were neceſſary to be known, and are the Subject of this Treatiſe. For as the motion of the ſun is *the meaſure of time; ſo no apter method could be thought on, than to ſhew the time of the day by the ſhadow of ſome index, properly fixt upon a plane.*

At the firſt, only three parts of the day were diſtinctly obſerved, which were ſun riſe, noon, and ſun ſet; but this diviſion being too general; they afterwards divided the natural day into 24 parts or hours, as a number very proper and commodious, for diſtinguiſhing the various times of the day, which were to be diſtinctly ſhewn by a dial.

The foundation of Dialling is entirely depending upon Aſtronomy. For the lines on a dial which ſhew the hours, are the interſections of the ſeveral hour circles, with the plane of the dial. And to project theſe hour lines, is the ſame thing as to project the ſphere, upon the dial plane. And therefore the making of dials depends upon projecting the ſphere, particularly the gnomonic projection, which is naturally adapted to this purpoſe; and which we have treated on before.

In the following Book, the firſt ſection contains the grounds of this art; by ſhewing, how the ſeveral requiſites are to be found, by the interſections of the circles of the ſphere, with the plane of the dial, from the principles of ſpherical trigonometry; from which the practical rules are deduced.

The ſecond ſection contains the practice, and that three different ways. 1. Geometrically, by rule and compaſs, which depends upon the gnomonic projection of the ſphere, before delivered. 2. By trigonometrical calculation, by the tables of ſines and tangents, which is the moſt exact way. 3. By the lines upon Collin's *Dialling ſcale, which is a method extremely eaſy and ready.*

The third ſection ſhews the way of making ſome other ſorts of dials; and drawing the furniture upon any common dial; that is the projection of the ſeveral circles of the ſphere; and inſerting therein, ſuch hours as have been uſed by other nations. And tho' theſe things are not abſolutely neceſſary, they may ſerve ſometimes as an ornament for a dial.

W. Emerſon.

DIALLING.

DEFINITIONS.

DEF. I.

DIALLING or *Gnomonicks*, is the art of making Dials, and,

DEF. II.

A Dial is an inſtrument for ſhewing the hour of the day, by the ſun's ſhining upon it.

The moſt common and uſeful ſort of dials, are thoſe where the hour lines are deſcribed on ſome plane ſurface, upon which the ſhadow of an index falling, ſhews the hour by the termination of its ſhadow.

DEF. III.

Dial planes, are thoſe planes on which dials are or may be deſcribed. And therefore any plane upon which the ſun can ſhine, may be a dial plane. All dials are denominated from that great circle of the ſphere, to which the dial plane is parallel. Hence,

DEF. IV.

A horizontal Dial, is a dial drawn on a plane parallel to the horizon.

DEF. V.

An equinoctial Dial, is upon a plane parallel to the equinoctial.

DEF. VI.

Erect or vertical Dials, are ſuch dials as are drawn on upright planes, or thoſe perpendicular to the horizon.

DEF. VII.

A direct Dial, is a dial drawn on a plane facing the eaſt, weſt, north, or ſouth; and is accordingly called an *Eaſt dial*, a *Weſt dial*, a *North dial*, and a *South dial*. The eaſt and weſt dials are parallel to the meridian; and the north and ſouth dials are parallel to the prime vertical.

DEF. VIII.

A declining Dial, is one that faces none of the cardinal points, but declines towards the eaſt or weſt. Theſe are *a ſouth-eaſt decliner*, *a ſouth-weſt decliner*, *a north-eaſt decliner*, and *a north-weſt decliner*.

DEF. IX.

An inclining or oblique Dial, is one whoſe plane ſtands at oblique angles upon the horizon. And theſe may be either direct ones or decliners.

DEF. X.

A reclining Dial, is one whoſe plane leans backwards, or from you.

DEF. XI.

A proclining or acclining Dial (ſometimes called an incliner), is one whoſe plane leans forwards or towards you. The planes on which theſe are drawn will be procliners (or incliners) on one ſide, and recliners on the other.

DEF. XII.

Declination of a plane, is an arch of the horizon, contained between the plane and the prime vertical; or between the meridian and a plane perpendicular to the dial plane; and is always reckoned from the ſouth or north.

DEF. XIII.

Reclination and proclination of a plane, is the angle it makes with a vertical plane; or it is the number of degrees that the plane leans from you or towards you, reckoned from the zenith; being the plane's diſtance from the zenith. But *Inclination* is properly the angle the plane makes with the horizon.

DEF. XIV.

The *center* of a Dial, is the point where all the hour lines meet, or towards which they tend.

DEF. XV.

The *Stile*, *Gnomon* or *Cock* of a Dial, is a pin, or piece of metal, &c. raiſed perpendicular upon the plane of the dial; by the ſhadow of this, as an index, the hour of the day is known.

DEF. XVI.

The *Subſtile*, is the line on which the ſtile is erected, perpendicular to the plane of the dial. This always goes thro' the center of the dial.

DEF. XVII.

Stile's height, is the angle which the top edge of the ſtile makes with the ſubſtile; that is, when the ſtile is in the form of a triangle. And then the angular point is at the centre of the dial. But if the ſtile is a pin, the *height* is the perpendicular length of it; if a parallelogram, the center is at an infinite diſtance.

DEF. XVIII.

The ſubſtile's diſtance from the meridian, is the angle which the ſubſtile makes with the 12 o'clock line.

DEF. XIX.

Meridian of the plane, is the meridian perpendicular to the plane of the dial; and therefore it is the ſame as the ſubſtile. This is quite different from the meridian of the place, which is that meridian which is perpendicular to the horizon.

DEF. XX.

Height of the meridian, is an arch of the great circle or dial plane, comprehended between the horizon and meridian of the place.

DEF. XXI.

Plane's difference of longitude, is the angle on the ſphere contained between the meridian of the place and the meridian of the plane; this is alſo called the *inclination of meridians*.

DEF. XXII.

Hour arch, is an arch of the equinoctial, anſwering to the time; or the angle at the pole. Thus 15 degrees anſwer to 1 hour, 30 deg. for 2 hours, 45 degrees for 3 hours, &c. This takes its beginning at the ſubſtile.

DEF. XXIII.

Hour angle, is the angle which any hour line upon the plane of the dial makes with the ſubſtile.

DEF. XXIV.

The horizontal line, is a line drawn parallel to the plane of the horizon in any dial; and is made by the horizontal plane cutting the dial plane, and paſſing thro' that point of the ſtile, whoſe ſhadow ſhews the hours. In like manner,

DEF. XXV.

The equinoctial line, is the interſection of the plane of the equinoctial and dial plane; an *azimuth line*, the interſection of any azimuth with the dial plane, &c.

DEF. XXVI.

The contingent line, is a line drawn thro' the foot of the ſtile perpendicular to the ſubſtile; and ſerves inſtead of the equinoctial for finding the hour points upon it, thro' which the hour lines are to be drawn. When the contingent does not paſs thro' the foot of the ſtile, it repreſents the equinoctial.

SECT. I.

The foundation of Dialling, and the general properties of Dials, and Dial Planes. The Rules for calculating all the Requiſites. To find the time of the ſun's ſhining on any Plane. Explanation of the lines on the Dialling Scale.

PROP. I.

IF a right line be fixt any where upon the earth, parallel to the earth's axis; the ſhadow thereof by the ſun, moves uniformly about it, deſcribing equal angles in equal times.

It is matter of obſervation that the ſun apparently moves about the earth in 24 hours, with a uniform motion, deſcribing 15 degrees of the equinoctial every hour. And altho' this apparent motion of the ſun is really to be aſcribed to the earth, which actually moves uniformly round its axis in 24 hours; yet we may refer this motion to the ſun, as it makes no manner of difference in the appearances, for the riſing, ſouthing, ſetting, and horary motions, will all be exactly the ſame whether the motion be in one or the other. This ſuppoſition of the ſun's uniform diurnal motion, is the foundation of all the meaſures we have of time;

and particularly the whole art of Dialling depends upon that ſuppoſition. Now ſuppoſe the ſun in motion round the earth's axis, and that this axis by the ſun's rays caſts a ſhadow; this ſhadow, being in a right line with the ſun which is the luminous body, and the axis which is the opaque body, will always be on oppoſite ſides of the axis, from the ſun; and therefore if the ſun moves uniformly, the ſhadow will likewiſe move uniformly about this axis. Fig.

Let AB be the earth's axis, CD any other line on the earth, parallel to it. Let ER be drawn perpendicular to AB, CD, and in the plane of the ſun's motion; to cut them in the points F and L. Suppoſe MFI, NLK drawn from the ſun. Then by reaſon of the vaſt diſtance of the ſun, and the ſmall diſtance of the points F and L upon the earth compared therewith; the lines MF and NL are to be looked upon as parallels. Therefore the angle RFM is equal to the angle RLN; and theſe being equal to the alternate angles GFI and HLK made by the ſhadows, theſe alternate angles will alſo be equal. Hence, whilſt the ſun ſeems to move thro' the angle RFM about F, it will alſo ſeem to move thro' the equal angle RLN about L. And at the ſame time the ſhadow of F will move thro' the angle GFI, whilſt the ſhadow of L will move thro' the equal angle HLK. And therefore both the ſun and the ſhadow of L move uniformly about CD. And thus the ſhadow of any other point, and conſequently the whole plane of the ſhadow of CD, moves uniformly about CD. 1.

Cor. 1. *Hence if the place of the ſhadow of this line* CD *be marked, or drawn with a black line, at any given hour; every day when the ſun returns, its ſhadow ſhall fall upon the ſame black line, at the ſame hour of the day.*

Fig. 1. Cor. 2. *Therefore if a line be fixed parallel to the earth's axis in any fixt plane; and the place of its ſhadow be marked at 12 o'clock; and likewiſe where it cuts the plane at every 15 degrees of revolution of the ſhadow, theſe marks will denote the hours of the day, and the ſhadow will meaſure time as truly upon that plane, as the ſun itſelf does in the heavens; that is, there will be conſtructed a true ſun dial.*

For as the ſun moves thro' 15 degrees every hour, the ſhadow will likewiſe move thro' 15 degrees every hour, in its revolution round CD; and therefore its interſection with the fixt plane, muſt needs point out the ſeveral hours.

Cor. 3. *Hence it is the very ſame thing, whether a dial be drawn upon any given plane, or upon that great circle of the ſphere which is parallel to it.*

PROP. II.

2. *If a line be erected perpendicular to a plane, and the top of it deſcribe any curve therein, by its ſhadow in the ſun. If the plane be removed to any other place on the earth, in a quite parallel ſituation; the ſhadow will deſcribe the very ſame curve as before, and at the ſame time.*

I call that ſituation of a plane *quite parallel*, when it not only continues parallel to itſelf, or to ſome original plane, to which it was parallel at firſt; but alſo when any right line drawn in it, continues parallel to itſelf, or to ſome original line, to which it was at firſt parallel. When only the paralleliſm of the plane is regarded, it may have an infinite number of poſitions, for any one ſide of it may be up or down. But when a certain line drawn in it, is to have the ſame poſition, and tend the ſame way, this fixes its ſituation, if the face of the plane looks the ſame way as before. So that if a plane is moved from one place to another, ſo that it continues always parallel to itſelf, and a right line drawn in it continues parallel to itſelf; that plane is always in the required ſituation. This being explained. Fig 2.

Let the line AB be erected perpendicular to the plane CD, and let A the top of it by its ſhade in the ſun, deſcribe the curve FGHI. Then if it be removed to *cd*, in ſuch a parallel ſituation; and the curve *fghi* be deſcribed by the ſhadow of the top *a*. This curve will be the very ſame as FGHI. For draw FB, GB, and *fb*, *gb*. And let SAF, TAG be the ſun's rays, when the ſhadow is at F and G; and *ſaf*, *ſag*, his rays when the ſhadow is at *f* and *g*. Then from the vaſt diſtance of the ſun, and conſequently from the paralleliſm of the rays SF, *ſf* at the ſame moment, and of TG, *tg*, alſo at the ſame moment, another time; we ſhall have the angle AFB = *afb*, and AGB = *agb*, alſo FAG = *fag*. And ſince the perpendicular *ab* = AB, therefore *af* = AF, *ag* = AG, whence *fg* = FG. After the ſame manner is proved that *gh* = GH, and *hi* = HI, &c. Therefore the whole curve *fghi* is the ſame as the curve FGHI. And becauſe the plane is in a parallel ſituation, the angle *cbf* = CBF; and therefore the curve is in the ſame poſition in reſpect of any line drawn upon the plane.

It is to be noted, that when the ſun is in the equinoctial, then FGHI or *fghi* will be a right line. For the equinoctial being a great circle, its plane will cut any other plane in a right line, ſuppoſing it either at the center of the earth, or any way on its ſurface. For any point of the earth, as well as the center, may be taken for the center of the equinoctial, or of the ſun's motion at that time. But at other times the earth is not in the center of the

Fig. 2. the ſun's motion. For the line drawn from the earth to the ſun continues not in one plane, but deſcribes a conic ſurface, and therefore the curve FGHI will be ſome conic ſection.

Cor. 1. *Hence the motion of the ſhadow of a point as* A, *upon a plane* CD, *is the very ſame, whether the plane be placed in the center of the earth, or any way on its ſurface, provided it be in a ſituation quite parallel.*

Cor. 2. *The place of the ſhadow at the ſame moment, will be at the ſame place* G *or* g, *whether the place be at* CD *or* cd.

PROP. III.

If a line be elevated above a plane and fixt there, its ſhadow will be the ſame, at the ſame moment of time, whatever part of the earth it is placed in, provided it be in a ſituation quite parallel.

3. Let the line AO be raiſed above the plane CD, and fixt there, interſecting the plane in O. From any point in it as A, let fall the perpendicular AB upon the plane. From the ſun at S, draw the line SAG interſecting the plane in G. Then (by Cor. 2. of the laſt Prop.), the ſhadow of A at that inſtant will fall upon G, wherever the plane CD is placed, ſo as to be in a poſition quite parallel. But if a plane be drawn thro' the ſun at S, and the line AO, it will cut the plane CD in the right line GO; where AGO is the plane of the ſhadow, and GO the line of ſhadow. Conſequently the ſhadow of AO, will at the ſame moment fall upon the ſame line GO wherever the plane CD is placed, in a quite parallel ſituation.

And if the ſtile AO be parallel to the plane CD, the ſhadow will fall on the ſame line, in all places; as will be evident by letting fall a perpendicular from any other point of the line; for the ſhadow of that point, will always fall upon the ſame point of the plane; and the line of ſhadow drawn thro' theſe two points will always be the ſame. Fig. 3.

PROP. IV.

In every dial that has a ſtile, the edge of it, that gives the ſhadow, muſt always be parallel to the earth's axis, and point directly to the two poles.

It has been ſhewn in Prop. I. that the ſun moves uniformly about the earth's axis; and conſequently, by reaſon of the ſun's great diſtance, it alſo moves uniformly about any line parallel thereto. That is, if a plane be ſuppoſed to be drawn perpendicular to the earth's axis, or to this line, and a circle be deſcribed about the ſtile, as the equinoctial circle is about the earth's axis; then the ſun, and conſequently the ſhadow, which is oppoſite to it, will move uniformly in this circle, deſcribing equal angles or equal arches in equal times. And this it will always do, from the parallelifm of the ſtile to the earth's axis. But alter the poſition of the ſtile, ſo that it may not be parallel to the earth's axis, as before; and then no ſuch regular motion will be made about it by the ſhadow. Therefore the poſition of the ſtile to ſhew this regular motion of the ſun, muſt be ſuch, as to be parallel to the earth's axis, or to point directly to the two poles.

Cor. *Hence the ſtile muſt be ſo fixt upon any dial plane, that its edge may point directly to that pole which is elevated above the dial plane.*

Fig. 3.

PROP. V.

In any dial, the angle of the ſtile's height above the ſubſtile, is equal to the height of the pole above the plane of the dial.

It has been proved in the laſt Prop. that the ſtile muſt be parallel to the earth's axis. Now the angle that the axis of the earth makes with a great circle of the ſphere parallel to the dial plane, is meaſured by the arch of a great circle, which is perpendicular to the former, and paſſing thro' the earth's axis, which arch is contained between the parallel great circle and the axis, or the angle which the axis makes with that parallel great circle. But this angle is the elevation of the pole above that great circle. And ſince the ſtile is parallel to the earth's axis, and the ſubſtile parallel to the interſection of the perpendicular circle with the parallel circle; therefore the contained angles will be equal; that is, the angle of the ſtile and ſubſtile is equal to the elevation of the pole above the dial plane, or above its parallel great circle.

Cor. 1. *If the dial plane be parallel to a great circle paſſing thro' the poles; the ſtile will have no angle of elevation at all above the dial plane, but will be parallel to the ſubſtile.*

Cor. 2. *If the ſtile be continued thro' the plane on the other ſide; its angle with the ſubſtile will be equal to the height of the other pole, above the other ſide of the plane.*

For the alternate angles being equal, the height of the ſtile above the ſubſtile, is the ſame on the other ſide of the plane. And the height of the pole being the ſame on both ſides its parallel great circle,

cle, it will alſo be the ſame on both ſides the dial plane; that is, equal to the angle of the ſtile and ſubſtile. Fig.

PROP. VI.

The interſection of the meridian of the place and the plane of the dial, is always the hour line of 12 *o'clock; and all the other hour lines are the interſections of the ſeveral meridians or hour circles with the plane of the dial, all paſſing thro' the ſtile.*

It is plain, that when the ſun is in the meridian of any place, that it is 12 o'clock in that place. But the plane of the meridian paſſes thro' the ſtile, and the ſhadow of the ſtile being in the ſame plane with the ſtile and the ſun; this ſhadow will appear on the dial plane where the meridian interſects it; and therefore this interſection is the hour line of 12.

Likewiſe ſince all the meridians interſect one another in the earth's axis, they may be ſuppoſed to interſect one another in the ſtile, which is parallel to the axis, ſince all the motions and revolutions are alike performed round both. And ſince the ſun, the ſtile, and the ſhadow, are all in one plane; the ſhadow muſt appear where this plane interſects the dial plane. And therefore when the ſun is in any hour circle, the ſhadow will fall upon that interſection, which therefore will be the proper hour line at that time.

Cor. 1. *All the hour lines meet in one point, which is the center of the dial.*

For as all the meridians paſs thro' the ſtile, they will all paſs thro' the point where the ſtile cuts the dial plane; that is, thro' the center of the dial. And all the interſections with the dial plane, that is, all the hour lines, will meet in that point, which is the center.

Cor. 2. *When the dial plane paſſes thro' the poles, the hour lines will be parallel to one another.*

For then the center, or the point where all the hour lines concur, is at an infinite diſtance.

PROP. VII.

In any dial with a center, if the hour lines be produced thro' the center, ſo as to appear on the other ſide of the plane; and if the ſtile be alſo produced thro' the plane; you will then have a dial for the back ſide of the plane; where the ſame hours belong to the ſame lines produced.

For ſince the meridians or hour circles interſect the dial plane in right lines, and the ſtile is in the plane of every hour circle; its ſhadow will always be in that interſection. Therefore in whatever part of the hour circle the ſun is, its ſhadow will always be in the interſection of it with the dial plane, and therefore in the ſame right line. But it will likewiſe be the ſame hour of the day, when the ſun is in the ſame hour circle, whether it ſhines upon one ſide of the plane or the other. Whence the ſame right line continued thro' the center will always denote the ſame hour. And if the ſun was in different or in oppoſite points of the ſame hour circle; any line will ſtill denote the ſame hour, tho' ſometimes it may be the morning hour, and ſometimes the afternoon hour.

PROP. VIII.

In all upright dials, the hour line of 12 is perpendicular to the horizon.

For the interſection of the meridian of the place with the dial plane, (by Prop. VI.) is the hour line of 12 o'clock. But the meridian of the place is perpendicular to the horizon; and the dial plane is alſo perpendicular to the horizon, therefore (Geom. V. 15.) their interſection is perpendicular to the horizon; that is, the 12 o'clock line is perpendicular to the horizon.

Fig.

PROP. IX.

In a direct eaſt or weſt reclining dial; the hour line of 12 *is parallel to the horizon.*

For the meridian of the place is perpendicular to the prime vertical; and the dial plane is perpendicular to the prime vertical; and therefore the common interſection of the meridian and dial plane is perpendicular to the prime vertical. But the interſection of the meridian and dial plane (by Prop. VI.) is the hour line of 12. Therefore the hour line of 12 is perpendicular to the prime vertical; and therefore is parallel to the horizon.

PROP. X.

A dial removed from its true place to any other, and placed in a ſituation quite parallel, and the ſun ſhineing on it; will always ſhew what a clock it is, at the place it came from.

For (by Prop. III.) the ſhadow of the ſtile of this dial, will fall upon the ſame hour line, at the ſame moment of time, wherever the dial is placed, in ſuch a parallel ſituation. Therefore at every particular hour of the day, at the original place, the ſhadow will fall on the ſame hour of the dial at the other place, juſt the ſame as if the dial had remained

Fig. remained at the firſt place. And therefore it always ſhews the hour of the day at the firſt place.

Cor. 1. *If any dial whatever be removed from its original place, to another place under the ſame meridian, and placed in a quite parallel ſituation, it will ſhew the hour of the day truly at this laſt place; that is, it will go truly there.*

For all places under the ſame meridian have each hour of the day, at the ſame moment of time. That is, it is 12 o'clock at one place, when it is 12 o'clock at the other; and one o'clock at the firſt place, when it is one at the other, &c.

Cor. 2. *But in other places, not under the ſame meridian, it will not ſhew the true time for this laſt place, in this ſituation, but will go faſter or ſlower, proportional to the difference of longitude from the original place, according as it is removed eaſtward or weſtward.*

P R O P. XI.

If a reclining plane be ſet ſo far back, (in a great circle perpendicular to that plane,) juſt as many degrees as it reclines, and parallel to its firſt poſition; it will then be an erect plane.

4. Let AB be the reclining plane, GB a great circle of the ſphere perpendicular to it, C the center; remove the plane to G, and draw CBD, and CG; then DB, being a vertical plane, the angle ABD will be the reclination of the plane AB. And AB being removed thro' the arch BG, into a parallel ſituation at G; ſo that the angle ABD may be equal to GB; or the angle GCB; then the line GC will be parallel to AB; and GC being a radius of the ſphere, it is perpendicular to the ſurface at G; but FG being alſo parallel to AB, FGC is one right line, and FG a vertical plane.

Cor. 1. *If a vertical plane* FG *be moved in a parallel direction thro' any arch of the great circle* GB, *perpendicular to it; its reclination* ABD *will be equal to that arch* GB. Fig. 4.

Cor. 2. *Likewiſe in a proclining plane* AB, *if it be ſet ſo far forward, that the arch* GB *may be equal to the degrees of proclination, and be placed parallel to itſelf; it will then be a vertical plane* FG.

Cor. 3. *If a reclining plane* AB, *continuing parallel to itſelf, be carry'd thro' the arch of a great circle perpendicular thereto* BH, *equal to the plane's inclination to the horizon,* ABG; *it will then be a horizontal plane at* H.

For ſince angle ABD = GCB, and ABG = BCH, GCH = DBG = a right angle, and AB or FG parallel to the horizon at H.

PROP. XII.

If any vertical plane ABID, *whoſe horizontal line is* BI, *be removed, to any place* b *in the plane of its great circle, in a quite parallel poſition. Its horizontal line* bi, *will be elevated above the horizon of the place* b, *the quantity of the arch* Bb. *Or angle* bbi *= arch* Bb. 5.

From the center C of the great circle B*b*, draw BC, C*b*, and draw *bf* parallel to BC, and the horizontal line *bb*, of the place *b*. Then ſince the angles CBI, *fbi*, C*bb* are right angles; we have angle C*bf* = *bbi*. But from the parallels BC, *bf*; angle C*bf* = BC*b*; therefore *bbi* = BC*b* = arch B*b*.

Cor. *Hence if the plane* BD *be removed thro' an arch of* 90 *degrees, the horizontal line* BI *will become a vertical line.*

Fig.

PROP. XIII.

If an erect declining dial plane be removed along the plane of its great circle, till its difference of longitude be equal to the plane's difference of longitude; it will become a full ſouth or north erect plane.

6. Let NESW be the horizon, Z the zenith, HZD the great circle wherein the dialling plane is placed, being at Z; NS the meridian, P the pole. Draw the meridian PB, perpendicular to the great circle HD; and B is the place where the dial plane is a full ſouth or north plane, becauſe the meridian of the place B is perpendicular to it. And (by Def. 19.) PB will be the meridian of the plane. Therefore (Def. 21) the plane's difference of longitude is the angle ZPB, equal to that thro' which the plane was moved.

Cor. 1. *The plane's difference of longitude is the difference between the longitude of the given place, and the longitude of the place where the dial plane is a direct ſouth or north plane.*

Cor. 2. *And to find the new latitude of* B. *In the ſpherical triangle* PZB; *rad: coſ. latitude* (S.ZP) : : *coſ. declination* (S.Z) : *coſ. new latitude* (S.PB.)

PROP. XIV.

A reclining eaſt or weſt dial plane will be an upright plane, under the ſame meridian, when placed 90 degrees from thence in a parallel ſituation.

6. For let Z be the zenith as before, WE the prime vertical, NAS the great circle parallel to the dial plane. Then the circle NAS cuts the meridian NS in the points N and S. And (by Prop. XI.), if

the plane be moved from Z to A in a parallel ſituation, it will be erect at A. And (by Prop. XII.) it will be erect in any place of its great circle NAS. And therefore at N and S, it will both be erect, and in the meridian, where ZN and ZS are 90 degrees. Fig. 6.

Cor. *The new latitude* N *or* S, *where an eaſt or weſt recliner is erect, will be the complement of the old latitude at* Z. *And the new declination is the complement of the reclination.*

For NP = complement of ZP, and ZP = comp. NP = new lat. Alſo the angle WNA or the declination = comp. ANZ or of AZ the declination.

PROP. XV. *Prob.*

The declination and reclination of a dial plane being given. To find the new latitude and declination, where it ſhall be an upright plane, upon the ſame meridian.

Let NASW repreſent the ſphere, NESW the horizon, P the pole, Z the zenith, NS the meridian, WQE the equinoctial, WZE the prime vertical, P1, P2, P3, &c. hour circles, HMA a great circle parallel to the dial plane. Draw the vertical ZB perp. to HMA. Then M will be the new place, upon the meridian NS, where the plane is upright, and PM the complement of its latitude. Therefore in the right angled ſpherical triangle ZBM, there is given the angle BZM the declination, and the ſide ZB the reclination; to find the angle BMZ the complement of the declination at M; and the hypothenuſe ZM, which ſubtracted from PZ the comp. of the latitude, when the plane runs between the zenith and pole, (otherwiſe PZ ſubtracted from it, when the plane is below the pole), leaves PM, the complement of the new latitude. 7.

Fig. 7. Cor. 1. *As coſine of the old declination*
Radius:
So tangent. reclination:
Tangent of an arch. And the difference between this arch and the complement of the latitude, is the complement of the new latitude.

For (by Caſe 9. right angled ſpherical triangles) coſ. BZM : rad : : tan. BZ : tan. ZM.

Cor. 2. *Radius:*
Sine of the declination: :
Coſine — reclination:
Sine — new declination.

For (by Caſe 8.) rad : S.BZM : : coſ. BZ : coſ. BMZ.

PROP. XVI. *Prob.*

The declination and reclination of a plane being given; to find the new latitude, and longitude, where it ſhall be a direct north or ſouth upright plane.

Let NS be the meridian as before, Z the zenith, P the pole, and HMA the great circle, to which the dial plane is parallel. Draw the meridian PF1, perpendicular to the given circle HMA, then F is the place where the plane becomes a direct north or ſouth vertical plane. And drawing ZB perp. to HMA. In the triangle BMZ there is given the reclination BZ, and declination = angle BZM, to find ZM and angle BMZ, as in the laſt Prop. Then in the triangle PMF right angled at F; there is given PM and angle PMF; to find the complement of new lat. PF, and difference of longitude MPF.

Cor. 1. *Let* B = PM (*the difference between* MZ *and the comp. of the latitude. Then*)
Radius:
Sine of B : :

Coſine — declination at M :
Coſine — new latitude.
For (by Caſe 2.) rad : S.PM : : S.PMF : S.PF.

Fig. 7.

Cor. 2. *Radius* :
Coſine of B : :
Tan. PMF :
Cotan. MPF *the diff. longitude,* (by Caſe 3.)

Or thus,

Radius :
Cotan. B : :
Cotan. new latitude :
Coſine — diff. longitude.

For (by Caſe 4.) Rad : Cotan. PM : : Tan. PF : Coſ. MPF.

PROP. XVII.

Any dial whatever, whether direct, declining, or inclining, &c. made for any latitude; will ſhew the time truly, in all places of the ſame latitude; provided it be placed in a like ſituation, in regard to the meridian of the place; that is, provided it have the ſame declination, reclination, and horizontal poſition.

For if the dial be placed alike in different places under the ſame parallel, the ſhadow of the gnomon, at the ſame hour of the day, cannot but fall upon the ſame hour lines. For imagine two dials to be made for two ſuch different places; there being exactly the ſame data for both; the ſame hour lines and every thing elſe will be the ſame in both. And that they may both go true in their reſpective places, for the apparent time, there is nothing more required, than to give them both the ſame ſituation.

Cor. 1. *Any ſort of dial made for one place, will go true in any other place, tho' in a different latitude, and longitude; provided it be ſet in a proper ſituation.*

Fig. For by this Prop. if it be removed to any place in the ſame parallel, or of a different longitude; and placed alike, in regard to the meridian, horizon, &c. it will go truly there, for that place. And by Cor. 1. Prop. X. if the ſame dial be removed from this ſecond place to a third place, under the ſame meridian; and ſet there in a quite parallel poſition, to that it had in the ſecond place; it will go truly at this third place. Therefore it would go alike at all the three places.

Cor. 2. *Any dial made for any place of the world, will go truly at any other place of the world, being placed in a proper poſition.*

PROP. XVIII. *Prob.*

To find the requiſites for a horizontal dial.

7. Here is nothing required, but the latitude of the place; from which the angles of the ſeveral hour lines with the 12 o'clock line muſt be found.

Let NESW be the horizon, NS the meridian, P the pole, Z the zenith; P*c*, P*d*, P*g*, &c. the ſeveral hour circles upon the ſphere. Then we are to find the arches N*c*, N*d*, N*g*, &c. or the angles at the center Z of the circle NESW. Therefore in the right angled ſpherical triangle PN*c*, there is given PN the latitude of the place, and angle NP*c*, which is 15 deg. for an hour, 30 deg. for 2 hours, 45 deg. for 3 hours, &c. to find the oppoſite ſide N*c*, or N*d*, or N*g*, &c. which are the hour angles.

Cor. 1. *Radius* :
Sine of the latitude : :
Tan. hour arch :
Tan. hour angle.

For (by Caſe 7. right angled ſpherical triangles,) Rad : S.PN : : tan. NP*c*, or the correſpondent arch of the equinoctial : tan. N*c*, which is the meaſure of Fig. the hour angle : : and ſo tan. NP*d* : tan. N*d*, &c. 7.

Cor. 2. *A horizontal dial made for ſouth latitude, will ſerve equally for the ſame degree of north latitude; turning the ſouth end to the north, and reckoning the hours the contrary way from the meridian.*

PROP. XIX. *Prob.*

To find the requiſites for an erect direct ſouth or north dial.

Here the height of the pole above the plane muſt 7. be had, and this is equal to the complement of the latitude. Therefore let NESW be the horizon, Z the zenith, P the pole; WZE the plane of the dial. Then the pole is elevated above the plane of the dial, the arch PZ, which is the complement of NP, or the complement of the latitude. Then if the hour circles P1, P2, P3, &c. be drawn, cutting the right circle WZE in *q*, *r*, *s*, &c. Then to find the arches Z*q*, Z*r*, Z*s*, &c. In the right angled ſpherical triangle PZ*q*, there is given PZ the complement of the latitude, and the angle ZP*q*; to find the arch Z*q*, or the angle at the center. And the like for the arches Z*r*, Z*s*, &c, in the triangles ZP*r*, ZP*s*, &c.

Cor. 1. *Radius* :
Coſine-latitude : :
Tangent-hour arch :
Tang. hour angle.

For (by Caſe 7.) rad : S.PZ : : tan. ZP*q* : tan. Z*q* : : tan. ZP*r* : tan. Z*r*, &c.

Cor. 2. *In a direct ſouth or north inclining dial, the ſame analogy muſt be uſed, only taking for the ſecond term, the ſine of the pole's height above the dial plane; to be found by* Prop. XI.

Fig. 7. Cor. 3. *A direct ſouth dial, is a horizontal dial on the ſouth ſide of the globe; and a north dial, on the north ſide.*

PROP. XX. *Prob.*

To find the requiſites, for erect declining dials.

Here the latitude of the place, and the declination of the plane muſt be given; and what is further wanted muſt be found from theſe. And that is, 1. the height of the pole above your dial plane. 2. The diſtance of the ſubſtile from 12 o'clock. And 3 the plane's difference of longitude.

Let NESW be the horizon, C its center, NS the meridian, P the pole, and Z the zenith; HA the declining vertical plane; P1, P2, P3, &c. meridians or hour circles, cutting the circle HA in *t*, *v*, &c. Let the meridian P*v*2 be perpendicular to HA, then the plane of the meridian P*v*2 will be perpendicular to the plane of the dial; and therefore the angle ZP*v* is the plane's difference of longitude (by Def. 21), and the arch P*v* is the height of the pole above the plane. And the arch Z*v* is the ſubſtile diſtance from the meridian. Therefore in the right angled ſpherical triangle PZ*v*, all the 3 requiſites will be found. For we have PZ the complement of the latitude, and the angle PZA the complement of the declination AZE; from whence will be found P*v*, Z*v*, and the angle ZP*v*.

Cor. 1. *As radius* :
Sine declination : :
Cotan. latitude :
Tan. ſubſtile's diſtance from 12 *o'clock, and lies the contrary way as the declination.*

For (by Caſe 1. of right angled ſpherical triangles), rad : coſ. PZ*v* : : tan. PZ : tan. Z*v*.

Cor. 2. *Radius*: Fig. 7.
Coſine declination: :
Coſine-latitude:
Sine ſtile's height.

For (by Caſe 2), rad : S.PZ : : S.PZ*v* : S.P*v*.

Cor. 3. *S. latitude*:
Radius : :
Tan. declination:
Tan. plane's diff. longitude.

For (by Caſe 3), rad : coſ. PZ : : tan. PZ*v* : cotan. ZP*v* : : tan. ZP*v* : cotan. PZ*v*.

Cor. 4. *For the hour angles, it will be,*
As radius:
Sin. pole's height : :
Tan. hour arch from the meridian of the plane:
Tan. hour angle from the ſubſtile.

For let P*t*1 be any hour circle; in the triangle P*tv*, there is given the pole's height P*v*, and the hour arch, equal to the angle *t*P*v*; to find *tv*, (by Caſe 7), rad : S.P*v* : : tan. *t*P*v* : tan. *tv* = angle at the center, or the hour angle.

That all the requiſites are rightly found from theſe proportions, will appear thus. The planes of the two meridians PZ, P*v*, interſect one another on the ſphere, in an angle ZP*v*, equal to the plane's difference of longitude. And the ſame meridians, interſect the plane of the dial in an angle equal to ZV, as it ſhould be. For they interſect one another in the center of the ſphere, which is the center of the dial. And they interſect the dial plane HA in Z and *v*. and Z*v* meaſures that angle at the center. And for the ſame reaſon, *tv* meaſures the hour angle from the ſubſtile.

Again P*v* meaſures the angle of the ſtile above the ſubſtile. For P*v* is perpendicular to HA, and the arch P*v* meaſures the angle formed at the center of the ſphere, which is the center of the dial;

Fig. 7. one ſide being the axis drawn from P, the other the ſubſtile drawn from V.

Alſo ZPv has been ſhewn to be the plane's difference of longitude.

Cor. 5. *The plane's dif. longitude on the ſphere, is converted into the ſubſtile's diſtance from* 12, *by the interſection of the two meridians (of the place and of the plane), with the dial plane.*

For the angle ZPv expreſſes one, and arch Zv the other.

Cor. 6. *And every hour angle on the ſphere, is converted into the hour angle in the dial, by the interſection of the ſame two hour circles, with the plane of the dial.*

For the angle *t*Pv expreſſes the hour angle on the ſphere, and *tp* the angle at the center.

SCHOLIUM.

Tho' the rules laid down in theſe Corollaries are ſufficient for finding the ſeveral requiſites; yet by having other data, they may be found by other proportions, ſome of which for variety, I ſhall here ſet down.

In Cor. 1. For finding the ſubſtile's diſtance.
Coſ. pole's height above the plane (Pv) :
: *Sine latitude* (*coſ.* PZ) : :
: *Radius* :
Coſ. ſubſtile's diſt. from 12.
By Caſe 6.

Or *Radius* :
Sin. declination (*coſ.* PZv) : :
Tan. pole's height above the plane (Pv) :
Sin. ſubſtile's diſtance (Zv).
By Caſe 10.

In Cor 2. For finding the ſtile's height. Fig. 7.

Coſ. ſubſtile's diſtance (*Zv*) :
Sin. latitude (*coſ.* PZ) : :
Radius :
Coſ. ſtile's height (P*v*).
By Caſe 6.

Or *Radius*
Sin. ſubſtile's diſtance (Z*v*) :
Cotan. declination (*tan.* PZ*v*) :
Tan. ſtile's height (P*v*).
By Caſe 7.

In Cor. 3. For the plane's diff. longitude.
Coſ. latitude (S.PZ) :
Radius : :
Sin. ſubſtile's diſtance (Z*v*) :
Sin. plane's diff. longitude (ZP*v*).
By Caſe 5.

Or *Radius* :
Tan. latitude (*cotan.* PZ) : :
Tan. ſtile's height (P*v*) :
Coſ. plane's dif. longitude (ZP*v*).
By Caſe 4.

Or *Radius* :
Coſ. declination (S.PZ*v*) : :
Coſ. ſubſtile's diſtance (Z*v*) :
Coſ. plane's dif. longitude (ZP*v*).
By Caſe 8.

Or *Coſ. ſtile's height* (P*v*) :
Sin. declination (*coſ.* PZ*v*) : :
Radius :
Sin. plane's dif. longitude (ZP*v*).
By Caſe 11.

Or *Sin. ſtile's height* (P*v*) :
Radius : :
Tan. ſubſtile's diſtance (Z*v*) :
Tan. plane's dif. longitude (ZP*v*).
By Caſe 13.

PROP.

Fig. 7.

PROP. XXI. *Prob.*

To find the requiſites for a direct eaſt or weſt inclining or reclining dial.

The latitude of the place and the reclination of the plane muſt be given. And from theſe muſt be found, 1. The height of the pole above the plane. 2. The ſubſtile's diſtance from 12 o'cock. 3. The plane's difference of longitude.

8. Let HZON be the meridian, HO the horizon, ZN the prime vertical, P the pole, Z the zenith. HFO the inclining plane. Let the meridian PFD be perpendicular to HFO. Then the plane of the meridian PFD will be perpendicular to the plane of the dial, and PF will be the meridian of the plane (Def. 19.), and OPF the plane's difference of longitude (Def. 21.) alſo PF (or angle PCF) will be the height of the pole above the plane. And FO (or angle FCO) the diſtance of the ſubſtile from the meridian. Therefore in the right angled ſpherical triangle PFO, there is given PO the latitude of the place, and the angle POF the reclination or proclination of the plane; to find FO, FP, and angle OPF.

Cor. 1. *Radius* :
Coſ. reclination (or proclination) : :
Tan. latitude : :
Tan. ſubſtile's diſt. from 12 *o'clock*; *running upwards towards the north, or downwards towards the ſouth.*

For (by Caſe 1), rad : coſ. POF : : tan. OP : tan. OF = angle OCF, which the meridian FC (or ſubſtile) makes with CO the 12 o'clock line, (or meridian of the place), at C the center of the dial.

Cor. 2. *Radius*:
Sin. latitude : :
Sin. reclination (*or proclination*) :
Sin. ſtile's height.

Fig. 8.

For (by Caſe 2.) rad : S.OP : : S.POF : S.PF = angle PCF, which the ſtile PC makes with the ſubſtile FC, at C the center of the dial.

Cor 3. *Radius* :
Coſ. latitude : :
Tan. reclination (*or proclination*) : :
Cotan. plane's dif. longitude.

For (by Caſe 3.) rad : : coſ. OP : : tan. POF : cotan. OPF = angle made by the meridian of the place, and that of the plane.

Cor. 4. *For the hour angles, it will be,*
As radius :
Sin. pole's height above the plane : :
Tan. hour arch, from the meridian of the plane :
Tan. hour angle from the ſubſtile.

For, drawing any hour circle PID, in the right angled ſpherical triangle PFI, there is given PF, and angle FPI, to find FI; by Caſe 7. Rad : S.PF : : tan. FPI : tan. FI = angle FCI, which the correſpondent hour line on the dial makes with the ſubſtile, at the center.

The calculation in the triangle OPF is for an eaſt dial; but for a weſt dial, let A be the pole, and draw the meridian AGB perpendicular to HGO; then there are the ſame data in the triangle AGH, as in PFO; whence the ſame things will be found for a weſt dial.

SCHOLIUM.

Other rules for finding the requiſites may be deduced from the ſame triangle CPF, as follows.

Cor. 1. For the ſubſtile's diſtance.
As *Coſ. pole's height above the plane* (PF) :

Fig. 8.
Coſ. latitude (PO) : :
Radius :
Coſ. ſubſt. diſt. from 12 o'clock (FO).
By Caſe 6.

Or *Radius* :
Cotan. reclination (POF) : :
Tan. ſtile's height, or pole's height (PF) :
Sin. ſubſtile's diſtance (FO).
By Caſe 10.

Cor. 2. For finding the ſtile's height.

As *Coſ. ſubſtile's diſtance* (FO) :
Coſ. latitude (PO) : :
Radius :
Coſ. ſtile's height (PF).
By Caſe 6.

Or *Radius* :
Sin. ſubſtile's diſtance (FO) : :
Tan. reclination (POF) :
Tan. ſtile's height (PF).
By Caſe 7.

Cor. 3. For the plane's diff. longitude.

As *Radius* :
Cotan. latitude (PO) : :
Tan. height of the pole above the plane (PF) :
Coſ. plane's dif. longitude (FPO).
By Caſe 4.

Or *Coſ. Pole's height above the plane* (PF) :
Coſ. reclination (POF) : :
Radius :
Sin. plane's dif. longitude (FPO).
By Caſe 11.

Or *Sin. latitude* (OP) :
Radius : :
Sin. ſubſtile's diſtance (FO) :
Sin. plane's diff. longitude (FPO).
By Caſe 5.

PROP.

PROP. XXII. *Prob.*

To find the requiſites for ſouth declining reclining dials.

Here muſt be given the latitude of the place, and the declination and reclination of the plane; and from theſe muſt be computed, 1. The height of the meridian. 2. The height of the pole above the plane. 3. The diſtance of the ſubſtile from the hour line of 12. And 4. The plane's difference of longitude.

Let WPASH repreſent the ſphere, C its center, WNES the horizon, NS the meridian, WQE the equinoctial, P the pole, Z the zenith, and HMA the plane of the dial. Draw the meridian PF1 perpendicular to the circle HBA; then the plane of this meridian will be perpendicular to the plane of the dial, and therefore PF will be the meridian of the plane, and the angle MPF the plane's difference of longitude, and PF the height of the pole above the plane, and MF the ſubſtile's diſtance from 12, and MA the height of the meridian. Therefore all the requiſites will be found in the triangles NAM and PMF, right angled at N and F. Therefore in the triangle NAM, there is given NA the complement of the declination AE, and angle NAM the complement of the reclination MAZ; to find AM, NM, and angle NMA. And the latitude NP being given, PM will be known. Then in the triangle MPF, there is given PM, and the angle PMF, to find MF, PF, and angle MPF, as required. 7.

Cor. 1. *Sin. reclination* :
Radius : :
Cotan. declination :
Tan. height of the meridian.

For

Fig. 7. For (by Caſe 9, right angled ſpherical triangles), coſ. MAN : rad :: tan. NA : tan. AM = angle ACM at the center of the ſphere, or of the dial, contained between the horizontal line CA, and the meridian CM or 12 a clock.

Cor. 2. *Radius* :
Coſ. declination ::
Cotan. reclination :
Tan. of an arch A.

Then put B = *difference between* A *and the latitude. If* A *is leſs, your pole is elevated; if greater, the oppoſite pole.*

Then *Cotangent of* B :
Coſ. reclination ::
Sin. declination :
Tan. ſubſtile's diſtance from 12. *And the ſubſtile runs upwards towards the north in reclining S.planes, and their oppoſite incliners.*

For in the triangle NAM (by Caſe 7), rad : S.NA :: tan. NAM : tan. NM = A; then NM − NP = PM = B. Alſo (by Caſe 8), rad : S.NAM :: coſ. NA : coſ. NMA $= \frac{\text{S.NAM} \times \text{coſ. NA}}{\text{rad}}$.

And in the triangle MFP, (by Caſe 1), cot. PM : rad :: rad : tan. PM :: coſ. NMA $\left(= \frac{\text{S.NAM} \times \text{coſ NA}}{\text{rad}}\right)$: tan. MF $= \frac{\text{S.NAM} \times \text{coſ. NA}}{\text{cotan. MP}}$ = angle MCF, at the center, and in the plane of the dial.

Cor. 3. *Coſ. ſubſtile's diſtance* :
Coſ. B ::
Radius :
Coſ. ſtile's height.

For (by Caſe 6), coſ. MF : coſ. MP (B) :: rad : coſ. PF = angle PCF at the center of the dial, CF being in its plane, is the ſubſtile.

Cor. 4. *Sine of* B : Fig. 7.
Radius : :
Sin. ſubſtile's diſtance :
Sin. plane's diff. longitude.

For (by Caſe 5), S.PM (B) : rad : : S.MF : S.MPF = angle on the ſphere, between the meridian of the plane, and the meridian of the place.

Cor. 5. *For the hour angles it will be*
As Radius :
Sin. pole's height above the plane : :
Tan. hour arch from the meridian of the plane :
Tan. hour angle from the ſubſtile.

For drawing any hour circle P*o*3, interſecting the circle HMA in *o*. Then there will be given the angle FP*o*, contained between the meridian of the plane PF1, and the meridian P*o*3. Therefore in the right angled triangle PF*o*, there is given PF the height of the pole above the plane, and angle FP*o*, to find F*o* by caſe 7. Rad : S.PF : : tan. FP*o* : tan. F*o* = angle FC*o*, at the center of the dial, made between the ſubſtile ZF, and the hour line C*o* correſponding to the hour circle P*o*.

Cor. 6. *The height of the meridian is equal to the angle made at the center of the dial, by the* 12 *o'clock line, and the horizontal line; and ſhews how much the* 12 *o'clock line is elevated above the horizon; and it runs upwards in reclining planes, and downwards in proclining ones, towards the ſame hand as the declination is.*

For MA is the meaſure of the angle MCA contained between the 12 o'clock hour line MC, and the horizontal line CA; being in the plane of the dial ACHM; and the angular point at C, the center of the dial.

For the ſame reaſon MF, or the angle MCF, ſhews what angle the 12 o'clock line MC makes

Fig. 7. with the ſubſtile CF, at the center C; the triangle MCF being in the plane of the dial. And likewiſe F*o* or angle FC*o* ſhews the angle contained between the ſubſtile FC and the hour line F*o*; the triangle FC*o* being alſo in the plain of the dial; and ſo of others.

Here the plane of the dial ABH falls between the zenith and the pole, but if it paſs between the pole and the horizon, the calculation will be the ſame; and then the pole P will be elevated on the other ſide of the plane. But if the dial plane paſs thro' the pole, then B will be o; and MF, PF, are o. Hence

Cor. 7. *If* A (NM) *is greater than the latitude* (NP), *the pole is depreſſed below the plane, and the oppoſite pole elevated above it. If* A *is leſs than the latitude, the pole is elevated above it. If they be equal, the pole falls in the plane of the dial; and then* MF, PF, *are nothing.*

For when A = latitude, B = o.

Cor. 8. *When the reclining plane falls in the pole; then to find the plane's difference of longitude, it will be,*

As radius :
Sin. latitude : :
Tan. declination :
Tan. plane's dif. longitude.

For when M approaches near to P, the triangle MPF will be a plane triangle, and the angle MPF (the plane's dif. longitnde) will be the complement of PMF or NMA, that is, when M falls upon P, the plane's dif. longitude will be the complement of NPA. Therefore in the triangle NPA, there are given NP the latitude, and NA the comp. declination, to find the angle NPA (by Caſe 13), S.NP : rad : : tan. NA : tan. NPA : : cotan. NPA : cotan. NA.

Otherwiſe thus. Fig. 7.

Radius :
S. declination : :
Coſ. reclination :
S. plane's dif. longitude.

For (by Caſe 8), rad : S.NAP : : coſ. NA : coſ. NPA. And other proportions may eaſily be found to do the ſame thing, as this which follows.

Coſ. latitude (NP) :
S. reclination (*coſ.* NAP) : :
Radius :
Coſ. dif. longitude (S.NPA).

Cor. 9. *The ſame rules ſerve equally for an inclining plane, uſing the proclination inſtead of the reclination; but here* A *will always be greater than* 90 *degrees; and the oppoſite pole always elevated above your plane.* For then the point M falls between Z and S.

SCHOLIUM.

From the ſame two triangles NAM, and PMF, other rules for finding the requiſites may be deduced; ſuch are the following.

In Cor. 2. For the ſubſtile's diſtance.

As *Radius* :
Coſ. reclination (S.NAM) :
Sin. declination (coſ. NA) : :
Coſ. an angle M (NMA).

Then *Radius* :
Coſ. M (PMF) : :
Tan. B (PM) :
Tan. ſubſtile's diſtance (MF).
By Caſe 1.

Or *Coſ. pole's height above the plane* (PF) :
Coſ. B (PM) : :
Radius :
Coſ. ſubſtile's diſtance (MF).
By Caſe 2.

ig. 7. Or *Radius* :
Cotan. M (PMF) : :
Tan. pole's height above the plane (PF) :
Sin. ſubſtile's diſtance (MF).
By Caſe 10.

Or *Cotan. declination* (*tan.* NA) :
Tan. pole's height above the plane (PF) : :
Sine of A (NM) :
Sin. ſubſtile's diſtance (MF).

This appears from Cor. 1. Prop. 27. Sect. III. Trigonometry.

In Cor. 3. for the angle of the ſtile's height.

As *Radius* :
Sine of B (PM) : :
Sine of M (PMF) :
Sin. ſtile's height above the plane (PF).
By Caſe 2.

Or *Coſ. ſubſtile's diſtance* (MF) :
Radius : :
Coſine of B (PM) :
Coſ. ſtile's height (PF).
By Caſe 6.

Or *Radius* :
Tangent of M (PMF) : :
Sin. ſubſtile's diſtance (MF) :
Tan. ſtile's height (PF).
By Caſe 7.

Or *Sine of* A (NM) :
Sin. ſubſtile's diſtance (MF) : :
Cotan. declination (*tan.* NA) :
Tan. ſtile's height (PF).

By Cor. 1. Prop. 27. Sect. III. Trigonometry.

Or *Sin. height of the meridian* (MA) :
Sine of B (PM) : :

Coſ. declination (S.NA) : Fig. 7
Sin. ſtile's height (PF).
By Cor. 1. Prop. 26. Sect. III. Trigonometry.

In Cor. 4. For the plane's dif. longitude.

As *Radius* :
Coſine of B (PM) : :
Tangent of M (PMF) :
Cotan. plane's dif. longitude (MPF).
By Caſe 3.

Or *Radius* :
Cotan. B (PM) : :
Tan. pole's height above the plane (PF) :
Coſ. plane's dif. longitude (MPF).
By Caſe 4.

Or *Coſ. ſtile's height* (PF) :
Coſ. M (PMF) : :
Radius :
Sin. plane's dif. longitude (MPF).
By Caſe 11.

Or *Coſ. pole's height above the plane* (PF) :
Sin. declination (*coſ.* NA) : :
Coſ. reclination (S.NAM) :
Sin. plane's dif. longitude (MPF).

For (by Caſe 8), rad : coſ. NA : S.NAM : coſ. $\text{NMA} = \frac{\text{coſ. NA} \times \text{S.NAM}}{r}$, and rad : coſ. PF : : S.MPF : coſ. PMF or $\text{NMA} = \frac{\text{coſ. PF} \times \text{S.MPF}}{r}$; therefore coſ. NA × S.NAM = coſ. PF × S.MPF.

It may be obſerved, that inſtead of the two triangles NAM, and MBF, the two triangles BZM and MBF may be made uſe of; the triangle BZM being complemental to the triangle NAM; from which two triangles the very ſame concluſions will follow as before.

Fig.

PROP. XXIII. *Prob.*

To find the requiſites for a north declining, reclining dial.

Here alſo the latitude of the place, and the declination and reclination of the plane muſt be given; to find, 1. the height of the meridian. 2. The height of the pole above the plane. 3. The diſtance of the ſubſtile from the hour line of 12. 4. The plane's difference of longitude, as before.

9. As in the laſt Prob. Let WNES be the horizon, NS the meridian, P the pole, Z the zenith, and HMA the plane of the dial, and PF an hour circle perpendicular to it, which will be the meridian of the plane. Let ZB be alſo perpendicular to HBA. Then all the requiſites will be found in the right angled triangles ZBM and PFM. Therefore,

In the triangle BMZ, there is given the angle MZB the declination equal to EZA, and ZB the reclination of the plane; to find BM, ZM, and angle ZMB; and having ZP the complement of the latitude, PM will be = PZ + ZM. Then in the triangle MPF, there is given PM and angle PMF; to find the ſubſtile's diſtance MF, the ſtile's height PF, and plane's dif. longitude MPF or MP*f*.

Cor. 1. *Sin. reclination* :
Radius : :
Cotan. declination :
Tan. height of the meridian, running upwards, towards the ſame hand as the declination. But to the contrary hand, in procliners.

For (by Caſe 7), rad : S.ZB : : tan. MZB : tan. BM : : cot. BM : cot. MZB. And here HM being the height of the meridian, lies now towards the weſt; which was towards the eaſt, when the reclination of the plane was northward. Fig. 9.

Cor. 2. *Radius* :
Coſ. declination : :
Cotan. reclination :
Cotan. of an arch A.

Then put B = A + *complement of the latitude. And the north pole is always elevated in recliners.*

Then *Cotan* B :
Coſ. reclination : :
Sin. declination :
Tan. ſubſtile's diſtance from 12; *which will be greater than* 90°, *when* B (PM) *is greater than* 90°.

For in the triangle ZMB (by Caſe 9), coſ. BZM : rad : : tan. ZB : tan. ZM (A) : : cot. ZM : cot. ZB. And in the triangle NAM (by Caſe 8), rad : S.NAM : : coſ. NA : coſ. NMA or PMf = $\frac{\text{S.NAM} \times \text{coſ. NA}}{\text{rad}}$. And in the triangle PM$f$ (by Caſe 1), rad : coſ. PMF : : tan. B : tan. FM. Or cotan. B : rad : : coſ. PMF $\left(\frac{\text{S.NAM} \times \text{coſ. NA}}{\text{rad}}\right)$: tan. FM or fM = tan. ſubſtile's diſtance.

Cor. 3. *Coſ. ſubſtile's diſtance* :
Coſ. arch B : :
Radius :
Coſ. ſtile's height. And if B *is greater than* 90°, *the ſtile's height is greater than* 90°; *or it will appear elevated towards the north part of the plane.*

For in the triangle MPf (by Caſe 6), coſ. MF : coſ. MP (B) : : rad : coſ. FP or fP.

Fig. 9. Cor. 4. *Sine of* B :
Radius : :
Sin. ſubſtile's diſtance :
Sin. plane's dif. longitude.

For in the triangle MP*f* (by Caſe 5), S.PM (B) : radius : : S.MF : S.MPF or MP*f* the dif. longitude, or the angle on the globe between the meridian of the place and that of the plane.

Cor. 5. *Radius* :
Sin. pole's height above the plane : :
Tan. hour arch from merid. of the plane :
Tan. hour angle from the ſubſtile.

This is demonſtrated as in Cor. 5. Prop. XXII.

Cor. 6. *When* A *is equal to the latitude, then* B *is equal to* 90 *degrees; and the ſubſtile's diſtance, and alſo the plane's difference of longitude, are each* 90 *degrees.*

For (by Cor. 2), cotan. B (0) : coſ. reclination : : ſin. declination : infinity = tan. ſubſt. diſtance = 90°. And (Cor. 4.) S.B (rad : radius : : S.ſubſt. diſtance (90) : S.plane's dif. longitude = radius, the ſine of 90.

Cor. 7. *The requiſites may alſo be found by the analogies laid down in the Schol. of the laſt Prop. firſt finding* A *and* B *by this Prop.*

Cor. 8. *When* A = *the latitude, then the ſtile's height will be equal to the angle* ZMB, *and therefore*
Radius :
Sin. declination : :
Coſ. reclination :
Coſ. ſtile's height.

For when ZM = latitude, M falls upon Q, the interſection of the equinoctial and meridian, and then PM = 90° = oppoſite angle PFM. Therefore the ſide PF (or ſtile's height) = its oppoſite angle PMF. Therefore in the triangle ZMB, to find the angle M (by Caſe 8), rad : S.MZB : : coſ. ZB : coſ. angle M = PF. Fig. 9.

Or thus.

Sin. latitude :
Radius : :
Sin. reclination :
Sin. ſtile's height.

For (by Caſe 5), S.ZM : rad : : S.ZB : S.ZMB = S.ſtile's height PF.

Or thus.

Radius :
Coſ. latitude : :
Tan. declination :
Cotan. ſtile's height.

For (by Caſe 3), rad : coſ. ZM : : tan. MZB : cot. ZMB or PMF = PF.

Cor. 9. *In all theſe dials, the ſubſtile lies to the contrary ſide in regard to the declination. Or the ſubſtile lies always northward (in north latitude) from the upper part of the 12 o'clock line.*

For the ſtile is a line fixt in poſition, and therefore a moveable plane being made to decline, to either hand, the ſubſtile (which is perpendicular under the ſtile) muſt needs lie to the other hand. And ſo the ſubſtile lies northward from the upper part of the meridian, or ſouthward from the lower part.

Cor. 10. *The ſame rules that ſerve for finding the requiſites in a reclining ſouth plane, will ſerve equally for finding the requiſites in its oppoſite north incliner or procliner; and lie on the ſame hand, if you face the plane. But the contrary pole will be elevated.*

For by Prop. VII. The ſame hours belong to both ſides of the plane; and therefore the ſame requiſites; ſince the inclination of the plane to

the

Fig. 9. the horizon continues the ſame, and likewiſe the declination; only in one, they lie eaſt, in the other weſt.

Cor. 11. *Hence alſo the ſame requiſites and the ſame hours belong equally to two dials, that have equal declinations, one eaſtward, the other weſtward; the inclination, if any, remaining the ſame. But then they all lie towards contrary hands.*

For as there are the ſame data in both, the quantity of theſe requiſites, and of the hour angles, muſt needs remain the ſame; and differ only in quality, or in their poſition towards the right and left.

PROP. XXIV.

In any dial whatever, if a line be drawn parallel to any hour line, to interſect the other hour lines, and note the ſixth hour line from this. Then any two hour lines on each ſide this ſixth, which are equidiſtant in hours, will alſo be equidiſtant along this parallel line.

11. Let the plane CGLH be perpendicular to the earth's axis paſſing thro' C, CA any hour line, and CD another at ſix hours diſtance. Then ſince in this caſe, all the hour angles about C, as BCD, DCF, are equal; therefore ACD being ſix hours, is a right angle.

Now ſuppoſe G*abdf*E is any other plane, paſſing alſo through GCH, cutting the planes of theſe hour circles, in the lines C*a*, C*b*, C*d*, C*f*; which will be the correſpondent hour lines in that plane. Alſo let the plane B*bf*F be parallel to the plane of the hour circle CA*a*, and therefore perpendicular to the plane GLH, cutting the former plane G*abdf*E in the line *bf*E, and the planes of the hour circles, in the lines B*b*, D*d*, F*f*; which conſequently will be perpendicular to the plane GLH, and therefore

parallel to one another. Alſo *b*E is parallel to C*a*, Fig. 11.
by Prop. 11. B. V. Geometry.

Now ſince CD is perpendicular to BF, and the angles BCD, DCF, equal; therefore BD = DF. Whence in the triangle B*b*E, ſince B*b*, D*d*, F*f* are parallels, and BD = DF, therefore *bd* = *df*. That is, in any dial plane C*abdf*E, if the line *b*E be drawn parallel to C*a*, to cut the hour line C*d*, ſix hours from it, and ſeveral more C*b*, C*f*, on each ſide; then thoſe that are equidiſtant on each ſide in hours, are alſo equidiſtant, in the line *bd*E, ſet contrary ways from C*d*.

Cor. *From hence you have a method of drawing all the hour lines in the dial, if you have ſeven of them drawn.*

As ſuppoſe you have the hour lines C8, C9, 12.
C10, C11, C12, C1, and C2, drawn; and you want to draw C7, C6, C5, C4. To the hour line C2 draw the parallel *kq*, cutting the given hour lines in *d*, *b*, *o*, *p*, *q*. From *d* in the hour line C8 (which is ſix hours from C2;) make *df* = *db*, *dg* = *do*, *dh* = *dp*, and *dk* = *dq*; and thro' *f*, *g*, *h*, *k*, draw the hour lines C7, C6, C5, C4.

Or if you want the hour lines on the right hand, it is but drawing a parallel to C*d*, or C8, and proceeding as before.

Alſo in any inclining or declining dial, when half the hour lines are drawn, the other half may be drawn, by drawing a parallel to the ſixth hour line, to cut the reſt; and transferring the diſtances of theſe hour lines, to the oppoſite ſide of the laſt hour line; which parallel ſerves for a line of contingence.

Having now ſhewn how the requiſites are found in all ſorts of dials, from the rules of ſpherical trigonometry, on which they depend; I ſhall now reſolve

ſome

Fig. 12. ſome problems relating to the time of the ſun's ſhining on theſe dial planes, which likewiſe depend on ſpherical trigonometry.

In the foregoing problems, I have all along ſuppoſed the dial was to be drawn upon its parallel great circle, inſtead of the plane of the dial; ſince it is the ſame thing, by Cor. 3. Prop. I. For by reaſon of the immenſe diſtance of the ſun, and the paralleliſm of its rays, the motion of the ſhadow of the ſtile among the hour lines, muſt be the very ſame in both. In what follows I ſhall alſo uſe the parallel great circle inſtead of the dial plane; for the ſun will be in the plane of both, at the ſame inſtant; and therefore will begin to ſhine on both, or leave ſhining on both, at the ſame moment of time.

PROP. XXV. *Prob.*

A direct north reclining dial plane being given; to find the time of the year, when the ſun will totally leave one ſide of the plane, and ſhine upon the other.

10. The ſun is in the plane of any great circle, at the points where his parallel of declination cuts that great circle, and therefore the time muſt be calculated when he is in theſe points. And at that time he is going off one ſide of the plane to go upon the other ſide. But when he wholly goes off one ſide of a plane, the ſun's parallel muſt touch the other ſide of the plane, in the meridian; if the plane is direct north or ſouth.

Let HZO be the meridian, EQ the equinoctial, Z the zenith, P the pole. Take the difference between the latitude of the place, and the reclination of the plane; and that will be the latitude of the place where the dial plane is perpendicular to the horizon. And this will be north latitude, when the latitude of the place is greater than the reclination; but ſouth, if leſs. And this new latitude of the plane will be the ſun's declination, when it quite leaves the plane; from whence the day of the year is known. Fig. 10.

For if ZA is the reclination, then the plane will be vertical at A, therefore AC is the dial plane, and the ſun's parallel AB touches it in A. Therefore whilſt the ſun advances from A towards P, it will ſhine altogether on the ſide of the plane towards P, having quite left the other ſide. Therefore the ſun's declination EA will ſhew the time of leaving the ſide towards H.

Examp.

Suppoſe a north plane leans ſouthward 35 degrees, in lat. 54½; to find the time of the ſun's leaving the ſouth ſide.

Here 35 ſubtracted from 54½ leaves 19½ for the ſun's declination EA; and this anſwers to May 18, when it quite leaves the ſouth ſide going northward. Alſo in its return towards the ſouth, on July 26, it begins again to ſhine on the ſouth ſide, leaving the ſame declination as before.

Cor. 1. *Hence if the difference between the latitude of the place and the reclination of the plane, exceeds the ſun's greateſt declination; the ſun ſhines on both ſides of the plane, till he paſſes the equator; and then he ſhines only on that ſide next the equator.*

For then the plane AC will be interſected by every parallel of declination; till he come to the equinoctial. But beyond the equinoctial, the interſection is below the horizon, when the ſun ſhines not. So when the ſun is between E and H, he ſhines only on the ſide AC towards E.

Cor. 2. *When the latitude is equal to the reclination, the ſun ſhines half a year on one ſide, and half a year on the other ſide of the plane.*

For

Fig. 10. For then A falls upon E, and the plane EC becomes parallel to the circles of declination.

Cor. 3. *When the latitude of the place is greater than 23½ degrees, a direct ſouth vertical or reclining dial plane, has the ſun ſhining on both ſides of the plane, every day, whilſt the ſun is in the northern ſigns. But only on the ſouth ſide, when in the ſouthern ſigns.*

Cor. 4. *When a north recliner, reclines ſo much, as that the point A falls more than 23½ deg. beyond E towards H; all the phænomena will be contrary.*

Cor. 5. *By the ſame means, in any dial plane, however ſituated, whether declining, inclining, &c. the time of the year may be found, when the ſun quite leaves one ſide of the plane.*

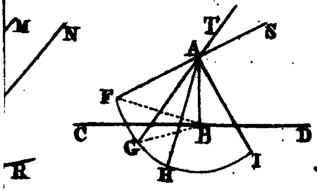
For if AC be the dial plane, which will be a direct ſouth plane in ſome longitude or other (by Prop. XVI.) Then by having the height of the pole above the plane, which is PA; you will find its diſtance from the equinoctial EA, by ſubſtracting from 90, then will EA be the ſun's declination, when the ſun leaves the ſouth ſide. And the ſun's declination being had, the time is known. And hence,

Cor. 6. *The ſun will always leave ſhining upon one ſide of a plane, at the ſame time of the year, when the pole's height above the plane is the ſame, let its poſition, in other reſpects, be what it will.*

PROP. XXVI. *Prob.*

Having the ſituation of any direct ſouth inclining dial plane; to find the hour of the day, when the ſun leaves the north ſide, to ſhine on the ſouth ſide of it.

The height of the pole above the plane muſt firſt be found; for which add the proclination to the



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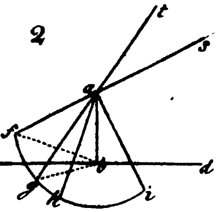

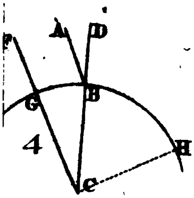

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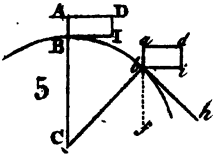

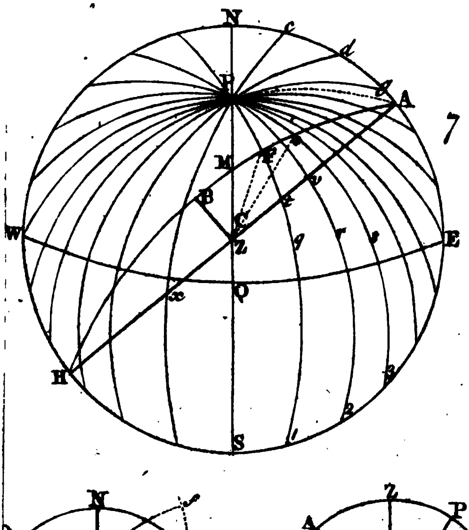

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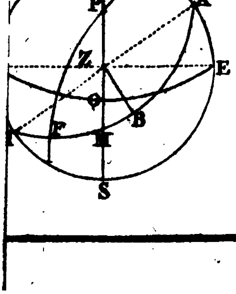

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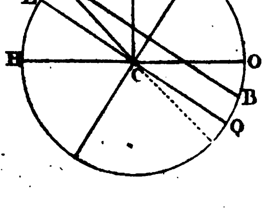

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Pl.I. *pa.* 46.

the complement of the latitude, when it leans ſouth. Fig. 13.
Then ſay,

As radius :
Tan. ſun's declination : :
Tan. height of the pole above the plane :
Coſ. hour angle at the pole, from noon.

Then that angle reduced to time, ſhews how long before or after noon, the ſun begins to ſhine on the ſouth ſide.

For let HAO be the meridian, EQ the equinoctial, Z the zenith, P the pole, AC the inclined dial plane. Draw the ſun's parallel of declination RL, to interſect the plane in S, and draw the hour circle SP. And when the ſun is in S, it is juſt going off the north ſide upon the ſouth ſide of the plane. Therefore in the right angled ſpherical triangle APS, it is (by Caſe 4), rad : cotan. SP : : tan. AP : coſ. angle APS; which converted into time gives the hour before and after noon, when the ſun is in S.

Cor. 1. *If the plane* AC *paſſes thro' the pole* P, *or the 6 o'clock hour circle. The ſun paſſes from the north ſide to the ſouth ſide always at 6 o'clock. And in the winter months, the ſun ſhines not at all on the north ſide.*

Cor. 2. *If the plane fall below the pole, or lyeth between* CP *and* CO; *the ſun goes off the north ſide before the hour of ſix in the morning; and goes on again after ſix in the evening.*

Cor. 3. *If the plane fall in with the equator, the ſun ſhines all the day on one ſide; being on that ſide towards which the declination is.*

For if EC be the plane, it ſhines on the north ſide only, when it has the north declination ER; and on the ſouth ſide, when it has ſouth declination.

Fig. Cor. 4. *In a north recliner, where the reclination is greater than the latitude; the ſun never ſhines on the ſouth ſide in the ſummer months.*

Cor. *In all dial planes, where the height of the pole above the plane is the ſame; the ſun goes off at the ſame hour of the day, reckoning time for each, at the place where it is a direct ſouth dial.*

PROP. XXVII. *Prob.*

To find the hour of the day when the ſun leaves one ſide of a vertical declining plane, to ſhine upon the other.

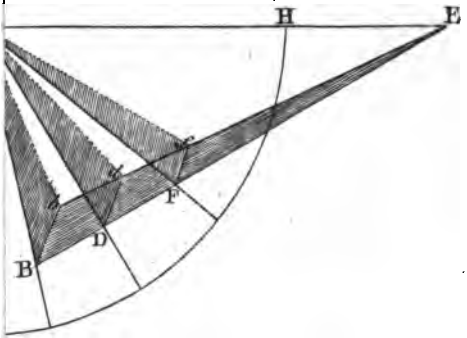
To do this we muſt firſt find the time from noon, when the ſun leaves one ſide of a direct north or ſouth plane, to ſhine on the other, by the laſt Prop. And if the plane declines eaſt; to the time found, add the plane's diff. longitude, in time, and the ſum will be the diſtance of time before noon, when the ſun leaves the north ſide. But ſubtract the plane's dif. longitude from the time found, and you will have the time of leaving the ſouth ſide. But if the plane declines weſt, the ſaid difference of time will ſhew the diſtance of time before noon, when the ſun begins to ſhine on the ſouth ſide; and the ſum will ſhew the time of leaving the ſouth ſide.

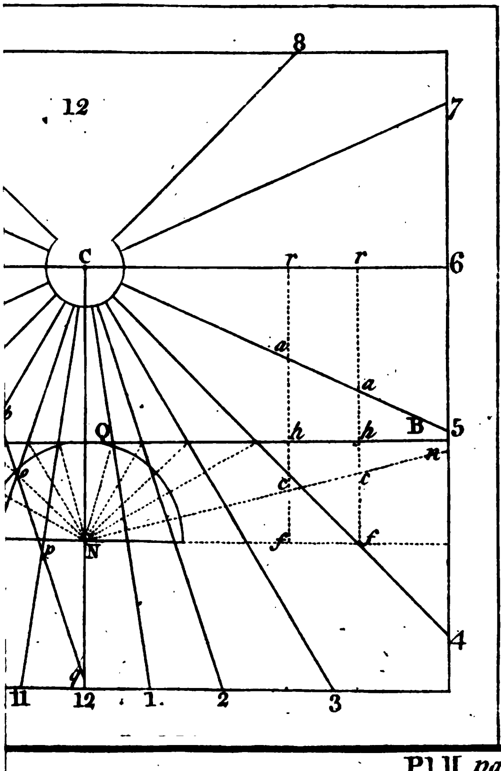
Examp.

6. *Suppoſe a plane declining eaſt* 25 *deg. in lat.* 54½. *Where the height of the pole* PB *above the plane is* 31° 45′, *and the plane's dif. longitude* ZPB *is* 29 48. *And the ſun's declination* 18° *north.*

As Radius — —	10.
Tan. ſun's declination (18)	9.51177
Tan. pole's height (31 45)	9.79156
Coſ. hour angle at the pole 78 24	9.30333

Dif.





Pl II. *pa. 48.*

Fig. 14.

Dif. longitude	29 48
ſum	108 12
dif.	48 36

Then 108 12 reduced to time gives 7^h 13^m before noon; that is, at 4^h 47^m in the morning, or a little after ſun riſe, when the ſun comes upon the ſouth ſide. And 48°. 36′ reduced to time is 3^h 14^m; and ſo it goes off the ſouth ſide again, at near a quarter paſt three.

But if the plane had declined weſt, the ſun would have come on, 3 14 before 12; that is, at 8^h 46^m; and gone off again at 7^h 13^m.

Otherwiſe thus.

Let PB be the meridian, P the pole, Z the zenith, and SZA the dial plane. Let SBA be the ſun's parallel, and draw PI perpendicular to SA. Then A will be the place of the ſun when it begins to ſhine on the ſouth ſide of the plane, and S his place when he goes off it.

Then in the oblique ſpherical triangle ZPA we have given, PA the complement of the ſun's declination, PZ the complement of the latitude, and angle PZA the complement of the plane's declination. Alſo in the triangle SPZ, the ſame things are given, viz. SP, PZ, and SZP; from hence to find the angles ZPA and ZPS, by Caſe 2, of oblique triangles.

Radius : ——	10.
S. latitude (*coſ* PZ) 54° 30′	9.91068
Cotan. plane's declination (*tan.* PZA) 25°	10.33133
Cotan. ZPI, 29 48 ——	10.24201

Fig. 14.

Then

Tan. lat. 54 30 (*cot.* ZP) —	10.14673
Tan. ſun's declin. 18° (*cot.* PA)	9.51177
Coſ. ZPI, 29 48 —	9.93840
	19.45017
Coſ. API *or* SPI, 78 24	9.30344

Then to API, 78 24. from SPI, 78 24
add ZPI, 29 48. ſubt. ZPI, 29 48
APZ = 108 12. SPZ = 48 36

which reduced to time, give the ſame anſwer as before.

PROP. XXVIII. *Prob.*

To find the hour of any day, when the ſun goes off the eaſt ſide of an eaſt reclining dial plane, to ſhine upon the weſt ſide.

The day being had, the ſun's declination is known; therefore,

As Radius :
Tan. ſun's declination : :
Tan. pole's height above the plane :
Coſ. hour angle from 12 *at the pole, which will be greater than* 90°, *where the ſun's declination is north. Then ſubtract the plane's difference of longitude from this angle, and the remaining angle, converted into time, gives the hour after* 12 *at noon, when the ſun leaves the eaſt ſide.*

8. For if HZO be the meridian, P the pole, Z the zenith, HFO the dial plane, PF an hour circle perpendicular to HFO. Then the plane's dif. longitude is RPZ equal to OPF, which is the dif. longitude from Z. But the hour angle found before, is the time of the ſun's going off the plane HFO, after the noon at the other place, that is, after that

hour in the morning that correſponds to that dif. Fig. longitude. Therefore ſubtracting the diff. longi- 8. tude RPZ from the hour angle found before; the remainder reduced to time, ſhews how long after 12 a clock the ſun goes off the plane.

And *if it be a weſt reclining plane, then the time laſt found will ſhew, how long before 12 the ſun comes upon the weſt ſide, or leaves the eaſt ſide.*

Example.

An eaſt plane reclines 35 degrees, in lat 51½. The pole's height above the plane, 26 41; the ſun's declination, 23½ north; and the plane's diff. longitude, 66 27.

Radius —	10.
Tan. ſun's decl. 23½	9.63830
Tan. pole's height, 26 41	9.70120
Coſ. hour angle, 102 38.	9.33950

Then from 102 38
ſubt. 66 27
rem. 36 11 = 2 hours 24 min. after 12,

when the ſun goes off the plane.

Otherwiſe thus.

Let NB be the meridian, N the north point of 15. the meridian, P the pole, BSA the ſun's parallel for that day. Then in the oblique ſpherical triangle NPS; there are given NP the latitude, PS the complement of the ſun's declination, and the angle PNS the reclination of the plane; to find the angle NPS or BPS, by Caſe 2. Let PI be perpendicular to NS. Then

Radius —	10.
Coſ. latitude (NP) 51½	9.79414
Tan. reclination (PNS) 35	9.84523
Cotan. NPI, 66 27	9.63937

Fig. Then *Cotan. latitude* (NP) 51½ 9.90060
15. *Coſ.* NPI, 66 27 — 9 60157
Tan. ſun's decl. (*cotan.* SP) 23½ 9.63830

Coſ. SPI, 77 22 19.23987
add NPI, 66 28 9.33927

143 50, which taken from 180, leaves BPS = 36 10, which reduced to time is 2^h 24^m, as before.

PROP. XXIX. *Prob.*

To find when the ſun goes off any eaſt declining plane, that paſſes thro' the pole; or comes upon a weſt declining one,

As ſine of the latitude :
Radius : :
Cotan. plane's declination :
Tan. hour angle (*from* 12) *at the pole.*

This converted into time gives the hour after 12, when the ſun goes off an eaſt declining plane. And ſo many hours before 12 it comes upon a weſt declining plane, paſſing thro' the poles.

16. For let NB be the meridian, P the Pole, Z the zenith, APD the declining plane, which is ſome hour circle, LS the ſun's parallel, S his place when he goes off the plane. Then in the right angled ſpherical triangle PNA, we have given PN the latitude, NA the complement of the declination; to find the angle NPA, by Caſe 13, S.NP · Radius : : Tan. NA : Tan. NPA or DPB, the hour angle after 12, that the ſun at S goes off the eaſt ſide.

Examp.

Fig. 16.

Let the plane decline 65 *degrees, in lat.* 54½.

S. latitude 54½		9.91068
Radius	—	10.
Cotan. declin. 65		9.66867
Tan. hour angle, 29 48		9.75799

And 29 48 reduced to time, is $1^h\ 59^m$, when the ſun goes off the eaſt ſide upon the weſt ſide. And if it was a weſt declining plane, the ſun would come on at $10^h\ 1^m$.

PROP. XXX. *Prob.*

Having given a ſouth reclining dial plane, declining eaſt or weſt; to find the hour of a given day, when the ſun comes on or goes off the ſouth ſide.

Find firſt the height of the pole above the plane, and the plane's difference of longitude, by Prop. XXII. Then by Prop. XXVI, find the hour angle, as if it was a direct ſouth plane; to this add the plane's dif. longitude, and it gives the hour angle from noon, when the ſun comes upon it; and from the hour angle ſubtract the plane's diff. longitude, for the hour angle of going off. This is for an eaſt decliner. And do the contrary for a weſt decliner.

Example.

Let the latitude be 51½, *the declination* 33° *eaſt, and reclination* 55. *Whence the height of the pole is* 19 25, *and longitude of the plane* 17 42; *and the ſun's declination* 23½.

Then *Radius*	—	10.
Tan. ſun's declin. 23½		9.63830
Tan. pole's height, 19 25		9.54713
Coſ. hour angle, 98 49,		9.18543

Fig. being greater than a right angle, when the plane falls below the pole.

Then hour angle	98 49	
diff. longitude	17 42	
ſum	116 31	= $7^{h}\ 46^{m}$.
diff.	81 7	= $5^{h}\ 25^{m}$.

So that the ſun comes upon plane $7^{h}\ 46^{m}$ before 12, and leaves it at $5^{h}\ 25^{m}$. But in a weſt decliner it comes on, $5^{h}\ 25^{m}$ before 12, and goes off at $7^{h}\ 46^{m}$.

17. For let NPB be the meridian, P the pole, Z the zenith, AMD the dial plane, SL*s* the ſun's parallel. Draw the hour circle FPL perpendicular to the circle AFD. Then the ſun comes upon the ſouth ſide of the plane at *s*, and goes off at S. Draw the meridians PS, P*s*. Then ſince the meridian PF is perpendicular to the plane, PF is the height of the pole, and MPF or ZPL is the plane's diff. longitude. But in the right angled triangle FPS, there is given FP the pole's height, and PS the complement of the ſun's declination; therefore by Caſe 4. rad : cotan. PS : : tan. PF : coſin. SPF or *s*PF, leſs than a right angle; therefore SPL is greater than a right angle. Whence *s*PL + LPZ = *s*PZ the angle before noon; and SPL — LPZ = SPZ the angle after noon. Theſe angles reduced to time, ſhew when the ſun comes upon the plane at *s*, and when he goes off it at S.

PROP. XXXI. *Prob.*

Having a north reclining dial plane, declining eaſt or weſt; to find the hour of any day, when the ſun goes upon the north ſide.

Find the pole's height above the plane, and the plane's difference of longitude. Then find the hour angle (by Prop. XXVI.) as if it was a di-

rect

reſt ſouth plane. Then the difference of the hour angle and plane's longitude, reduced to time, ſhews the diſtance of time from noon, when the ſun comes upon the north ſide of the weſt decliner; which is before noon, when the plane's longitude is bigger, otherwiſe after noon. But in an eaſt decliner, that diſtance of time muſt be taken after noon when the plane's longitude is greater, otherwiſe before noon; and ſhews when the ſun comes upon the ſouth ſide of it. Fig. 18.

Example.

Suppoſe the latitude 51½. *Pole's height* 54 45. *Plane's dif. longitude* 121 15. *Sun's declination ſouth* 15°; *the plane declining weſt.*

Then, Radius —	10.
Tan. ſun's decl. 15	9.42805
Tan. pole's height, 54 45	10.15074
Coſ. hour angle 112 17	9.57879

from 121 15
take 112 17

8 58 = 36 minutes.

So the ſun comes upon the north ſide 36 minutes before noon. If the plane had declined eaſt, it would go off 36 minutes after noon, and go upon the ſouth ſide.

Let NPB be the meridian, P the pole, Z the zenith, AMD the dial plane, SL the ſun's parallel, S the ſun's place when in the plane. Draw the ſun's hour circles, PS; and LPF, perpendicular to the dial plane AFS. Then PF is the pole's height, and MPF the plane's dif. longitude. And in the right angled triangle FPS, we have FP, and PS, the ſun's diſtance from the pole; to find the angle SPF (by Caſe 4), rad. : cotan. PS : : tan. PF : coſ. SPF, which will be greater than a right

Fig angle, when PS is greater, and PF leſs, than a
18. right angle. Then $MPF - SPF = MPS$, which is before noon, as S is on the eaſt ſide of the meridian.

PROP. XXXII. *Prob.*

To deſcribe the conſtruction, nature, and uſe of the common dialling ſcales; or of ſuch lines and ſcales as are commonly made uſe of, in drawing dials.

The lines on the common dialling ſcales are theſe.

1. A ſcale or line of hours; marked *hours* on the ſcale.

2. A line of degrees anſwerable to the hours, for it is no more than the hours in degrees. This is marked *Inclin. M.* or inclination of meridians.

3. The next is the line of latitudes, marked *Latit.* And at the end of it, there is a ſmall line of chords to 60.

4. Then there is a large *line of chords*, the whole length of the ſcale.

5 and 6. Theſe are the two polar lines, a greater and a leſs, and marked *G. Pol.* and *L. Pol.* Theſe two lines may be continued ad infinitum.

Their Conſtruction.

19. Draw the line GB, and on any point C as a center, deſcribe the circle ADB; and draw CDI perpendicular to it, to divide the ſemicircle into two quadrants AD and DB. Then,

1. Let the quadrant AD be divided into degrees, make $AH = HD = 45$ degrees, and draw the radius CH, and GHI perpendicular to it. Then GH and HI are tangents at H, and GH, or HI $=$ radius CH. Thro' the points r, r, r, &c. at 15, 30, 45, &c. degrees, draw the lines C1, C2, C3, C4, &c. to cut GI in 1, 2, 3, 4, 5, 6. Then

GI is the line of hours. But if lines be drawn Fig. thro' s, s, s, &c. at 10, 20, 30, &c. degrees, 19. (and alſo thro' the intermediate degrees), to cut the line GI; and numbered 10, 20, 30, &c. then you have the line, *incil. mer.* or the ſecond line on the ſcale, which is only the firſt line reduced to degrees.

2. For the line of latitudes. Thro' s, s, s, &c. or the points of 10, 20, 30, &c. degrees, draw parallels to AC, as sm, sm, &c. cutting CD in m, m, &c. Then thro' m, m, m, &c. draw Am, Am, Am, &c. cutting the quadrant BD in n, n, n, &c. Draw BF parallel to CD; and transfer the lengths Bn, Bn, Bn, &c. upon the line BF, to 10, 20, 30, &c. And if the ſame be done with all the intermediate degrees, then BF will be a line of latitudes. And the whole length of this line of latitudes, is equal to BD or AD the chord of 90°.

3. The line G. Pol. and L. Pol. are no more than two lines of tangents, reduced to hours. The radius of the greater, is the radius of the ſmall line of chords, at the end of the line of latitudes; and this radius is half AC.

Cor. 1. *Put radius* AC $= r$, *ſin. latitude* C$m = s$, *then the length on the line of latitude* $s = \dfrac{2rs}{\sqrt{rr + ss}}$, *for that latitude.*

For draw Bn. Then the triangles ACm, ABn are ſimilar; whence Am $(\sqrt{rr + ss})$: Cm (s) :: AB $(2r)$: Bn or B20 $= \dfrac{2rs}{\sqrt{rr + ss}}$.

Cor. 2. *If* $t =$ *tan. latitude; then the length on the line of latitudes* $= \dfrac{2rt}{\sqrt{rr + 2tt}}$, *for any latitude.*

For

Fig. 19. For the ſecant ($\sqrt{rr+tt}$) : tangent (t) : : radius (r) : the ſine $s = \frac{tr}{\sqrt{rr+tt}} = Cm$. And $Am = (\sqrt{AC^2 + Cm^2} =) \frac{\sqrt{rr+2tt}}{\sqrt{rr+tt}}$, whence

$$Am \left(\frac{r\sqrt{rr+2tt}}{\sqrt{rr+tt}}\right) : Cm \left(\frac{tr}{\sqrt{rr+tt}}\right) :: AB\,(2r) : Bn = \frac{2tr}{\sqrt{rr+2tt}}.$$

Cor. 3. *Put y = ſine of the arch, whoſe tangent is $t\sqrt{2}$; then $y\sqrt{2}$ = length on the line, Bn.*

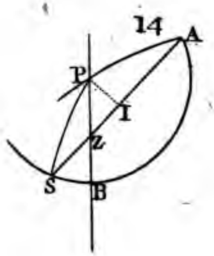
For $y = \frac{rt\sqrt{2}}{\sqrt{rr+2tt}}$ = ſine of the arch whoſe tangent is $t\sqrt{2}$.

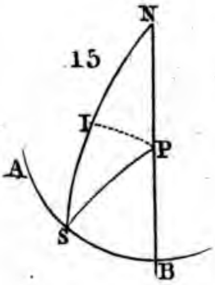
The Uſe.

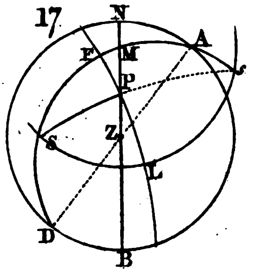
18. To draw a dial by the ſcale. Let C be the center of the dial, draw the ſubſtile CD, and ACB perpendicular to it. Then having the height of the pole above the plane, take it from the line of latitudes, and ſet it from C to A and from C to B. Then take the whole length of the line of hours, and ſetting one foot of the compaſſes in A or B, with the other croſs the line CD in D. Draw the lines AD and BD. Then get the ſeveral hour arches from the meridian of the plane, and counting them on the ſcale of hours, ſet them from D towards A, and from D towards B, in the lines DA, DB. And from C, draw lines thro' all theſe points, for the hour lines, as C1, C2, C3, C4, C5, and C11, C10, C9, C8, C7.

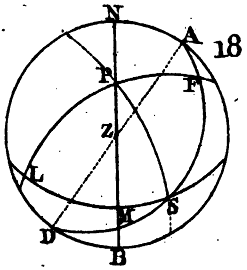
20. The uſe of the two polar lines is only for drawing ſuch dials as have no centers; or where the dial plane goes thro' the poles. And the radius of each is the length of 3 hours on the ſcale, to be uſed. To ſhew the truth of this operation,

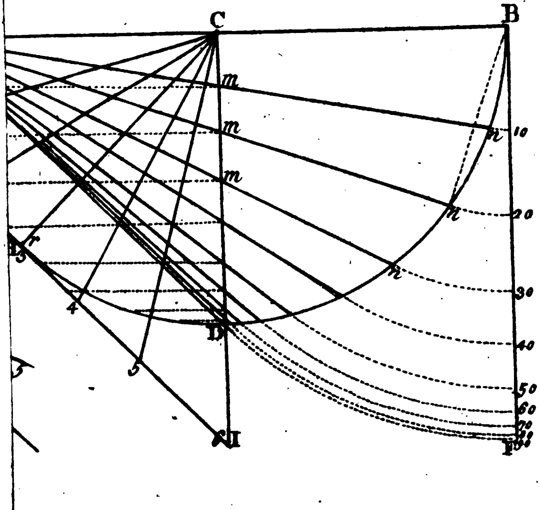
Biſect











Pl.III. pa. 58.

UNIV

Biſect BD in O, then will BO or OD $= r$, and Fig. 21. ſince BD $= 2r$, and CB $= \frac{2rs}{\sqrt{rr + ss}}$, and CD $= \sqrt{\overline{BD^2 - CB^2}} = \frac{2rr}{\sqrt{rr + ss}}$. Whence $\frac{BC}{DC} = \frac{s}{r}$. Now by conſtruction DO is a ſcale of tangents of the hour arches; and the numbers on the line expreſs theſe hour arches. And as OD is the tangent of 45° (or radius), and OA the tangent of an other arch; therefore AD is the difference of the tangents of theſe two arches; which being known, the difference of the arches will be known; and this is the hour arch correſponding to DA. For (by Cor. 2. Prop. 9. S. I. B. I. Trigon.) $\frac{DO - OA}{DO + OA} \times r$, or $\frac{DA}{BA} \times r =$ tan. this hour arch. But CD : CB : : $r : s$, and in the triangle CDB, CD : DB : : r : tan. CDB; therefore $s =$ tan. CDB. Let fall AF perpendicular to CD. Then in the triangles CFA and DFA, r : AF : : cotan. FDA : DF : cotan. FCA : CF; and alternately, CF : DF : : cotan. FCA : cot. FDA : : tan. FDA : tan. FCA : : BA : DA, (becauſe DFA, DCB are ſimilar triangles). Whence, tan. FCA $= \frac{DA}{BA} \times$ tan. FDA; and $r \times$ tan. FCA $= \frac{DA}{BA} r \times s =$ tan. hour arch $\times s$. Therefore r : s : : tan. hour arch : tan. FCA the hour angle, which is the ſame proportion, as in Cor. Prop. XVIII. and therefore is truly found.

If Ca be an hour line, and Da greater than DO; then Da is the ſum of the tangents OD, Oa; and (by Cor. Prop. VIII. ib.) the tan. ſum of the arches $= \frac{DO + Oa}{DO - Oa} r = \frac{Da}{Ba} r =$ tan. hour arch, correſponding

Fig. 21. ſponding to Da, on the line. Alſo tan. ſCa = $\frac{Da}{Ba}$ × tan. ſDa, and r × tan. ſCa = $\frac{Da}{Ba} \times rs$ = tan. hour arch × s. Whence as before, it will be, $r : s ::$ tan. hour arch : tan. hour angle ſCa.

Alſo if A or a fall upon O, then r = tan. hour arch, and tan. hour angle DCO = s.

Laſtly, if OA = Oa, then Ba = DA; and CA, Ca will be hour lines, whoſe hour arches are equidiſtant from CO, or the third hour.

SCHOLIUM.

There are ſeveral other lines put upon ſcales for dialling, but are only particular, as being made for ſome particular latitude only. Theſe lines are, 1. A line of chords. 2. A line of the ſubſtile's diſtance from the meridian. 3. A line for the height of the ſtile above the plane. 4. A line of the angle of the hours of 12 and 6. 5. A line of the plane's diff. longitude, or inclination of meridians. All theſe are calculated for every degree of declination, by Prop. XX. for ſome particular latitude, as that of London; and put on a ſcale together. And thus the ſeveral requiſites for any declining dial are had by inſpection. Its uſe is no more than this, count the plane's declination in the line of chords; and a line drawn directly croſs, will cut all the other lines in their proper points, which the numbers will ſhew; and ſo give all the requiſites without calculation. And theſe requiſites being had, the dial may be drawn by this Prop.

When a ſcale of this ſort is not to be had, the requiſites may be found by Gunter's ſcale; extending upon the ſeveral lines, according to the ſeveral rules and proportions laid down for that purpoſe.

SECT. II.

The Practical Part of Dialling, or the Art of drawing Dials upon all ſorts of Planes. How to find the Situation of a Plane; and how to place any Dial truly.

PROB. I.

To find the meridian line of any plane.

IF the plane be even, erect a pin on it perpendicular to the plane, but if not, procure a plain board GH of an equal thickneſs, and faſten it to the plane; upon which erect an iron pin CF about the middle, nearly perpendicular, and of a proper length; and let it be ſharp at the top; then take a wooden ruler, with a ſharp iron pin in one end; and lay the other end upon the ſharp end of the iron pin F, which is faſt in the board; then carry the other end ſteadily about; the ſmall pin in the ruler will deſcribe a circle ADB, upon the board. Obſerve two times in the day, when the ſhadow of the top of the iron pin F, falls upon the circumference of the circle, and mark the two points as at A and B. Biſect the arch AB, in the point D; and from D, draw a line DE thro' the center of the circle, which is eaſily found. Then DE is the meridian of the plane. 22.

Or you may deſcribe a circle round the center C, and raiſe the pin CF upon that point C, to caſt a ſhadow;

Fig. 22. ſhadow; but then the pin CF muſt be exactly perpendicular to the plane at C.

If the day be cloudy, the ſhadow may happen not to fall twice upon one circle; therefore it may be proper to deſcribe ſeveral circles, that it may fall upon ſome of them twice. And if two points in ſeveral circles be marked, they will one correct another, and the operation will be more exact.

To know whether the pin is perpendicular to the plane or not; ſet one point of the compaſſes in ſome point of the circle, and extend the other to the top of the pin F; then if you ſet it in any point of the circle, the ſame extent will always reach to the point F, when the pin is truly perpendicular.

It is certain, when the ſun does not change his declination, (as he does not ſenſibly in 12 hours), that he will have equal altitudes at equal diſtances on each ſide the meridian. And therefore if CD be the meridian of the plane, the arches AD and BD muſt be equal.

PROB. II.

To find the inclination of any plane to the horizon.

23. Let ABC be the plane, apply one ſide of a quadrant to the plane, ſo that the plummet may fall exactly on the other ſide; the firſt ſide will be horizontal; therefore draw a line DF along that ſide, which will be the horizontal line of the plane. If you cannot apply a quadrant, then uſe a carpenter's ſquare or a level. To the line EF, draw the perpendicular EL, which will be the vertical line of the plane. To this vertical line EL, apply a ſtreight ruler EK; and to the end K, beyond the plane, apply the quadrant GH, on the under ſide; and obſerve what number of degrees are cut by

the line and plummet at I, reckoning from the ruler; as GI, and that will be the reclination of the plane; or its proclination, if it lean forward. For inclining planes, a quadrant may be applied directly, to the vertical line. Fig. 23.

Or thus.

Apply the ſide of a ſquare to the line EL on the plane; and apply the ſide of a quadrant to the other ſide as KH; then ſee what angle your line and plummet makes with the firſt ſide EK, as LKI, and that is the reclination or proclination of the plane. If the line KI hang along the line KH, then your plane is horizontal; if along the ſide KL, it is vertical.

If by turning the rule and quadrant all manner of ways on the plane, the perp. line KI always falls on the ſide KH; then the plane is truly horizontal. And the like for the ſquare and quadrant.

PROB. III.

To find the declination of any plane.

24. Place a board DM with one ſide of it cloſe to the wall or plane EF, and to lie with its upper ſurface exactly horizontal; there fix it. Upon this board, as it lies, find a meridian line MC (by Prob. I.), and draw a line on the board to repreſent it. Then lay a quadrant AMB flat upon the board, with one ſide cloſe to the plane, and ſo, that the center C may be in the meridian line, and the body of the quadrant, to that hand where the acute angle lies. Then note the degrees of the quadrant, cut by the meridian line, reckoning from the perpendicular ſide of the quadrant CB, then that, viz. the angle MCB will be the declination. And the

Fig. 24. the declination will be to the right hand, if the quadrant lie to the left; and the contrary.

If you know when it is juſt noon, you may get a meridian line thus, hold up a plumb line, ſo as the ſhadow of it may fall upon your plane; mark two points of the ſhadow, and draw a line thro' them, and that ſhall be the meridian line of the place. And if the plane be horizontal or direct north or ſouth; it will be the meridian of the plane too.

Otherwiſe thus.

This may alſo be done by a compaſs and magnetical needle. This is fixt in a box, ſquare on all ſides; and to uſe it, apply the north ſide of the box to a ſouth wall or plane; and the ſouth ſide to a north wall, ſo as the box may lie horizontally. Then obſerve how many degrees the needle is from the flower-de-luce when it reſts, (having due regard to the variation); and ſo much will the declination be. And if the needle points towards the right hand, the declination is to the right hand, and the contrary, in ſouth planes. But in north planes, if the needle points toward the right hand, the declination is to the left, and the contrary. You muſt mind that no iron be near the compaſs. The variation is 17$\frac{1}{2}$ deg. weſt in *England.*

Or thus.

This method is aſtronomical, and the moſt exact; but it is proper to have an aſſiſtant; for two obſervations are required to be made at the ſame time; the firſt to find the diſtance of the ſun's azimuth, from the vertical of the plane; and the ſecond, to find his altitude.

For the firſt, draw a horizontal line upon the plane, and apply one edge of a quadrant to it, ſo as the limb of the quadrant be towards the ſun;

and holding it horizontal, let an aſſiſtant hold up a thread and plummet in the ſun, ſo that the ſhadow of the thread may paſs thro' the center of the quadrant; then obſerve the degrees which the ſhadow of the quadrant falls upon, counting them from the perpendicular ſide, and you have the ſun's azimuth, in reſpect of the plane's vertical circle. Fig.

Again to find his azimuth in reſpect of the meridian, his altitude muſt immediately be taken; and his declination being known, from the day of the month; his azimuth will be found by Caſe 11, of oblique ſpherical triangles.

Theſe two azimuths being had, it is eaſy to know, whether the vertical of the plane lies to the eaſt or weſt of the meridian. For if the ſun's azimuth from the meridian be greater, the plane declines towards the ſun; but if leſs, it declines from the ſun; that is, when both azimuths lye one way. And then the declination is the difference of theſe azimuths.

But if the azimuths lie contrary ways, the plane declines from the ſun; and the ſum of theſe azimuths is the declination.

If you know the exact time of noon, and hold up a line and plummet, before the quadrant at that time, you will have the plane's declination at once, by obſerving the ſhadow, which muſt paſs thro' the center of the quadrant.

If you obſerve the ſun's altitude, juſt when he comes into that plane; then the ſun's azimuth being found, gives you the poſition of that plane.

If your plane be a coarſe wall, nail a long ruler to it, to which apply your inſtruments.

Fig.

PROB. IV.

To draw a dial geometrically upon any plane.

Project the ſphere by any method, upon that great circle of the ſphere which is parallel to the dial plane; and deſcribe the ſeveral hour circles, and note the places where they interſect the primitive circle. Draw lines from the center thro' all theſe points of interſection, and they will be the hour lines for your dial.

7. For let NWSE be that great circle, which will biſect the globe, paſſing thro' the center C. Let fig. 7. repreſent the globe; P the pole; then CP will be the earth's axis, elevated above the plane of the circle, the quantity of the angle or arch NP. Then ſince the hour circles do all paſs thro' the axis PC, they all paſs thro' C, which is in the plane of that circle; and the points of interſection with the primitive, 1, 2, 3, *c*, *d*, *g*, &c. being alſo in the plane of that circle; therefore the hour circles interſect the primitive in lines which paſs thro' the center C, and thro' theſe ſeveral interſections. And ſince the ſhadow will be in any line of interſection, when the ſun is in that hour circle; therefore theſe lines of interſection will be the hour lines; that is, lines drawn from C to 1, 2, 3, &c. will be the ſeveral hour lines of the dial.

As all the hour circles in the orthographic projection of the ſphere will be ellipſes; and as theſe are difficult to deſcribe, their interſections with the primitive will not be got exact enough. Therefore this ſort of projection is no way commodious for the drawing of dials, and therefore is never uſed.

In the ſtereographic projection, all the circles of the ſphere are circles in the projection, which are far more eaſily deſcribed than ellipſes, and far more correct;

Fig. 7.

correct; therefore the interſections with the primitive will be more exactly obtained; which makes this method more commodious for drawing dials; and therefore it is often uſed for this purpoſe.

But in the gnomonical projection, all the great circles are projected into right lines, which are ſtill eaſier to draw than circles; therefore dials are beſt and eaſieſt drawn by this ſort of projection; for which reaſon, in what follows, I ſhall make uſe of it in the geometrical conſtruction of dials. And the reaſon of the ſeveral operations, will be obvious to any body that underſtands the nature of this projection; and needs no other demonſtration. And the rules of this ſort of projection, you have in the laſt ſection of my Treatiſe on Projection; and therefore the dialiſt ought to be acquainted therewith, as it is the foundation of all theſe operations.

To proceed in the eaſieſt method, imagine the perpendicular height of the ſtile to be the radius of the ſphere, and that the dial plane touches the ſphere at the foot of the ſtile; then if the ſphere be projected on that plane, by the eye (or projecting point) at the top of the ſtile, which is the center of the ſphere; all the hour circles will be projected into their proper hour lines, which will tend to the projected pole, which will be the center of the dial. And the projections of the leſſer circles will make the reſt of the furniture; and will be conic ſections. And a line drawn from the top of the perpendicular ſtile to the center of the dial, will repreſent the earth's axis, and be the ſtile or gnomon.

In theſe conſtructions, the ſtile is all along ſuppoſed to be a perpendicular pin; which afterwards is eaſily changed into a triangular form, by drawing a line from the top of it, to the center of the dial.

PROB. V.

To draw an equinoctial dial, or an horizontal dial under the poles.

25. Since the ſtile here muſt be a perpendicular pin; from C the center of the dial, aſſumed about the middle of the plane, deſcribe the ſmall circle *abd*, of the ſame diameter as the pin, which muſt be cylindrical; and that circle is to be the foot of the ſtile. Alſo from the ſame center C, deſcribe another circle 1, 2, 3, &c. Draw C*o* for the meridian, and parallel thereto, draw 12 *a* for the 12 o'clock line, touching the circle *abd* in *a*. Then divide the outer circle into 24 equal parts, beginning at 12; and to the points of diviſion, 1, 2, 3, &c. draw the hour lines, to touch the inner circle towards the ſame hand as 12 *a* was drawn. Then mark the hour lines with 1, 2, 3, 4, &c. on the right hand, and 11, 10, 9, &c. on the left hand; and draw another circle including the whole.

Upon the circle *abd* erect the pin or ſtile, which muſt be perpendicular to the dial plane, and of any length. Then this will ſhew the hour, by the ſhadow on the left ſide.

To ſet up this dial in any other latitude, it muſt be placed with the 6 o'clock hour line horizontal, the 12 o'clock line pointing to the north, and the ſouth edge S raiſed above the horizon, to make an angle with it equal to the complement of the latitude, or the height of the equinoctial; and then the plane of the dial will be parallel to the equinoctial, as it ought to be.

If you draw the hour lines from the center C, the pin muſt diminiſh to a point at the top. And the diviſion of the hours muſt then begin at O; and the ſhade of the top will ſhew the hour of the day, among the hour lines. In this dial you need Fig. draw no more hour lines, than the ſun ſhines on in 25. ſummer, if the dial is drawn on the north ſide. But if it is drawn on the ſouth ſide, you need but draw 12 hours, which is the longeſt that the ſun can ſhine upon it.

If you have a mind to note the half hours and quarters, you muſt divide each hour, 12 1, 12, 23, &c. into two equal parts for half hours, or into four equal parts for quarters.

PROB. VI.

To draw a horizontal dial under the equinoctial.

1. *Geometrically.*

Draw two parallel lines upon your plane, AB 26. and DE, at a convenient diſtance, for inſcribing the hour lines; make MCF perpendicular thereto. Take CF for the height of the ſtile, and C*c* for the thickneſs thereof; and draw *ncf* parallel to MCF. With the center F, deſcribe the quadrant CH; and with the center *f*, the quadrant *c*G. Divide each quadrant into ſix equal parts at *n*, *n*, &c. and draw lines from the reſpective centers thro' all the points of diviſion, to interſect the line DE in the points 1, 2, 3, &c. Then thro' theſe points 1, 2, 3, 11, 10, 9, &c. draw lines parallel to MC or *mc*, and theſe will be the hour lines of the dial.

If you divide each hour *nn*, *nn*, &c. into two or four equal parts; and draw lines thro' theſe points from the two centers, to interſect DE as before, then theſe interſections will ſhew the half hours or quarters.

Make MK = CF, then will MK3C be the figure of the ſtile, and C*c* or M*m* its thickneſs; which ſtile muſt be erected upon the baſe CM*mc*,

perpen-

Fig. 26. perpendicular to the plane of the dial; and the ſhadow of the top will ſhew the hour of the day.

If inſtead of leaving a ſpace C*c* for the ſtile, you deſcribe a ſemicircle round F, and divide all from the center F; then ſince C and *c* coincide, your ſtile muſt be exceeding thin, or at leaſt it muſt be brought to an edge at the top. This is the readieſt way, but then your dial will not be ſo exact in the middle of the day.

Or inſtead of plate for the ſtile, it may be an iron pin, erected perpendicular upon any point of CM, and its height muſt be equal to CF.

Or you may have a plate of any height for the ſtile, with a ſmall hole made thro' it at the height of CF, for the ſun to ſhine through among the hour lines.

The hours muſt be numbered on the weſt ſide 11, 10, 9, 8, &c. and on the eaſt ſide 1, 2, 3, 4, &c.

This dial is to be placed ſo, as its plane may be parallel to the horizon, and DE pointing eaſt and weſt, or CM north and ſouth. Or to fix it in any other latitude, the north ſide at M muſt be raiſed to make an angle above the horizon equal to the latitude of the place; and CM pointing to the north.

2. *By Calculation.*

As radius :
Stile's height CF : :
So *tan. hour arch* (15°) :
Diſtance of the hour from MC (C1).
So *tan* 30^d ;
Diſtance C2, &c.

Then the diſtances C1, C2, C3, &c. ſet off from C to 1, 2, 3, &c. gives the points thro' which the afternoon hours are to paſs; and the ſame ſet off from *c* to 11, 10, 9, &c. give the points where the forenoon hours are to paſs. Therefore lines drawn thro' all theſe points parallel to MC will be the hour lines. Fig. 26.

If you would calculate for the half hours and quarters, you muſt ſay, as rad : ſtile's height CF : : ſo tan. 3° 45′ : to the diſtance from C, on the line CE for a quarter : : and ſo tan. 7° 30′ : diſtance for half an hour : : and ſo tan. 11° 15′ : diſtance for three quarters : : tan. 18° 45′ : diſtance for an hour and a quarter, and ſo on.

3. *By the Scale.*

Having drawn the meridian CM, and *cm* parallel to it at a diſtance equal to the thickneſs of the ſtile; from either polar line, which ſuits beſt, take the extent from the beginning of the line to I the firſt hour, and ſet from C to 1, and from C to 11 on the line DE; then take the extent from the beginning, to II, the ſecond hour, and ſet it from C to 2, and from *c* to 10; likewiſe take the extent to III, the third hour, and ſet from C and *c* to 3 and 9. Do the like with the reſt of the hours.

Then lines drawn thro' theſe points 1, 2, 3, 11, 10, &c. parallel to CM, will be the hour lines of the dial. And the extent to III, or C3, is the height of the ſtile.

If you would have the hour lines of 7 in the morning, and 5 at night to come in; you muſt find the height of your ſtile for that purpoſe, thus. 5 hours = 75°. Then ſay, as Radius : CE : : Cotan 75° : CF the height of the ſtile; and to this height the dial muſt be made.

Fig.

PROB. VII.

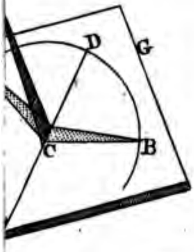
To draw a horizontal dial for any latitude.

1. *Geometrically.*

27. Take any convenient point P for the center of your dial, thro' P draw the meridian DE; at P make the angle DPR equal to the latitude. Draw a line parallel to DE, at a diſtance equal to the perpendicular height of the ſtile, to cut PR in F. Otherwiſe take any point F you pleaſe in the line PR, and draw FC perpendicular to DE, then FC is the height, and C the foot of the ſtile. Draw FG perpendicular to FP. Thro' G draw the contingent line AGB perpendicular to DE, which here repreſents the equinoctial. Let G*g* be the thickneſs of the ſtile, and draw *dge* parallel to DGE.

Or you may proceed thus, having made the angle DPR equal to the latitude, take any point G at pleaſure, thro' which draw the contingent line AGB, perpendicular to DE. From G take GF, the neareſt diſtance to PR, and ſet from G to D, and from *g* to *d*; and from the centers G and *g*, with any radius, deſcribe two quadrants, which divide into ſix equal parts, or hours, beginning at G and *g*. And thro' the points of diviſion, draw lines from the centers D and *d* to interſect AB in the hour point S, *m*, *m*; *n*, *n*, &c. Then from the centers P and *p*, draw lines thro' the points *m*, *n*, &c. as P*m*, P*m*; and P*n*, P*n*, &c. and theſe will be the hour lines. The 6 o'clock hour lines muſt be drawn parallel to AB; and the hours before and after ſix, are had by producing the oppoſite hour lines thro' the center.

Then PFC will be the form of the ſtile, which may be made longer or ſhorter by producing PF. And



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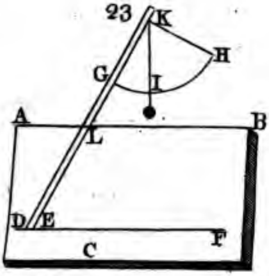

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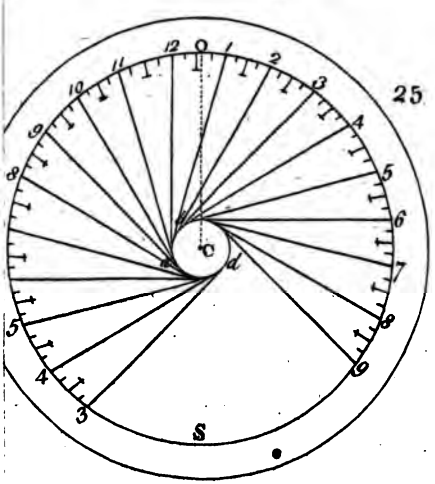

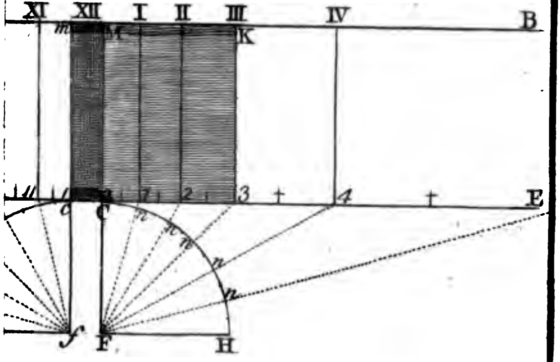

Pl.IV. *pa*. 72.

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And the ſtile is to be placed perpendicular upon the plane of the dial; that is, one ſide upon the ſubſtile PG, and the other upon the ſubſtile *pg*; each ſide of the ſtile being perpendicular upon the ſubſtile; for the plane of the ſtile ought always to be ſet perpendicular on the ſubſtile, in any dial. And then the top edge will ſhew the hour. Fig. 27.

If you allow no ſpace G*g* or P*p* for the thickneſs of the ſtile, it muſt be made exceeding thin to take up no room; or elſe it muſt be made ſharp on the upper edge. And that edge muſt tend directly to the center.

Inſtead of the triangular gnomon PFC, you may have a perpendicular pin FC, and then the ſhadow of the top F, will ſhew the hour. But the other is better, becauſe ſome part of the ſhadow will always be upon the dial; but the ſhadow of the top F, ſoon goes off.

You may alſo have the ſtile to be a plate of any form, with a hole in it for the ſun to ſhine through; which hole muſt be at the height CF, perpendicularly over C.

If you would have the half hours and quarters; you muſt divide every hour in the quadrant into two or four parts, and draw lines from the centers D, *d*, as before, to interſect AB, and thro' theſe points of interſection, draw lines from the center of the dial, which will give theſe halves or quarters. But the quarters may be had near enough, by dividing each half hour into two equal parts.

And to ſave labour, after you have drawn all the hour lines on one ſide of the dial, or for one half of the day; you may transfer them to the other ſide, for the other half of the day.

As to numbering the hours, thoſe on the eaſt ſide muſt be numbered 1, 2, 3, &c. from PD or

Fig. 27. 12 for the afternoon hours. And thoſe on the weſt ſide 11, 10, 9, &c. which are the forenoon hours. And the figures are moſt commodiouſly placed with their tops towards the center; becauſe we ſtand oppoſite to the ſun, to ſee the hour.

A horizontal dial is the only one, that ſhews the hours quite thro' the day, at all ſeaſons of the year. For if the ſun is up, or above the horizon, he muſt needs ſhine upon a horizontal plane, if nothing ſtands in the way.

The outſide of the dial terminating the hour lines, may be round or ſquare or of any form you will, for it makes no difference whether the hour lines be long or ſhort.

2. *By Calculation.*

As radius :
Sin. latitude : :
Tan. hour arch :
Tan. hour angle.

The hour arches are 15°, 30°, 45°, 60°, 75°, 90°; and if the quarters are taken in, they are 3° 45′, 7° 30′, 11°. 15′, 15° 0′; 18° 45′, &c. Suppoſe the latitude 54½; then the hour angles, found by the foregoing rule, for every half hour, will be as in the following table.

Fig. 27.

A Table of the hour arches and hour angles.

Hours.		hour arches.		hour angles.	
		0°	0′	0	0′
XII					
11½	12½	7	30	6	7
XI	I	15	0	12	18
10½	1½	22	30	18	38
X	II	30	0	25	10
9½	2½	37	30	32	00
IX	III	45	0	39	9
8½	3½	52	30	46	42
VIII	IV	60	0	54	39
7½	4½	67	30	63	02
VII	V	75	0	71	47
6½	5½	82	30	80	49
VI		90	0	90	00

Conſtruction.

Take any point P for the center, and draw PD for the meridian. Take P*p* = thickneſs of the ſtile, and draw *pd* parallel to PD. Make the angles DP XI, and *d*P1 = 12° 18′. The angles DPX and *dp*II = 25 10. And ſo of the reſt, as you find them in the table.

Laſtly, make the angle DPR = the latitude, for the ſtile.

3. *By the Scale.*

Aſſume any convenient point P for the center of the dial, thro' which draw the line VI P VI for the ſix a clock hour lines. Make P*p* the thickneſs of the ſtile; and thro' P and *p* draw the lines PD, *pd* perpendicular to VI VI, for the meridian or 12 a clock line. Then on the line of latitudes on the ſcale, ſetting one foot at the beginning, Fig. 27. ginning, extend the other to the latitude of the place; and ſet that extent from P to *b* and from *p* to *d*, in the ſix o'clock line. Then take in your compaſſes the whole length of the line of hours, and ſetting one foot in *b*, with the other cut the meridian PD in O; likewiſe ſet one foot in *d*, and with the other cut the meridian *pd* in *o*; and draw the lines *b*O, *do*. Then extend your compaſſes from the beginning of the line of hours to I (or 15°), and ſet that extent from O to 11, and from *o* to 1; and if you will, from *b* to 7, and from *d* to 5; in the lines *b*O, *do*.

Again extend from the beginning, to II (or 30°); and ſet it from O to 10, and from *o* to 2; and alſo (if you pleaſe) from *b* to 8, and from *d* to 4. Likewiſe take the extent to III (or 45°), and ſet from O to 9, and from *o* to 3. After this manner go thro' all the hours. Then lines drawn from P, thro' the points 7, 8, 9, 10, 11, give the forenoon hours. And lines drawn from *p*, thro' 1, 2, 3, 4, 5, will be the afternoon hour lines. The hour lines before and after 6, are had by producing the reſt thro' the center.

If you would have the half hours and quarters ſet off by the ſcale; you muſt extend to theſe half hours and quarters on the line of hours, and ſet them on the lines O*b* and *od*, as you did for the whole hours.

The ſtile DPR muſt be raiſed at right angles, to the dial, upon the ſubſtile which here is the ſpace PO*op*; and muſt make an angle with the ſubſtile equal to the latitude, as was ſaid before.

There is alſo a way to conſtruct dials, by the help of dialling tables, fitted to a certain latitude; which you may ſee in ſome books of dialling.

SCHOLIUM.

Fig. 27.

SCHOLIUM.

It may be obſerved here, that if a horizontal dial be made for a place in the torrid zone, to ſhew the hour by the top of the perpendicular ſtile CF. And when the ſun is in the neareſt tropic, or between the parallel of the place and that tropic ADL; the ſhadow of the ſtile CF, upon the dial NA*dgl*, will go back twice in the day; once in the forenoon, and once in the afternoon. For whilſt the ſun in the tropic, or parallel circle, moves thro' AGBDIKL, the ſhadow will move thro' *agadklk*, and then ſets; ſo that he goes back thro' *ag*, and thro' *kl*.

In this caſe the ſun is twice upon one azimuth in the morning, and twice in the evening; and then the ſhadow falls twice upon one line. And the arch *ag* or *kl* may be found, by calculating the ſun's azimuth at A and G, by right angled ſpherical triangles.

The like happens to any ſtar here, whoſe declination is greater than our latitude. For ſuch a ſtar will be twice upon one azimuth in the eaſtern part of its orbit, and twice in the weſtern part. In great latitudes ſuch a ſtar never ſets; but in its revolution, whilſt it is aſcending in the eaſt, it moves towards the ſouth, till ſuch time as it riſes perpendicularly, and then its azimuth is the greateſt. Then it returns again towards the north; ſo that it never gets to the ſouth of us.

PROB. VIII.

To draw an erect direct ſouth dial.

1. *Geometrically.*

Take a convenient point C for the foot of the ſtile, thro' which draw the meridian CDE. Raiſe 28.

Fig. 28. CF perpendicular to CE, and equal to the perpendicular height of the ſtile. Make the angle CFP upwards equal to the latitude of the place, to interſect the meridian in P, then is P the center of the dial. Draw FG perpendicular to FP, and AGB perp. to PG.

Or proceed thus, take the point P near the top of the plane, for the center of the dial; thro' P draw the meridian PE; at P make the angle EPR equal to the complement of the latitude. Take any point G in the line PE, and draw GF perpendicular to PR; or take the neareſt diſtance from G to the line PR; thro' G draw the contingent line AB, perpendicular to PE, repreſenting here the equinoctial.

Take G*g* equal to the thickneſs of the ſtile, and draw *pgq* parallel to PGE. Make GD and *gq* = GF, and from the centers D and *q*, with any radius, deſcribe two quadrants LG, M*g*. Divide each quadrant into ſix equal parts or hours, each 15°, beginning at G and *g*. From the centers D, *q*, draw lines thro' the points of diviſion, to cut AB in the hour points *m*, *m*; and *n*, *n*, &c. Then lines drawn from the centers P and *p*, thro' all the points *m*, *n*, &c. will be the hour lines. The half hours, &c. are drawn the ſame way.

And the perpendicular pin FC will be the ſtile, or which is better, the triangle RPG; which muſt be ſet perpendicular upon the hour lines of 12, PG and *pg*; the ſtile PR pointing downwards towards the ſouth pole.

Note, all the lines except the hour lines, ſhould be obſcure lines, or ſuch as may be rubbed out.

When the lines do not interſect within the plane, as at the firſt diviſion towards L and M. Take half the line *qg* in your compaſſes, and running one foot along the line *g*B, till the other foot fall upon the line *qc* at *c*, (keeping the feet of the compaſſes in a parallel poſition), thro' *c* draw *cba* parallel to *pq*; and ſet off *ba* = half *gp*; then draw *pa* for the hour line. Fig. 28.

Set the forenoon hours on the left hand and the afternoon hours on the right, with the tops of the figures upwards, in which poſition only, they can be read.

2. *By Calculation.*

As radius :
Coſ. latitude : :
Tan. hour arch :
Tan. hour angle.

Here the height of the pole above the plane is the complement of the latitude. Here as in the laſt Prob. the hour arches are 15°, 30°, 45°, &c. for whole hours; and 3° 45', 7° 30', 11° 15', 18° 45', &c. for quarters. Then having found the hour angles, by the foregoing rule, put them orderly in a table, as in the laſt Prob. which for the lat. 54½ will be as follows, for whole hours.

A Table of hour arches and angles.

Hours.		hour arches.		hour angles.	
XII		0°	0'	0°	0'
XI	I	15	0	8	51
X	II	30	0	18	32
IX	III	45	0	30	9
VIII	IV	60	0	45	10
VII	V	75	0	65	14
VI		90	0	90	0

Conſtruction.

Take any point P near the top of the plane, for the center of the dial; draw the meridian or 12 o'clock

Fig. 28. o'clock line PE, and *pe* parallel to it, at a diſtance equal to the thickneſs of the ſtile. Then with extent of 60 from a line of chords, deſcribe two quadrants, from the centers P and *p*. Then ſet off the ſeveral hour angles as you find them in the table, both ways from the lines PE and *pe*, upon theſe quadrants. And thro' the points, draw lines, from the centers P, *p*, for the hour lines. Thus EPXI, and *ep*I, is the firſt hour angle. EPX, and *ep*II the ſecond hour angle, &c.

Laſtly, make the angle EPR = complement of the latitude, for the ſtile, to be ſet upright upon the hour of 12.

3. *By the Scale.*

Take the point P near the top of the plane for the center, and draw the meridian PE, and *pe* parallel to it, at a diſtance equal to the thickneſs of the ſtile; and thro' P draw the 6 o'clock lines perpendicular to PE. Then from the line of latitudes on the ſcale, take off the extent from the beginning to the complement of latitude, and ſet that extent from P to *b*, and from *p* to *d*. Then take the whole length of the line of hours, or inclination of meridians; and ſetting one foot in *b*, croſs the meridian with the other foot at O. Likewiſe with one foot in *d*, croſs the line *pe* in *o*; and draw *b*O, *do*. Then take the extent from the beginning, to I, II, III, &c. hours (or to 15°, 30°, 45°, &c.) and ſet from O to 11, 10, 9, &c. and from *o* to 1, 2, 3, &c. reſpectively. Then draw lines from P thro' all the points 11, 10, 9, &c. and from *p* thro' 1, 2, 3, &c. and theſe will be the hour lines of the dial. The forenoon hours are to be numbered towards the weſt; and the afternoon hours towards the eaſt. Then make the angle EPF equal to the complement of the latitude, for the ſtile.

PROB.

Fig. 28.

PROB. IX.

To draw a direct north Dial.

1. *Geometrically.*

Draw a direct ſouth-dial for the ſame latitude, and turn it upſide down, and ſetting its face northward, it will be a direct north dial; and the ſtile will point upwards towards the north pole. But the hours muſt be numbered the contrary way from the ſubſtile; thoſe to weſt, are the morning hours, and thoſe to the eaſt, the evening hours. And the midnight hours muſt be left out, becauſe the ſun ſhines not then; and the hour lines of 7 and 8 in the morning, and 4 and 5 at night muſt be drawn thro' the center, as in the horizontal dial. It is evident the ſun never ſhines upon this dial, but only in the ſummer half year.

2. *By Calculation.*

As radius :
Coſ. latitude : :
Tan. hour arch :
Tan. hour angle.

The ſame as for a direct ſouth dial. And the conſtruction the ſame.

3. *By the Scale.*

Here the center H muſt be taken near the middle of the plane. And all the reſt of the work muſt be done, as in the ſouth dial. Only the hour lines after ſix in the morning, and before ſix in the evening, are had by producing the oppoſite ones thro' the center.

You may if you pleaſe, make a horizontal dial for the complement of the latitude; and turn the north end up, and ſet it to face the north, and it

Fig. 28. will be a true north dial. And the hour lines will be rightly numbered; but the mid-day hours muſt be left out, which now become the midnight hours.

A direct north dial is only the back ſide of a ſouth direct one; for both dials lie in the prime vertical; the meridian in the ſouth dial is the hour line for 12 at noon; but the ſame meridian continued thro' the plane, is the hour line for 12 at night, in the north dial. And all the hour lines in the ſouth dial being produced thro' the plane, will be the hour lines in the north dial, for then the ſun is in the oppoſite part of the meridian. And the ſtile of the ſouth dial continued thro' the plane, will be the ſtile of the north dial.

PROB. X.

To draw an erect direct eaſt Dial.

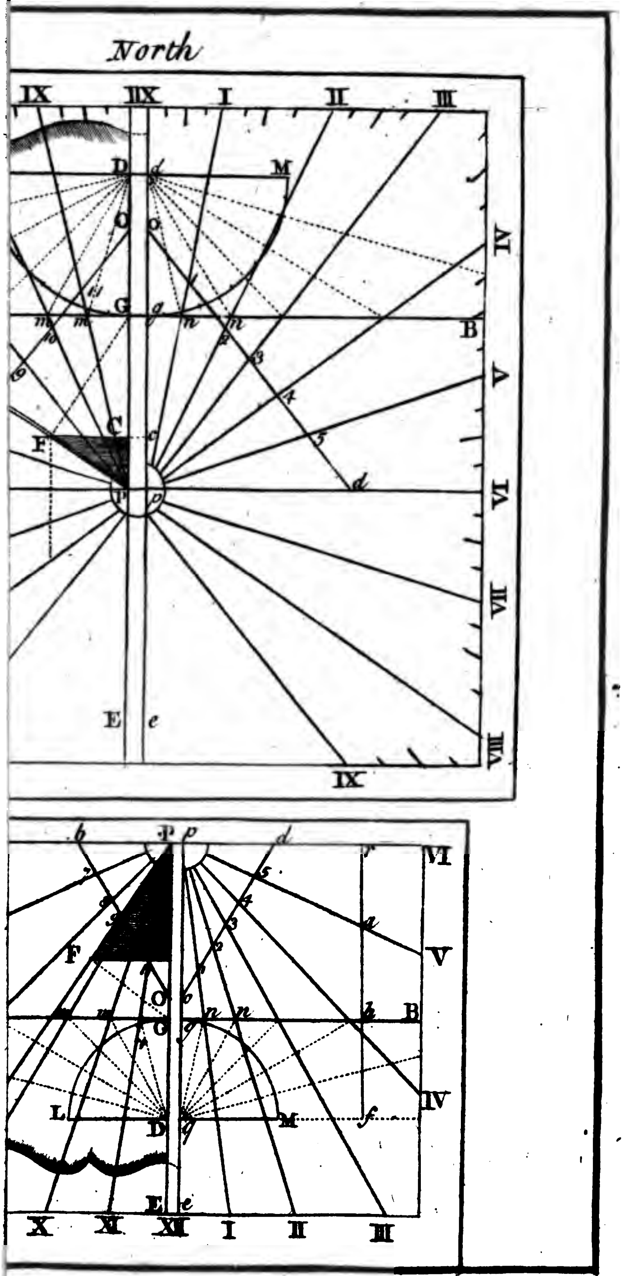
1. *Geometrically.*

29. Draw the horizontal line AB, and in AB take a point Q, on the left hand. Thro' Q draw the contingent line EQ, making an angle with AB, as AQE, equal to the complement of the latitude. Thro' any point E of the line EQ, draw FEG perpendicular to EQ, for the ſix o'clock hour line, and the ſubſtile.

Take any length EF for the height of the ſtile, and with the center F, taken in the line FG, and any radius, deſcribe the ſemicircle LEM. Divide each quadrant EL, EM into ſix equal parts for hours, beginning at E. Thro' theſe points draw lines from the center F, to cut the line EQ (which repreſents the equinoctial) in the hour points *n*, *n*, *n*, &c.

Thro' the points *n*, *n*, &c. draw lines parallel to GEF, and theſe will be the hour lines.

On



Pl.V.pa.82.

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On GF raiſe the perpendicular plane GHMR, whoſe height RM or GH is equal to EF; and that plane, which is a parallelogram, will be the ſtile, which muſt be made very thin on the top HM. Or you may have a perpendicular pin fixt at E, whoſe height is EF, for the ſtile. Fig. 29.

But it is better to leave a ſpace at EG equal to the thickneſs of the gnomon; and deſcribe two quadrants LEF, MEF, from two centers, and ſo find the hour lines on each ſide, ſeverally by theſe centers, as was done in the former Problems. And then the ſtile muſt be of a thickneſs equal to that ſpace.

Then the hours muſt be marked towards the ſouth, 5, 4, 3, leaving out the hours before ſun riſe; and towards the north 7, 8, 9, 10, 11; the ſubſtile being 6.

If you would have half hours and quarters, divide each hour of the ſemicircle into two or four parts, and proceed as in the whole hours.

If you would have your dial to take in 11 o'clock, you muſt find the ſtile's height to anſwer that, after this manner. If 11 be the point for 11 o'clock, draw 11 F to make an angle of 15° with 11 E, and to interſect FEG in F. Then EF is the height of the ſtile.

2. *By Calculation.*

Radius :
Height of the ſtile EF *in inches* : :
Tan. hour arch from 6 :
Diſtance from EG *the* 6 *o'clock line.*

Theſe diſtances being found for all the hours, muſt be put regularly in a table. But if you would have 11 o'clock juſt to come in, you muſt find the height of the ſtile by this proportion; as radius : length 11 E : : tan. 15° : FE, the height of the ſtile. Suppoſe FE to be 3 inches, then the hour diſtances from E, will be as in the following table.

Fig. 29.

	Hours.	diſtances.
	III	3.00
	IV	1.73
	V	0.80
Subſt.	VI	0.
	VII	0.80
	VIII	1.73
	IX	3.00
	X	5.20
	XI	11.20

Conſtruction.

Make the angle AQE equal to the complement of the latitude, AB being parallel to the horizon. Draw the 6 o'clock line EG perp. to QE. Then ſet off the ſeveral diſtances from E as in the table; that is, make E 5 and E7 = 0.80; E4 and E8 = 1.73; E3 and E9 = 3; E10 = 5.20; and E11 = 11.20; and thro' the points 7, 8, 9, &c. draw the hour lines parallel to EG. Laſtly, make RM the height of the ſtile = E9 = 3.

3. *By the Scale.*

Having drawn the horizontal line ABQ, and QE making an angle with it of the complement of the latitude; near the top, draw EG perpendicular to EQ for the ſubſtile and 6 o'clock line. Then in either of the polar lines, ſet one foot of the compaſſes at the beginning, and extend the other to I, ſet that extent from E to 5, and from E to 7, on the line EQ. Then extend from the beginning to II, and ſet it from E to 4, and from E to 8. In like manner extend from the beginning to III, IV and V; and ſet theſe extents, from E (to 3 and) 9, 10, 11, reſpectively. Then thro' all theſe points draw lines parallel to EG, for the hour lines.

And the diſtance from the beginning to III on the line, gives the height of the ſtile; to be ſet perp. upon the hour line of ſix. Fig. 29.

PROB. XI.

To make an erect direct weſt Dial.

This dial is made by the ſame rules as the eaſt dial, only changing the poſition of it. For in a weſt dial, the point Q and angle AQE muſt be taken on the left hand. And when the hour lines are drawn, they muſt be numbered the contrary way. The hours from the ſubſtile or 6 o'clock, muſt be numbered towards the ſouth 7, 8, 9; and from the ſubſtile towards the north, 5, 4, 3, 2, 1. If you look at the back ſide of the paper, turning it to the light; you will ſee the true figure of a weſt dial thro' the paper. Therefore if all the lines in an eaſt dial were to paſs thro' the plane, they will make a weſt dial, on the back ſide of it.

PROB. XII.

To draw a ſouth erect declining Dial.

Example 1.

Suppoſe a dial declines weſt 36 degrees, in the lat. 54½.

1. *Geometrically.*

Take the point C about the middle of the plane for the foot of the ſtile, thro' which draw the horizontal line AB, and CF perpendicular to it, and equal to the perpendicular height of the ſtile. 30.

Make the angle CFZ equal to the declination (36°), to the right hand if it decline eaſt; or to the left, if weſt; to cut the horizontal line in Z.

Thro' Z draw PZE perpendicular to AB, for the meridian.

Take

Fig. 30. Take ZF and ſet it from Z to X, in the line AB. Make the angle ZXP equal to the latitude (54½), to cut the meridian in P. Then P is the center of the dial.

Thro' C and P draw PCD for the ſubſtile.

Thro' C draw QQ perpendicular to PD, cutting the meridian in N, for the contingent line.

In CQ make CT equal to CF, and draw PT.

Take the neareſt diſtance from C to PT, and ſet it from C to D, in the line PD, and draw DN.

With the center D, and any radius as DC, deſcribe the ſemicircle SCW.

Divide the ſemicircle SCW into parts of 15 degrees for hours, beginning at the line DN, or where it cuts the circle.

Thro' theſe points draw lines from the center D, to interſect the equinoctial QQ, in the hour points 11, 12, 1, 2, 3, 4, &c.

From P the center of the dial, draw lines thro' the hour points, as P11, P12, P1, P2, P3, &c. for the hour lines.

Then the ſtile may either be the perpendicular pin FC, or the triangle PCT ſet upright on the ſubſtile PC. Here the edge of the ſtile PT muſt be made very thin; or which is better, a ſpace muſt be left at PC, equal to the thickneſs of the ſtile, for it to ſtand on, juſt as if this dial was cut in two, thro' the line PCD, and one part ſeparated from the other to that diſtance.

If you would have half hours and quarters, divide each hour of the ſemicircle into two or four parts, and draw lines thro' theſe points and the center, to cut the equinoctial, as before.

The meridian PE muſt be numbered 12, and the hours on the left muſt be 11, 10, 9, &c. and thoſe on the right 1, 2, 3, 4, &c.

If you had rather begin at the center of the dial, you may proceed thus. Take the point P near the

top,

Fig. 30. top, for the center; thro' which draw the meridian PE; in PE take any point Z, thro' which draw the horizontal line AZB. Make the angle ZPX equal to the complement of the latitude, cutting the horizontal line in X. Make the angle BZF equal to the complement of the declination, towards the right hand, if the declination be weſt; but to the left, if eaſt. Make ZF equal to ZX; from F draw FC parallel to ZP, to cut the horizontal line in C; then CF is the height of the ſtile, and C the foot of it. Thro' P and C draw PCD for the ſubſtile. Then draw the contingent QQ, and finiſh the reſt, as before.

2. *By Calculation.*

Here we have given the latitude of the place (54°½), and the declination of the plane (36°); to find the ſubſtile's diſtance, the height of the pole above the plane, and the plane's dif. longitude.

Radius	—	10.
Sin. declination (36)		9.76921
Cotan. latitude (54½)	—	9.85326
Tan. ſubſtile's diſtance 22 45,		9.62247

Again,

Radius	—	10.
Coſ. declination (36)		9.90795
Coſ. latitude (54½)	—	9.76395
Sin. ſtile's height, 28 1		9.67190

And,

S. latitude (54½)	—	9.91068
Radius	—	10.
Tan. declination (36)	-	9.86126
Tan. dif. longit. 41 45,		9 95058

Fig. 30. Then add 15 degrees continually to 41 45 (the diff. longitude), for the hour arches before 12: And ſubſtract 15 deg. continually, till you come at the ſubſtile; beyond which you muſt add 15 deg. continually, to the remainder of the hour where the ſubſtile ſtands; for the hour arches after 12. And theſe hour arches are all reckoned from the ſubſtile. All theſe muſt be regularly put down in a table againſt the reſpective hours. If you calculate for quarters, you muſt add and ſubſtract 3° 45′ continually. Then you muſt calculate for the hour angles, by the following analogy, and place them againſt the ſeveral hours, as in the following table.

As radius ——	10.
S. ſtile's height (28 1)	9.67185
Tan. hour arch (86 45)	11.24577
Tan. hour angle (83 6)	10.91762
Tan. 71 45	10.48181
Tan. 54 55	10.15366

And ſo on.

Hours.	hour arches.		hour angles.	
IX	86°	45′	83°	6′
X	71	45	54	55
XI	56	45	35	37
XII	41	45	22	45
I	26	45	13	19
II	11	45	5	35
	ſubſtile			
III	3	15	1	32
IV	18	15	8	49
V	33	15	17	7
VI	48	15	27	45
VII	63	15	42	59
VIII	78	15	66	7

Fig. 30.

Conſtruction.

Take P near the top, for the center of the dial; and draw the meridian PE, for the 12 o'clock line. By the line of chords make the angle EPC, 22° 45′ the ſubſtile's diſtance, on the right hand, as the declination is weſt; and draw the ſubſtilar line PCD. Make the hour angles from the ſubſtile, as you find them in the table, viz. CPIX = 83 6; CPX = 54 55; CP11 = 35 37; CP12 = 22 45; CP1 = 13 19; CP2 = 5 35; and CP3 = 1 32; CP4 = 8 49, &c. and PIX, PX, P11, P12, P1, P2, P3, &c. will be the hour lines. Make the angle CPN = 28 1, for the ſtile.

3. *By the Scale.*

If you have a ſcale fitted to your latitude; find the declination on the line of chords; againſt which, on the other lines, you will find all the requiſites, viz. the ſubſtile's diſtance, the ſtile's height, and plane's dif. longitude. If you have no ſuch ſcale, you muſt find the requiſites by Gunter's ſcale, if you would work the whole inſtrumentally; otherwiſe by calculation, as before.

Having made a table of the horary diſtances from the ſubſtile, chooſe any point H near the top for the center, thro' which draw the meridian or 12 o'clock line perpendicular to the horizon. Make the angle EPC = 22° 45′, the ſubſtile's diſtance from the meridian; thro' C, draw PCD for the ſubſtile, on the right hand, becauſe the declination is weſt. Draw *b*P*d* perpendicular to PC. Then from the line of latitudes, take the height of the pole above the plane, 28° 1′, and ſet it from P to *b* and *d*; and take the whole length of the line of the inclination of meridians, and ſet from *b* to cut PC in O, and draw *b*O and *d*O. Then from the ſame line of inclination, take theſe horary diſtances

(or

Fig. (or hour arches) as you have them in the table,
30. viz. 11° 45′, 26° 45′, 41 45, 56 45, &c. and ſet them ſeverally from O to *a*, *a*, *a*, &c. till you come at *b*. Alſo take the horary diſtances 3 15, 18 15, 33 15, &c. and ſet from O towards *d*, as at *r*, *r*, *r*, &c. till you come at *d*. Then from P, thro' all the points *a*, *a*, *a*, and *r*, *r*, *r*, &c. draw lines quite thro' the dial, which will be the hour lines required. If any point falls above the horizontal line, draw a line from it, thro' the center, to the other ſide. You may obſerve, that any diſtance O*a* is equal to its correſpondent *dr*; and any O*r* is equal to *ba*.

Make the angle CPT = 28 1 for the ſtile. And the forenoon hours muſt be markt towards the left hand, and the afternoon hours towards the right hand, of the meridian.

If you would have the half hours, they muſt be taken off the line of inclination, as before, and ſet on the lines O*b* and O*d*.

As this is a very common and uſeful caſe, I ſhall give another example of it.

Ex. 2.

Suppoſe a S. plane declines eaſt 49°, *in lat.* 51½.

1. *Geometrically.*

31. Take C for the foot of the ſtile, thro' which draw the horizontal line AB; and CF perpendicular to it, and equal to the height of the ſtile.

Make the angle CFZ equal to the declination (49°), to the right hand, becauſe the plane declines eaſt; and it will cut the horizontal line in Z.

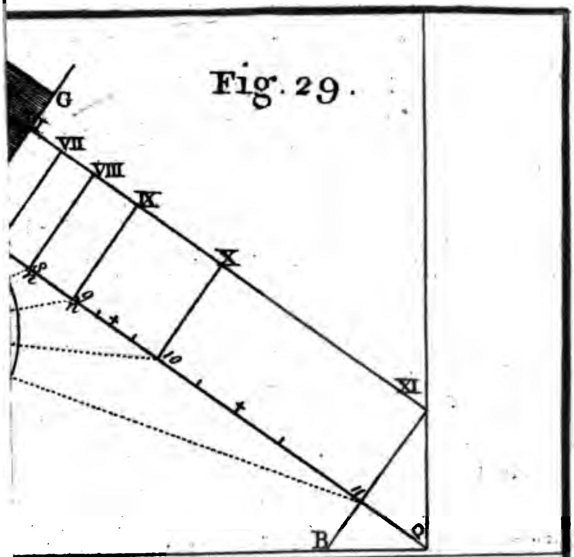
Thro' Z, draw PZE perpendicular to AB, for the meridian.

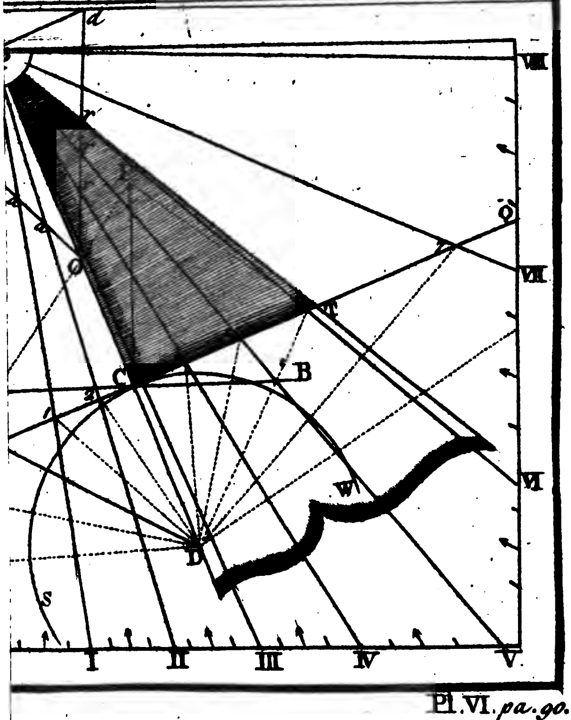
Set ZF from Z to X, in the horizontal line AB, either way.

Make the angle ZXP upwards, equal to the latitude (51½); then P is the center of the dial.

Thro'

Fig. 29.





Pl. VI. *pa. 90.*

Thro' C and P draw PCD for the ſubſtile. Fig. 31.

Thro' C draw the contingent line QQ perpendicular to the ſubſtile PC, to cut the meridian in N.

Make CT equal to CF, and draw PT.

Take the neareſt diſtance from C to PT, and ſet from C to V in the ſubſtile PD.

From the center V with any radius, deſcribe the ſemicircle SCW.

Divide the ſemicircle into parts of 15 degrees for hours, beginning at the line DN.

Thro' all theſe points of diviſion, draw lines from the center V, to cut the contingent QQ in the hour points, 5, 6, 7, 8, 9, &c.

From the center P of the dial, draw lines thro' all the hour points, as P5, P6, P7, P8, &c. for the hour lines.

Then the triangle PCT will be the ſtile, which muſt ſtand perpendicular to the ſubſtile PC.

2. *By Calculation.*

As radius —	10.
S. declination (49°)	9.87777
Cotan. lat. (51½)	9.90060
Tan. ſubſt. diſt. (30 59)	9.77831

Radius —	10.
Coſ. declination (49°)	9.81694
Coſ. latitude (51½)	9.79414
S. ſtile's height (24 6)	9.61108

S. latitude (51½)	9.89354
Radius —	10.
Tan. declination (49)	10.06083
Tan. dif. longit. (55 47)	10.16739

Then

Fig. 31. Then having the plane's dif. longitude 55 47; by adding and ſubtracting 15 degrees continually to and from it; the hour arches, reckoned from the ſubſtile, will be had, as in the following table.

Hours.	hour arches.		hour angles.	
III	79°	13′	65	0
IV	64	13	40	13
V	49	13	25	20
VI	34	13	15	31
VII	19	13	8	6
VIII	4	13	1	44
	Subſtile			
IX	10	47	4	27
X	25	47	11	10
XI	40	47	19	24
XII	55	47	30	59
I	70	47	49	31
II	85	47	79	46

Then for the hour angles,

Radius — 10.
S. ſtile's height (24 6) 9.61108
T. hour arch (4 13) 8.86763

T. hour angle (1 44) 8.47871

and ſo of the reſt, as in the table.

Conſtruction.

Take ſome point P near the top for the center of the dial, thro' which draw the meridian PE, for the 12 o'clock line. And make the angle EPC = 30 59 to the left hand, as the declination is eaſt, and draw the ſubſtile PC. Then ſet off the ſeveral hour angles from the ſubſtile, as you have them in the table: CP8 = 1 44; CP7 = 8 6; Fig. CP6 = 15 31, &c. and you have the hour lines. 31. Make CPT = 24 6, and CPT will be the ſtile.

3. *By the Scale.*

Having found the requiſites by Gunter's ſcale, or by calculation, take ſome point P for the center, from which draw the meridian or 12 o'clock line PE; make the angle EPC = 30 59, to the left hand, becauſe the declination is eaſt, and PC will be the ſubſtile. Thro' P draw a perpendicular to PC, on which ſet the ſtile's height 24 6, taken from the line of latitudes, from P to *b* and *d*. Then ſet the whole length of the line of hours or inclination of meridians, from *b* or *d* to O in the line PC, and draw *b*O and *d*O. Then take the ſeveral hour arches, which you have in the table, off the line of inclination, and ſet them from O towards *b*, and from O towards *d*. Thoſe on the left of the ſubſtile (4 13, 19 13, 34 13, &c.) muſt be ſet on O*b* from O to *a*, *a*, *a*, &c. And thoſe on the right hand (10 47, 25 47, 40 47, &c.) ſet from O to *r*, *r*, *r*, &c. Then from P, thro' all the points *a*, *a*, *a*, draw the hour lines P8, P7, P6, &c; and thro' all the points *r*, *r*, *r*, &c. draw the hour lines P9, P10, P11, &c. Then make the angle CPT = 24 6 for the ſtile.

PROB. XIII.

To draw a north erect declining Dial.

To do this make a ſouth declining dial, whoſe declination is the ſame, and lies the ſame way, then turn it upſide down, and it will be a north declining dial. But the hours muſt be numbered the contrary way.

Thus,

Fig. 31. Thus, if you want a north-eaſt decliner, make a ſouth-eaſt decliner, if a north-weſt decliner, make a ſouth-weſt decliner.

Likewiſe, by extending the ſtile and the hour lines quite thro' the center; ſo as the hours may be ſeen on the other ſide of the plane; the ſouth-eaſt decliner will then produce a north-weſt decliner; and a ſouth-weſt decliner, a north-eaſt decliner. But the hours muſt be numbered 1, 2, 3, &c. from the meridian on the right hand; and 11, 10, 9, &c. on the left. But the midnight hours, or thoſe before ſun riſe, and after ſun ſet, muſt be left out; as the ſun, being then below the horizon, does not ſhine on them.

And in drawing the dial, the center H muſt be taken about the middle of the plane, becauſe ſome hours more are required, than in a ſouth decliner; and theſe are had by producing the other hour lines thro' the center. I judge it needleſs to draw a figure for it; but the calculation of ſuch a dial will be as follows.

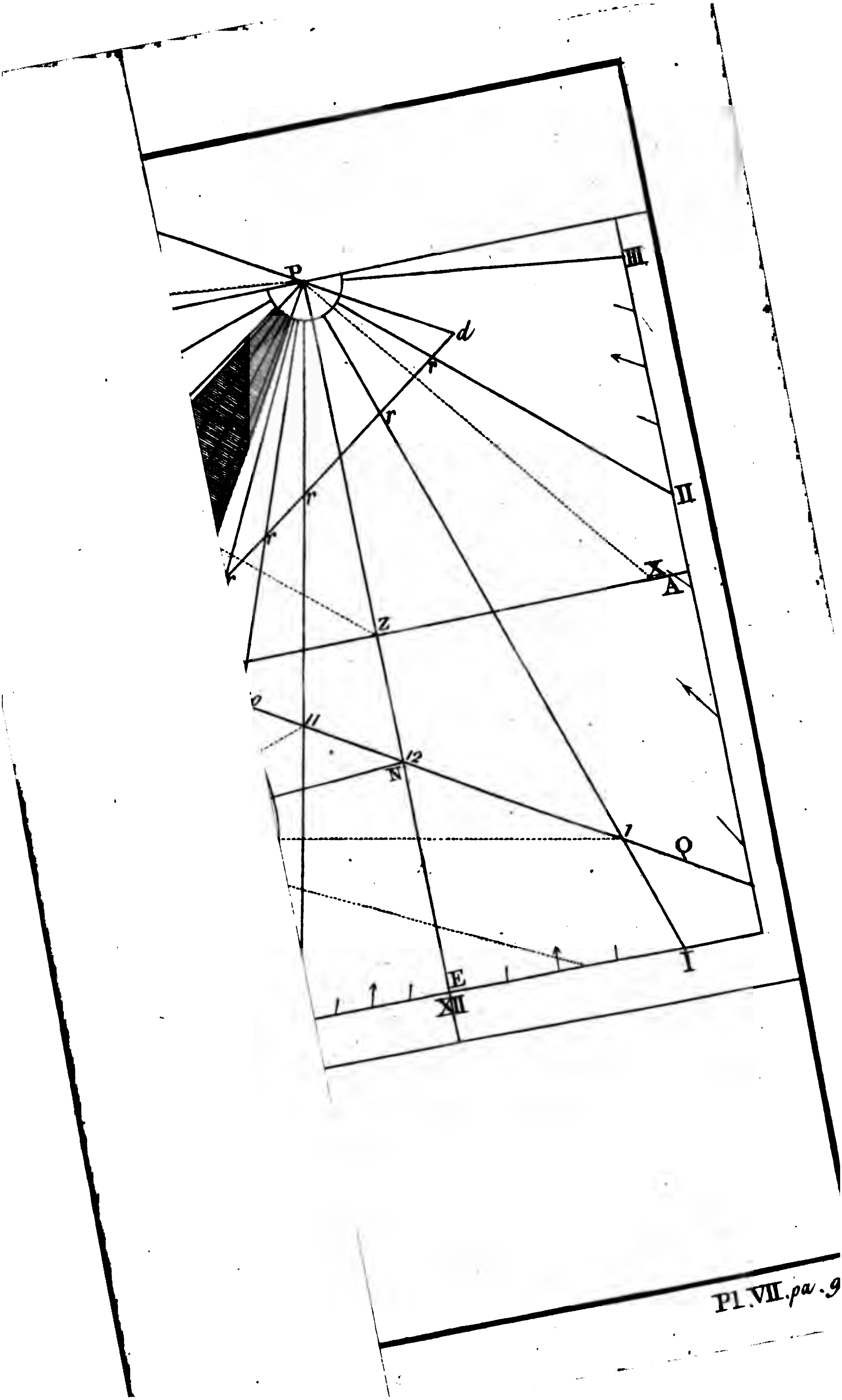
Suppoſe a north plane declining eaſt 21 *degrees, in lat.* 54½.

Then we ſhall find, by calculation, or Gunter's ſcale,

The ſubſtile's diſtance	14° 20′
Stile's height	32 50
Plane's dif: longitude,	25 15

And the hour arches, and hour angles will be as in the following table.

Hours



Pl. VII. pa. 9

Hours.	hour arches.	hour angles.
8	85 15	81 19
9	70 15	56 30
10	55 15	
11	40 15	
12	25 15	
1	10 15	
	Subſtile	
2	4 45	
3	19 45	11 06
4	34 45	20 37
5	49 45	32 38
6	64 45	48 59
7	79 45	71 33

PROB. XIV.

To draw an upright dial without a center; or a far declining dial.

Examp.

Suppoſe a plane declines 70 degrees eaſt, in the lat. $54\frac{1}{2}$.

1. *Geometrically.*

In ſuch dial planes, on which the pole has but ſmall elevation, if the center of the dial be taken within the plane; the hour lines, eſpecially near the ſubſtile, will be ſo cloſe together, as not to be readily diſtinguiſhed; and therefore ſuch a dial would be uſeleſs. But by making the ſtile higher at pleaſure, the hours will become wider; but then the center will run out of the plane; and therefore we muſt uſe the following method to draw the dial without a center.

Suppoſe

Fig. 32. Suppoſe the preſent center of the dial to be H, thro' H draw the meridian HE, in which take ſome point Z, thro' which draw the horizontal line XZB. Make the angle ZHX equal to the complement of the latitude, cutting the horizontal line in X. Make the angle BZF equal to the complement of the declination, towards the right hand, when the declination is weſt; but to the left, when it is eaſt. Make ZF equal to ZX; and from F draw FC parallel to HZ, to cut the horizontal line in C. Then CF is the preſent height of the ſtile, and C the foot of it. Thro' H and C draw the ſubſtile HCD. Thro' C draw ICI perpendicular to the ſubſtile HD; and thro' ſome other point S, another perpendicular KSK, to cut HE in A. In CI take CN equal to CF, and draw HNO for the ſtile. From S take the neareſt diſtance to HO, and ſet from S to G in the ſubſtile, and draw GA. Then ſince the ſtile HO has but ſmall elevation, draw T*t* parallel to HO, at a convenient diſtance, for the new ſtile. Then take the neareſt diſtance from S to T*t*, and ſet from S to D in the ſubſtile, and draw the line DL parallel to GA. From the center D, deſcribe a ſemicircle, cutting DL in L. Divide this ſemicircle into hours beginning at L, and draw lines from the center D, thro' theſe points, to cut the contingent line KK in the hour points, that of L being 12 o'clock.

Again, take the neareſt diſtance from C to T*t*, and ſet from C to *d* in the ſubſtile. From *d* deſcribe a ſemicircle, and make the angle C*dl* = SDL. Then divide this ſemicircle into hours beginning at *l*; and thro' all the points draw lines from the center *d*, to cut the contingent line II in the hour points, *l* being 12 o'clock.

Laſtly, thro' the correſpondent hour points, in both contingent lines, draw right lines, and theſe will be the hour lines. And to number the hours, obſerve the line DL runs to 12 o'clock, (which is generally out of the dial) and this directs all the reſt; for the next hour point is 11, the next 10, &c. Fig. 32.

2. *By Calculation.*

The requiſites being found as in the laſt Problem, will be as follows.

Subſtile's diſtance	33° 50′
Stile's height	11 28
Plane's dif. longitude	73 31

Then ſuppoſe HC to be 2.7 inches; HS, 7.3; then by plane trigonometry to find the leaſt diſtances from C and S to HO,

As *radius*	10.
S. SHO (11 28)	9.29841
So HC (2.7)	0.43136
diſtance from C (.537)	—1.72977
So HS (7.3)	0.86332
diſtance from S.(1.45)	0.16173

Theſe diſtances being too little, increaſe them with any quantity you think proper, ſuppoſe 1.1; then the neareſt diſtances from C and S to T*t*, will be 1.64 and 2.55; that is, the radius SD and C*d* will be 2.55 and 1.64. Then to find the hour points. After the hour arches are had by help of the dif. longitude 73 31, and put into a table; the correſponding diſtances, along the lines KK and II, from the ſubſtile, are had by this proportion.

As *radius*	10.
Tan. hour arch (61 29)	10.26493
So SD (2.55)	0.40654
diſtance from S (4.70)	0.67147
So C*d* (1.64)	0.21484
diſtance from C (3.02)	0.47977

Fig. 32. and ſo on thro' the whole, which muſt be put down in the table, againſt their correſponding hour arches, as follows.

Hours.	hour arches.	diſtance from S.	diſtance from C.
3	61 29	4.70	3.02
4	46 29	2.69	1.73
5	31 29	1.56	1.01
6	16 29	0.75	0.48
7	1 29	0.06	0.04
	Subſtile		
8	13 31	0.61	0.39
9	28 31	1.38	0.89
10	43 31	2.42	1.56
11	58 31	4.16	2.68

Conſtruction.

Draw an obſcure line HE perpendicular to the horizon, and make the angle EHD = 33 50, and HD is the ſubſtile. Make the angle DHO = 11 28, the height of the ſtile, and draw HO, then draw T*t* parallel to HO, at a diſtance 1.1 from it; then HST*t* will be the ſtile, to be ſet perpendicular upon the ſubſtile HD. Make HC = 2.7; and HS = 7.3. And thro' C and S, draw two perpendiculars to the ſubſtile HD, as II and KK. Then ſet off upon II and KK, from the points C and S, the diſtances you find in the table; thoſe above the ſubſtile, to the left; and thoſe below, to the right of it. Laſtly, thro' the correſpondent points in the two lines, draw the hour lines of the dial.

If you look thro' the back of the paper, you will ſee a dial declining as far to the weſt.

Fig. 32.

3. *By the Scale.*

Having found the requiſites by Gunter's ſcale, or by calculation, draw HE perpendicular to the horizon; and make the angle EHD the ſubſtile's diſtance, and DHO the ſtile's height, and draw HD and HO for the ſubſtile, and ſtilar line. Chuſe a point in the ſubſtile as S, thro' which draw a line KK perpendicular to the ſubſtile.

Then take the diſtance from the beginning to the hour III, on the greater polar line, and ſet it from S to T, and draw T*t* parallel to HO; and T*t* is the new ſtile. Then open the compaſſes from the beginning to the hour III, on the leſſer polar line, and ſetting one foot at S in the ſubſtile HD, move it along the ſubſtile (keeping the feet of the compaſſes parallel to KK,) till the other foot touch the line T*t*; as at *r*; thro' that point *r*, draw a line ICI parallel to KK.

Then from the table of hour arches, take the ſeveral arches from the greater polar line, and ſet them from S, upon the line KK, firſt to the left hand, and then to the right, as they lie in the table, in reſpect of the ſubſtile; then you have the hour points on the line KK.

Again, from the ſame table of hour arches take every arch from off the leſſer polar line, and ſet both ways from C, on the line II, as before. Then lines drawn thro' the correſpondent points in both lines KK and II, will be the hour lines of the dial.

The ſtile CST*t* muſt be placed perpendicular on the ſubſtile CS, and made thin at the top; unleſs you have a mind to leave a ſpace there, the thickneſs of the ſtile.

Fig.

PROB. XV.

To draw a dial upon a direct ſouth reclining plane, or upon a direct north inclining one.

1. If a ſouth plane reclines, and the reclination is leſs than the complement of the latitude, add the reclination to the latitude, the ſum is the latitude, for which you muſt make an erect ſouth dial, by Prob. VIII.

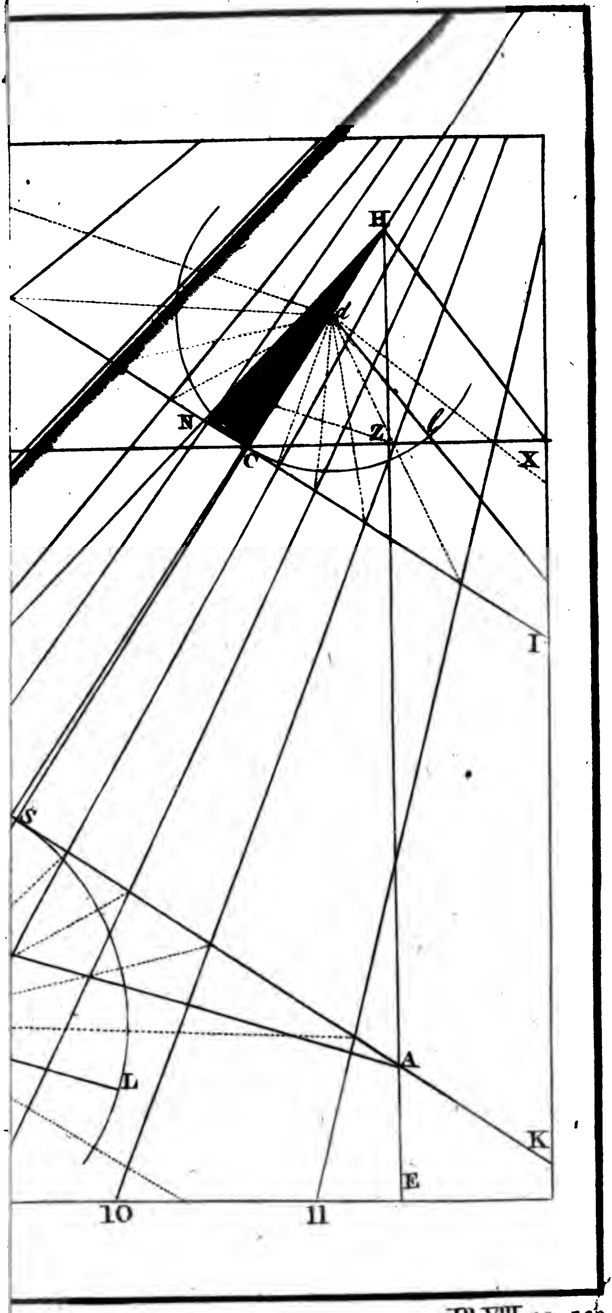
2. If the reclination be equal to the complement of the latitude, then the pole falls in the plane of the dial; and then you muſt make an horizontal dial under the equinoctial, by Prob. VI.

3. If the reclination be greater than the complement of the latitude; ſubtract the complement of the latitude from the reclination; and the remainder is the latitude, for which you muſt make a horizontal dial, by Prob. VII. And theſe are all the varieties that can happen in a reclining ſouth dial.

In north inclining planes, if a dial be made on the ſouth reclining ſide, and the ſtile and hour lines continued thro' the plane; there will be made a north inclining or proclining dial, as required. Otherwiſe thus, which comes to the ſame thing; make ſuch a dial as is before directed for a reclining plane, then turn it upſide down, and it will be a dial for its correſponding north inclining plane. But in all of them, the hours muſt be numbered the contrary way, from the meridian or 12 o'clock line.

Theſe inclining dials are the worſt ſort of dials, by reaſon the ſun does not ſhine ſo long on them, as upon other dials.

PROB.



Pl. VIII pa. 100.

PROB. XVI.

To make a direct north reclining dial, or a direct ſouth inclining one.

1. If a north plane reclines, and the reclination is leſs than the latitude; ſubtract the reclination from the latitude, and the remainder will be the latitude, for which you muſt make a direct ſouth dial; and turn it upſide down to face the north, and let the hours be numbered weſtward, 1, 2, 3, &c. and eaſtward, 11, 10, 9, &c. but ſome of theſe midnight hours muſt be left out; and ſome taken in, after 6 in the morning, and before 6 at night.

Otherwiſe thus, add the reclination to the complement of the latitude; and the ſum is the latitude for which a horizontal dial is to be made; then the north end is to be turned upward, and the thing is done. But leave out the midnight hours.

2. If the reclination be equal to the latitude, then the plane coincides with the equinoctial; and conſequently a horizontal dial under the poles muſt be made by Prob. V.

3. If the reclination be greater than the latitude, add the complement of the reclination to the latitude; and the ſum is the latitude, for which a horizontal dial muſt be made, by Prob. VII. which will ſuit that plane.

In ſouth inclining planes, (by Prob. VIII.) make a direct ſouth dial, for a latitude which is the difference between your latitude and proclination. Only, if that difference is 0, it will be an upright dial under the equinoctial, and the ſtile perpendicular; and made by Prob. V. And if the proclination be greater than the latitude, the dial will belong to ſouth latitude; and is made by Prob. VII.

but

Fig. 33. but the angle of the ſtile will lie the contrary way upon the meridian, as in fig. 33. A north reclining dial becomes a ſouth inclining one, when turned upſide down.

There need be no more than 12 hours in any of theſe incliners.

Example.

To make a north dial in lat. 60°, *that reclines* 5½ *degrees.*

To the comp. lat.	30
add the reclin.	5½
new latitude	35½ for a horizontal dial.

1. *Geometrically.*

34. Take the point P about the middle of the plane, for the center of the dial, and draw the meridian DPE, and a line perpendicular to it thro' P for the 6 o'clock line. Make the angle DPR = 35½ the new latitude. Take any point G in the meridian, thro' which draw the perpendicular AGB for the contingent line. Then ſet one foot of the compaſſes in G, and take the neareſt diſtance to the line PR, and ſet it from G to D. With the center D, and any radius, deſcribe a ſemicircle LSM; divide each quadrant LS, SM into 6 equal parts, thro' which and the center D draw obſcure lines to cut AB in the hour points. Thro' all theſe points or marks in AB, draw lines from the center P of the dial, and theſe will be the hour lines. But the midnight hours are left out, when the ſun does not ſhine. And the hours after 6 in the morning, and before 6 at night, are drawn thro' the center; and on theſe the ſun will ſhine in ſummer.

DPR is the ſtile, which will point to the pole when the dial is rightly placed.

Fig. 34.

2. *By Calculation.*

As radius :
Sin. new latitude (35½) : :
Tan. hour arch :
Tan. hour angle.

Hence the following table.

Hours.		hour arches.		hour angles.	
3	9	45	0	30	9
4	8	60	0	45	10
5	7	75	0	65	14
6		90	0	90	0

Conſtruction.

Draw the meridian DE, and take P for the center; at P make angles with PD and PE, as you find them in the table, and draw the hour lines.

Make DPR = 35½, and you have the ſtile.

3. *By the Scale.*

Let P be the center, about the middle of the plane; and draw the meridian PE, and P*d* perpendicular to it, for the 6 o'clock line; on which ſet the extent to the new latitude (35½), taken from the line of latitudes, from P to *b* and *d*. Then ſet the whole length of the hour line from *b* to *a* and *q*, on the meridian DE; and draw *ba*, *da*, *bq*, *dq*. Then on the line of hours, take the extent to I the firſt hour, and ſet from *b* to 5 and 7, and from *d* to 5 and 7. Then take the extent to the ſecond hour II, and ſet from *b* to 4 and 8, and from *d* to 4 and 8. Alſo take the extent to III, and ſet from *b* to 9, and from *d* to 3. Then from P draw lines

Fig. 34. thro' all theſe points, for the hour lines. Make DPR = 35½ for the ſtile.

The mid-day or midnight hours need not be drawn, as the ſun never ſhines on them.

PROB. XVII.

To make an inclining or reclining eaſt dial.

Example.

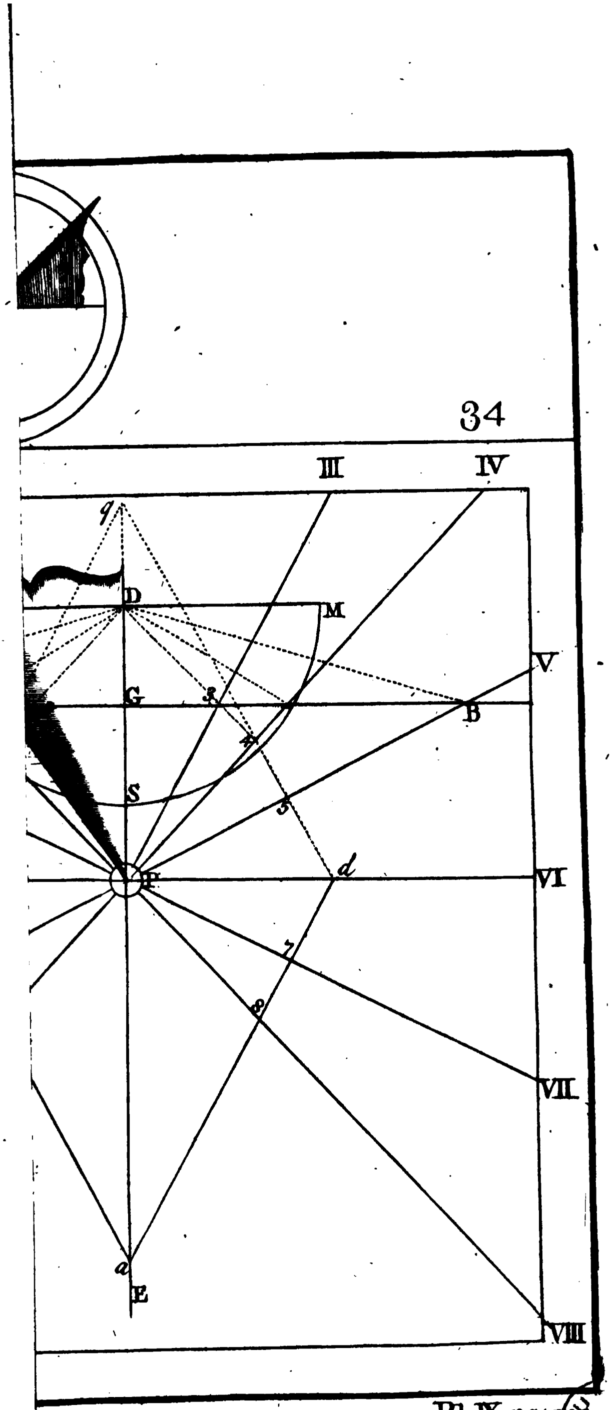
Let an eaſt dial recline 21 deg. in lat. 54½.

1. *Geometrically.*

35. Take ſome point C for the foot of the ſtile, near the middle of the plane; thro' which draw the perpendicular line DE. Draw CF perpendicular to DE, or parallel to the horizon, and equal to the height of the ſtile. Make the angle CFH equal to the complement of the reclination or proclination; downwards, if it recline; or upwards, if it incline, to cut the vertical line DE in H. Thro' H draw the meridian or 12 o'clock line PHI perpendicular to DE. Make HG = HF, and make the angle HGP = the complement of the latitude, to the left hand when the plane reclines, and to the right hand when it inclines; and P is the center of the dial, which will be at the top, as well as the meridian PI, when the plane inclines. Thro' P and C draw PCM for the ſubſtile.

Thro' C draw QQ perpendicular to the ſubſtile PM for the contingent, in which make CT = CF, and draw PT for the ſtile. Take the neareſt diſtance from C to the ſtile PT, and ſet it from C to V. Draw the line VI, from V to the interſection I of the contingent and meridian.

From the center V, with any radius, deſcribe a ſemicircle, which divide into hours, beginning at VI. Draw lines from the center V, thro' theſe divi-



Pl. IX. *pa. 204.*

Fig. 35. diviſions, to cut the contingent QQ in the ſeveral hour points. Then lines drawn from the center of the dial P, thro' theſe points, will be the hour lines. And the triangle CPT, erected on the ſubſtile, PV will be the ſtile.

For the hours, PI being 12 o'clock, the reſt proceed towards the left hand, being 11, 10, 9, 8, &c. in order.

2. By Calculation.

1. *As radius* ——		10.
Coſ. reclination (21)		9.97015
Tan. latitude (54½)		10.14673
Tan. ſubſtile's diſt.	52 37,	10.11688

2. *Radius* ——		10.
S. latitude (54½)		9.91068
S. reclination (21)		9.55432
S. ſtile's height,	16 58,	9.46500

3. *Radius* ——		10.
Coſ. latitude (54½)		9.76395
Tan. reclination (21)		9.58417
Cotan. dif. longitude,	77 26,	9.34812

Then make a table for the ſeveral hour arches, by continually adding and ſubſtracting 15 degrees, to and from the difference of longitude 77 26. And then to find the hour angles,

Radius ——	10.
S. ſtile's height (16 58)	9.46500
Tan. hour arch from the ſubſtile (57 34)	10.19692
Tan. hour angle 24 40,	9.66192

And ſo of the reſt, as in the following table.

Hours

Fig. 35.

Hours.	hour arches.		hour angles.	
3	57	34	24	40
4	42	34	15	0
5	27	34	8	40
6	12	34	3	43
	Subſtile			
7	2	26	0	43
8	17	26	5	14
9	32	26	10	30
10	47	26	17	37
11	62	26	29	12
12	77	26	52	37

Conſtruction.

Draw PI parallel to the horizon at the bottom of the plane, for the 12 o'clock line; or at top of the plane if it inclines; in which take any point P for the center of the dial, toward the left hand.

Make the angle IPV on the right hand, equal to the ſubſtile's diſtance from the meridian 52 37, and draw the ſubſtile PV. Alſo make the angle VPT equal to the ſtile's height 16 58, and draw the ſtile PT. Then make the hour angles on each ſide of the ſubſtile, as you have them in the table. as VP7 = 0 43, VP8 = 5 14, &c; and VP6 = 3 43, VP5 = 8 40, &c. And draw the hour lines.

3. *By the Scale.*

Having found the requiſites as above, by Gunter's ſcale, or by calculation; and made a table of the horary diſtances (or hour arches,) from the ſubſtile; take a point P near the bottom for the center, thro' which draw the 12 o'clock line PI parallel to the horizon. Make the angle IP*a* = ſubſtile's

ſtile's diſtance 52 37, and draw the ſubſtile P*a*. Fig. 35.
Thro' P draw *bd* perpendicular to P*a*, in which make P*b* and P*d* 16 58, the pole height, taken from the line of latitudes. Then take the whole line of hours, or the line of inclination, and ſet from *b* to *a* in the ſubſtile, and draw *ba* and *da*; then from the ſame line of inclination, take the hour arches as you have them in the table, and ſet thoſe above the ſubſtile from *a* towards *b*; and thoſe below, from *a* towards *d*, on the lines *ab* and *ad*. Thus, make *a*6 = 12 34, *a*5 = 27 34, *a*4 = 42 34, *a*3 = 57 34; alſo *a*7 = 2 26, *a*8 = 17 26, *a*9 = 32 26, &c. And draw the hour lines *a*3 *a*4, *a*5, *a*6, *a*7, &c. And make the angle *a*PT = 16 58, for the ſtile.

PROB. XVIII.

To make an inclining or reclining weſt dial.

This dial is made by the ſame rules as the eaſt incliner or recliner; only the angle PGH, which is made equal to the complement of the latitude, muſt be taken to the right hand, for a weſt recliner; but to the left hand for a weſt incliner. And the meridian PI will be at the bottom of the plane, if it reclines; but at top, if it inclines.

The hours muſt be reckoned the contrary way in the weſt dial; but the meridian or twelve o'clock line will guide all the reſt. If you turn the back of the paper towards you, and look thro' it, the eaſt recliner will be changed into a weſt recliner, for you will then ſee the figure of a weſt recliner thro' the paper. And ſo the back ſide of an eaſt incliner, becomes a weſt incliner.

Examp.

Fig. *Examp.*

35. *Suppoſe a weſt dial reclines* 50°, *in lat.* 54½.

As all the work is conſtructed as before, I ſhall only give the calculation of it, as follows.

Subſtile's diſtance	42° 1′
Stile's height	38 35
Plane's dif. longitude.	55 19

Then the table of hour angles and hour arches, will be as follows.

Hours.	hour arches.		hour angles.	
10	85	19	82	41
11	70	19	60	10
12	55	19	42	1
1	40	19	27	53
2	25	19	16	26
3	10	19	6	29
	Subſtile			
4	4	41	2	56
5	19	41	12	34
6	34	41	23	21
7	49	41	36	19
8	64	41	52	51
9	79	41	73	44

And if an eaſt dial inclines 50°, in lat. 54½; the hour angles will be the ſame as above.

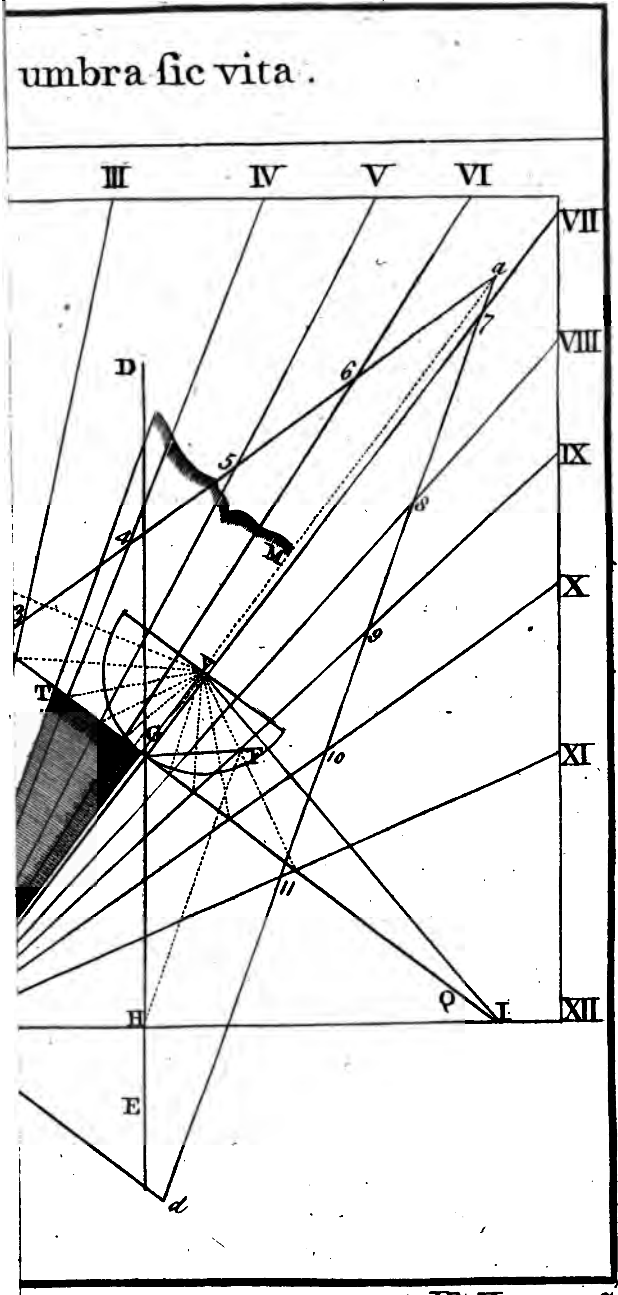
PROB. XIX.

To draw a ſouth declining, reclining or inclining dial.

Example 1.

Suppoſe a S.*plane declines weſtward* 25°, *and reclines* 15°, *in lat.* 54½.

1. *Geo-*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Pl.X.pa.108.

Fig. 36.

1. *Geometrically.*

Take any point C for the foot of the ſtile, about the middle of the plane, thro' which draw DE perpendicular to the horizon; and CF perpendicular to it, and equal to the ſtile's height.

Make the angle CFG the reclination of the plane; that is, upwards if it recline, or downwards if it incline; and make CFH the complement of it; and H is the nadir.

Thro' G draw the horizontal line AGB, perpendicular to DE.

In GD make GR = GF, and the angle GRI = the declination of the plane; to the right hand, if it declines eaſt, but to the left, if weſt; R being the dividing center of the horizon AB.

Thro' I and H, draw the meridian or 12 o'clock line ZIH.

Thro' C draw CMK perpendicular to ZH.

Set CM from C to L on the line CH, and extend from L to F, and ſet it from M to O, in the line MK; then O is the dividing center of ZH.

Draw OI, and make the angle IOP upwards, equal to the latitude; and draw OP interſecting ZH in P the pole, for the center of the dial.

Thro' P and C, draw PCV for the ſubſtile.

Thro' C draw the contingent QQ, perpendicular to the ſubſtile PC, interſecting the meridian in N.

Take CT equal to CF, and draw PT, then CPT is the ſtile.

Take the neareſt diſtance from C to the line PT, and ſet it from C to V in the ſubſtile PCV, and draw VN.

From the center V, with any radius, deſcribe a ſemicircle SCX, which divide into hours beginning at the line VN, which goes to 12.

Draw lines from V thro' all theſe points, to interſect the contingent QQ in the hour points.

And

Fig. 36. And thro' theſe points of interſection, draw lines from the center, which will be the hour lines of the dial.

Note, if OP run parallel to the meridian, PH; the ſubſtile PV, and all the hour lines (drawn thro' the ſeveral interſections of QO), will be parallel to ZH.

2. *By Calculation.*

As ſin. reclination (15°)	9.41299
Cotan. declination (25)	10.33132
Radius —	10.
Tan. heig. meridian, 83 9,	10.91833

As *Radius* ——	10.
Coſ. declination (25)	9.95727
Cotan. reclination (15)	10.57194
Tan. arch A, 73 32	10.52921
lat. 54 30	

Arch B 19 2; here A being greater than the latitude; the oppoſite pole is elevated.

As *Cotan. arch* B (19 2)	10.46220
Coſ. reclination (15)	9.98494
Sin. declination (25)	9.62594
	19.61088
Tan. ſubſtile's diſt. 8 1	9.14868

As *Coſ. ſubſtile's diſtance* (8 1)	9.99573
Coſ. arch B (19 2) —	9.97558
Radius ——	10.
Coſ. ſtile's height, 17 19	9.97985

As *Sin. arch* B. (19 2)	—	9.51337
Radius	—	10.
Sin. ſubſt. diſt. (8 1)	—	9.14445
Sin. plane's dif. longitude, 25 19		9.63108

Fig. 36.

Then make a table for the ſeveral hours, and place the plane's dif. longitude 25 19, againſt 12. To or from which add and ſubtract 15, 30, 45, 60, &c. to get the other hour arches, which put againſt their reſpective hours. Then for finding the hour angles,

Radius —	10.
S. ſtile's height (17 19)	9.47370
Tan. hour arch (79 41)	10.73985
Tan. hour angle, 58 33,	10,21355 for VII.

And ſo on for all the reſt, which will be as in the following table.

Hours.	hour arches.		hour angles.	
7	79	41	58	33
6	64	41	32	11
5	49	41	19	20
4	34	41	11	38
3	19	41	6	5
2	4	41	1	24
	Subſtile			
1	10	19	3	7
12	25	19	8	1
11	40	19	14	11
10	55	19	23	16
9	70	19	39	46
8	85	19	74	37

Fig. 36.

Conſtruction.

Draw the horizontal line *b*H near the bottom of the plane, in which take a point H towards the right hand, if the declination be weſt; but to the left if eaſt. Draw the line ZH making the angle *b*HZ = 83 9, the height of the meridian, for the 12 o'clock line. Take any point P near the top, when A is greater than the latitude, otherwiſe near the bottom, and draw the ſubſtile PV to the right hand for weſt declination, making the angle HPV = 8° 1′, the ſubſtile's diſtance. Then from P draw the hour lines, making angles with the ſubſtile at P, as you find them in the table. Thus VPII = 1° 24′. VPIII = 6° 5′, &c. And draw PT making the angle VPT = 17 19, the ſtile's height, and PTC is the ſtile.

3. *By the Scale.*

Having found the requiſites by Gunter's ſcale, or otherwiſe, draw the meridian PH, making an angle with a horizontal line of 83° 9′; draw the ſubſtile PV, making an angle at P with the meridian, of 8° 1′. Thro' the center P, draw the line *bd* perpendicular to PV; then take 17 19 (the pole's height) from the line of latitudes, and ſet from P to *b* and *d*. Then take the ſcale of hours or inclination, and ſet from *b* to *a*, in the line PV, and draw *ba*, *da*. Then from the line of inclination, take the ſeveral hour arches, againſt 2, 3, 4, 5, &c. in the table, and ſet them in the line *ad*, from *a* to 2, 3, 4, 5, 6, 7. Again, take the hour arches againſt 1, 12, 11, 10, &c. and ſet them in the line *ab*, from *a* to 1, 12, 11, 10, 9, 8. Then thro' all theſe points 2, 3, 4, &c. and 1, 12, 11, &c. draw lines from the center P for the hour lines.

Note, *a*2, *a*3, *a*4, &c. is equal to *b*8, *b*9, *b*10, &c.

Laſtly, draw PT, making the angle VPT = 17 19, and VPT will be the ſtile. Fig. 36.

Ex. 2.

Suppoſe a S. plane declines eaſt 50°, and reclines 42°, in lat. 54½.

1. *Geometrically.*

About the middle of the plane, take C for the foot of the ſtile, thro' which draw DE perpendicular to the horizon, and CF perpendicular to DE, and equal to the height of the ſtile. 37.

Make the angle CFG the reclination of the plane 42′, upwards, and CFH its complement, cutting DE in G and H, H being the nadir.

Thro' G draw the horizontal line GB, perpendicular to DE.

In GD take GR = GF, either way, and make the angle GRI = the declination 50°, to the right hand as it declines eaſt, to cut GB in I.

Thro' I and H draw the meridian PHIZ, or the 12 o'clock line.

Thro' C draw CMK perpendicular to the meridian HI.

Set CM from C to L on the line CH, and extend from L to F, and ſet it from M to O, in the line CMK.

Draw OI and make the angle IO*p* upwards, equal to the latitude 54½. Draw *p*OP interſecting the meridian HI in P, for the center of the dial, which repreſents the pole.

Thro' P and C draw PCV for the ſubſtile.

Thro' C draw the contingent QQ perpendicular to the ſubſtile PC, interſecting the meridian PI in N.

Take CT = CF, and draw PT, and CPT is the ſtile.

Fig. 37. Take the neareſt diſtance from C to the line PT, and ſet it from C to V in the ſubſtile PCV, and draw the line VN.

From the center V, with any radius, deſcribe the ſemicircle SCX, which divide into hours, beginning at the line VN, where it interſects the circle.

Draw lines from the center V thro' all theſe points, to cut the contingent line, as uſual, in the hour points; thro' which again draw lines from the center of the dial P, for the hour lines.

2. *By Calculation.*

As *Sin. reclination* (42)	9.82551
Cotan. declination (50)	9.92381
Radius —	10.
Tan. heig. meridian, 51 26,	10.09830

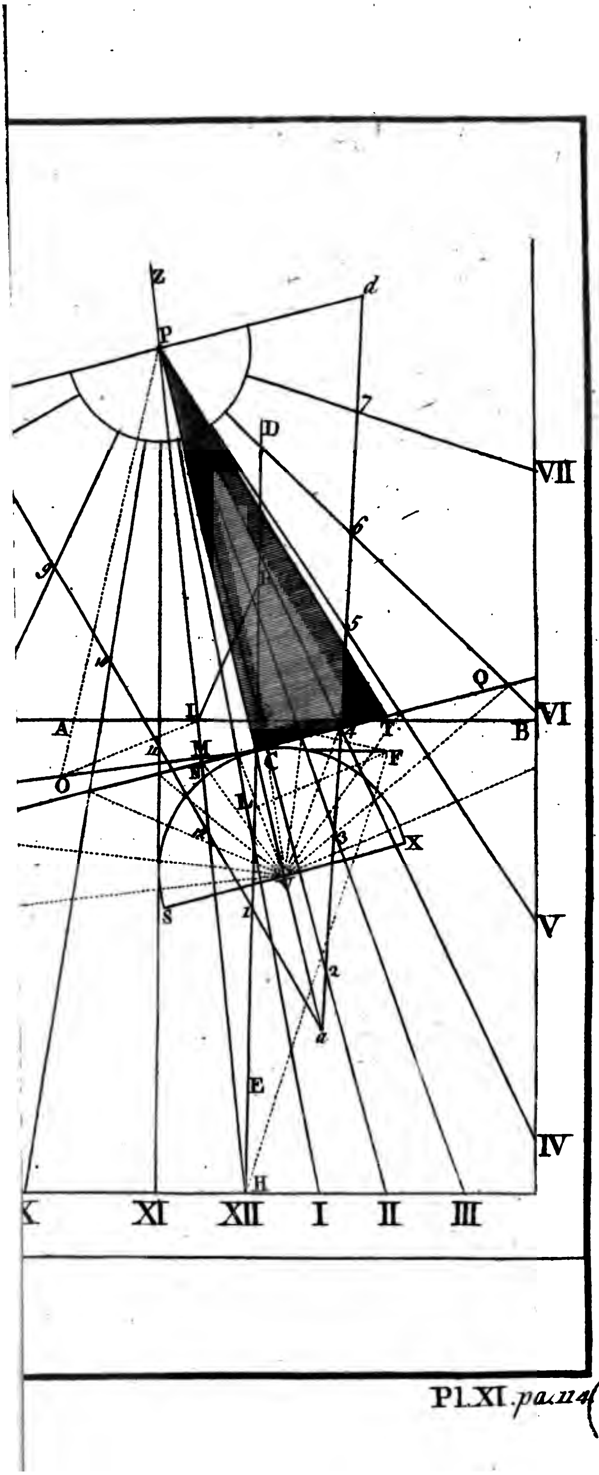
As *Radius* —	10.
Coſ. declination (50)	9.80806
Cot. reclination (42)	10.04556
Tan. arch A, 35 31	9.85362
Latitude 54 30	

Arch B, 18 59. Here A being leſs than the latitude, the pole is elevated above the plane.

As *Cotan. arch* B (18 59)	10.46343
Coſ. reclination (42)	9.87107
Sin. declination (50)	9.88425
	19.75532
Tan. ſubſtile's diſt. 11 5,	9.29189

As *Coſ. ſubſt. diſt.* (11 5)	9.99182
Coſ. arch B (18 59)	9.97571
Radius —	10.
Coſ. ſtile's height 15 30	9.98389

As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Pl.XI. pa.114

As *Sin. arch* B (18 59) 9.51227 Fig. 37.
Radius —— 10.
Sin. ſubſtile's diſtance (11 5) 9.28383

S.plane's dif. longit. 36 13, 9.77156

Then make a table of hour arches, placing 36 13 againſt 12; and find the reſt as uſual. From which find the hour angles by this analogy, and put them into the table.

Radius
Sin. ſtile's height;
Tan. any hour arch,
Tan. correſpondent hour angle.

Hours.	hour arches.		hour angles.	
4	83	47	67	49
5	68	47	34	33
6	53	47	20	3
7	38	47	12	7
8	23	47	6	43
9	8	47	2	22
	Subſtile			
10	6	13	1	40
11	21	13	5	55
12	36	13	11	5
1	51	13	18	24
2	66	13	31	14
3	81	13	59	58

Conſtruction.

Draw PY near the bottom of the plane, parallel to the horizon, the lat. being greater than A; and take P on the left hand for the center, being eaſt

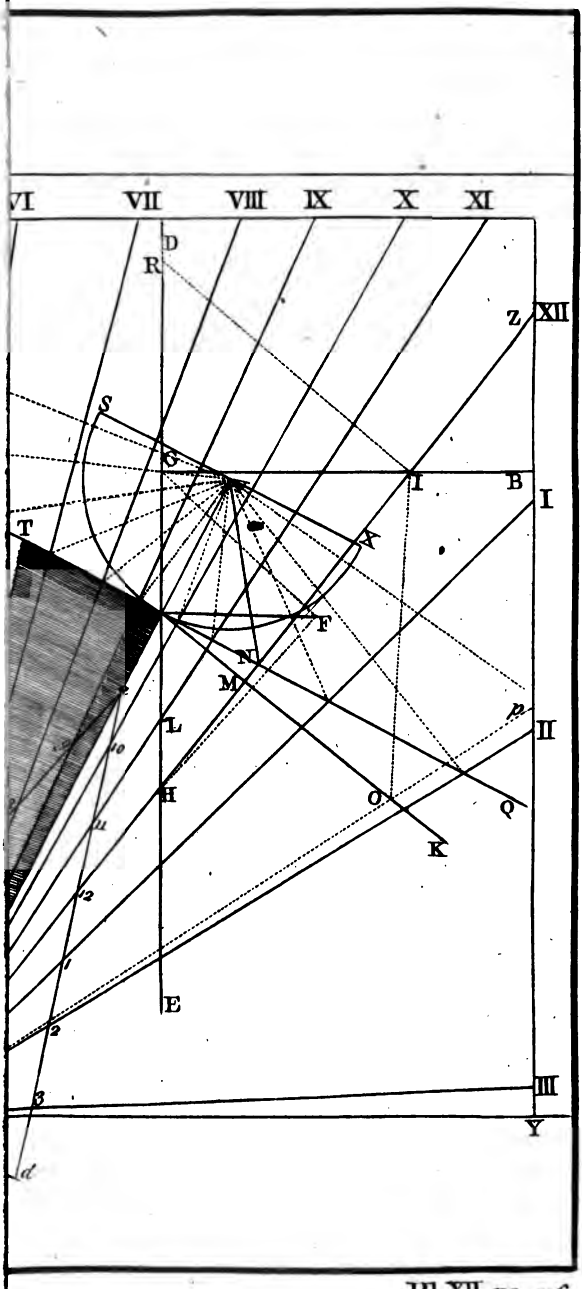
Fig. 37. declination. Make the angle YPI = 51 26, the height of the meridian; and to the left IPV = 11 5, the ſubſtile's diſtance; and VPT = 15 30, the ſtile's height. Then make the angles with the ſubſtile PV, on the left, 2 22, 6 43, 12, 7, &c. and on the right, 1 40, 5 55, &c. as in the table; and draw the hour lines. And CPT is the ſtile.

3. *By the Scale.*

Having found the requiſites by Gunter, draw the horizontal line PY, and meridian PI, making the angle 51 26, to the right. Make the angle IPV 11 5, and PV is the ſubſtile. Thro' the center P draw bd perpendicular to PV. And take 15 30 the pole's height, from the line of latitudes, and ſet from P to b and d; then take the whole line of inclination, and ſet from b to a, in the line PV, and draw ba, da. Then having made a table of the hour arches, take them ſeverally, from the line of inclination, and ſet them from a in the lines ab and ad. That is, ſet 6 13, 21 13, 36 13, &c. to 10, 11, 12, 1, 2, 3; and ſet 8 47, 23 47, 38 47, &c. to 9, 8, 7, 6, 5, 4. And thro' theſe points, draw the hour lines from P. And PCT will be the ſtile, making the angle CPT = 15 30.

In all ſuch dials, $a10 = b4$, $a11 = b5$, $a12 = b6$, &c. and $a9 = d3$, $a8 = d2$, $a7 = d1$, $a6 = d12$, &c.

If it happen that the arch A is equal to the latitude, then the dial plane will paſs thro' the pole; and the dial will become a horizontal equinoctial dial, as in the following Prob.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Pl.XII. pa. 116.

PROB. XX.

To draw a dial upon a declining reclining plane, paſſing thro' the pole.

Examp.

Suppoſe a dial plane declines 50° *eaſt, and reclines* 24° 38′, *in the lat.* 54½.

1. *Geometrically.*

38. The proceſs here is exactly the ſame as in the laſt Prob. till you draw PO or *p*O, which will be parallel to IH the 12 o'clock line. Then proceed thus, draw the contingent QMK perpendicular to the meridian, and to the ſubſtile *p*C, which muſt be parallel to the meridian. Set CF the height of the ſtile from C to V in the ſubſtile; and draw VM. Then from the center V deſcribe a ſemicircle, which divide into hours as uſual, beginning at VM; then lines drawn from the center thro' all theſe points of diviſion will cut the contingent line QK in the hour points. Thro' which points, lines drawn parallel to the meridian, will be the hour lines; and CV the ſtile's height, which may be a parallelogram.

2. *By Calculation.*

Here A = latitude, and the pole's height above the plane is *o*; and the other requiſites are found as before, except the plane's difference of longitude, which is found thus.

Radius	10.
Sin. latitude (54 30)	9.91068
Tan. declination (50)	10.07618
Tan. dif. longitude, 44 8,	9.98686
and height of the meridian	63 35
The arch A	54 30

Fig. The arch B = 0, and ſubſtile's diſtance is 0 or
38. no angle, becauſe the ſtile, ſubſtile and meridian are all parallel. Therefore aſſume the height CF = 2.7 inches, and putting the ſeveral hour arches into a table, find the hour diſtances, by the following proportion, which put into the table.

As radius :
Stile's height in inches : :
Tan. hour arch :
Hour diſtance.

Hours.	hour arches.		hour diſtances.
3	89	8	108.0
2	74	8	9.50
1	59	8	4.52
12	44	8	2.62
11	29	8	1.50
10	14	8	0.68
	Subſtile		
9	0	52	0.08
8	15	52	0.77
7	30	52	1.61
6	45	52	2.78
5	60	52	4.84
4	75	52	10.83

Conſtruction.

Draw the horizontal line AY; make the angle YAC = 63 35, and draw AC for the ſubſtile, in which take C for the foot of the ſtile, and thro' C draw the contingent QQ perpendicular to AC. Then ſet off the hour diſtances from C, in the line QQ, as they are in the table; and thro' theſe points draw lines parallel to AC for the hour lines. Make CL and BN = 2.7, and LNBC is the ſtile.

Fig. 38.

3. *By the Scale.*

Having found the requiſites, and made a table of the hour arches. Draw the horizontal line AY, and draw the ſubſtile AC, making an angle of 63 35 with AY; draw QCK perpendicular to AC. Then from either polar line, take the ſeveral hour arches, as in the table, and ſet them from C, in the lines CK and CQ. Then thro' theſe points draw lines parallel to the ſubſtile AC, for the hour lines. The ſtile muſt be a parallelogram CBNL, whoſe height CL is the diſtance from the beginning to the hour III upon that polar line.

For inclining dials.

1. In the *Geometrical work* all the difference is in the triangle GFH; for in an inclining dial, you muſt take the point G below C, and H above it, ſee fig. 37, and then the horizontal line GI will be below, and the meridian IH will run upwards from I. And the angle IOP or IO*p* muſt always tend to that part of the meridian IZ, above I; and made equal to the latitude; and the interſection of O*p* or OP, will always find the center; then all the reſt is plain.

2. For the *Calculation*, the ſame rules ſerve as for recliners; only the arch A is always more than 90°.

3. The working by the *Scale* is alſo the ſame, when the requiſites are found.

We may here take notice, that a ſouth weſt recliner turned upſide down, becomes a north weſt incliner; and a ſouth eaſt recliner turned upſide down, becomes a north eaſt incliner; the quantity of the declination and inclination remaining the ſame; but the hours muſt be numbered contrary.

Alſo if a dial be turned upſide down, the backſide (the hours appearing thro' the plane,) will be

Fig. 38. its oppoſite incliner. But the midnight hours muſt be left out, and ſome others put in, by producing the reſt thro' the center.

PROB. XXI.

To draw a north declining, reclining or inclining dial.

Examp. 1.

Suppoſe a N. *dial plane to decline eaſtward* 25°, *and to incline* 15°, *in lat.* 54½.

1. *Geometrically.*

39. Let C be the foot of the ſtile, DCE perpendicular to the horizon, CF the height of the ſtile perpendicular to DE.

Make the angle CFG = the proclination; downwards, becauſe it inclines, and CFH the complement of it, to cut DE in H the zenith.

Thro' G draw the horizontal line AG perp. to DE.

In GE take GR = GF, and make the angle GRI equal to the declination of the plane, to the ſame hand it declines, that is, eaſt.

Thro' I and H draw the meridian or 12 o'clock line HI.

Thro' C draw CMK perpendicular to HI.

In the line CH ſet CM from C to L, and extend from L to F, and ſet it from M to O, in the line MK; and O is the dividing center of HP.

Draw OI, and make the angle IOP downwards, equal to the latitude, and draw OP interſecting HI in P the pole for the center.

Thro' P and C draw PCV for the ſubſtile.

Thro' C draw the contingent QQ perpendicular to the ſubſtile PC, interſecting the meridian PH in N.

Take CT = CF, and draw PT, and CPT is the ſtile.

Take the neareſt diſtance from C to PT, and ſet Fig.
it from C to V, in the ſubſtile PCV, and draw VN. 39.
From the center V deſcribe a ſemicircle, which divide into hours, beginning at VN. Then draw lines from V thro' theſe points to cut QQ; and then from P thro' the points in QQ, draw the hour lines.

2. *By Calculation.*

Here the data being the ſame as in ex. 1. Prob. XIX, all the requiſites will be the ſame, being found by the ſame rules for an incliner, as for the oppoſite recliner; and are as follows.

Height of the meridian	83°	9′
Arch A	73	32
Arch B	19	2
Subſtile's diſtance	8	1
Stile's height	17	19
Plane's dif. longitude	25	19

Here A being greater than the latitude, your pole is elevated.

Then ſee the table belonging to that example.

Conſtruction.

At the bottom of the plane, draw the horizontal line PY, and make the angle YPH = 83 9, to the right hand, and PH is the meridian, or 12 o'clock. Make the angle HPV = 8 1, to the right hand, becauſe the declination is to the left, and PV is the ſubſtile. Then from P draw the hour lines, making the angles with the ſubſtile PV, as you find them in the table. Thus VP1 = 10 19, VP12 = 25 19, and VP2 = 4 41, VP3 = 19 41, &c. Make the angle CPT = 17 19, and CPT is the ſtile.

3. *By the Scale.*

Find the requiſites by Gunter. At the bottom of the plane, make the angle YPH = 83 9, and angle

Fig. 39. angle HPV = 8 1, and PV is the ſubſtile. Draw *bPd* perpendicular to PV. Set the pole's height (17 19), taken from the line of latitudes, from P to *b* and *d*, and ſet the whole line of inclination from *b* to *a*, and draw *ba*, *da*. Then take the ſeveral hour angles (as in the table), and ſet from *a* towards *b* and *d*. Thus take 4 41 and ſet from *a* to 2, and from *b* to 8. Set 19 41 from *a* to 3, and from *b* to 9. Set 34 41 from *a* to 4, and from *b* to 10, &c. Then from P draw lines thro' all theſe points for the hour lines. Make the angle VPT = 17 19, for the ſtile.

Example 2.

Suppoſe a north plane to decline weſtward 60°, *and recline* 52°, *in lat.* 54½.

1. *Geometrically.*

40. About the middle of the plane take C for the foot of the ſtile; thro' which draw DCE perpendicular, and CF parallel to the horizon, and make CF = height of the ſtile.

Make the angle CFG = reclination, upwards becauſe the plane reclines, and CFH the complement of it, cutting DE in G and H the nadir.

Thro' G draw AG perpendicular to DE, for a horizontal line.

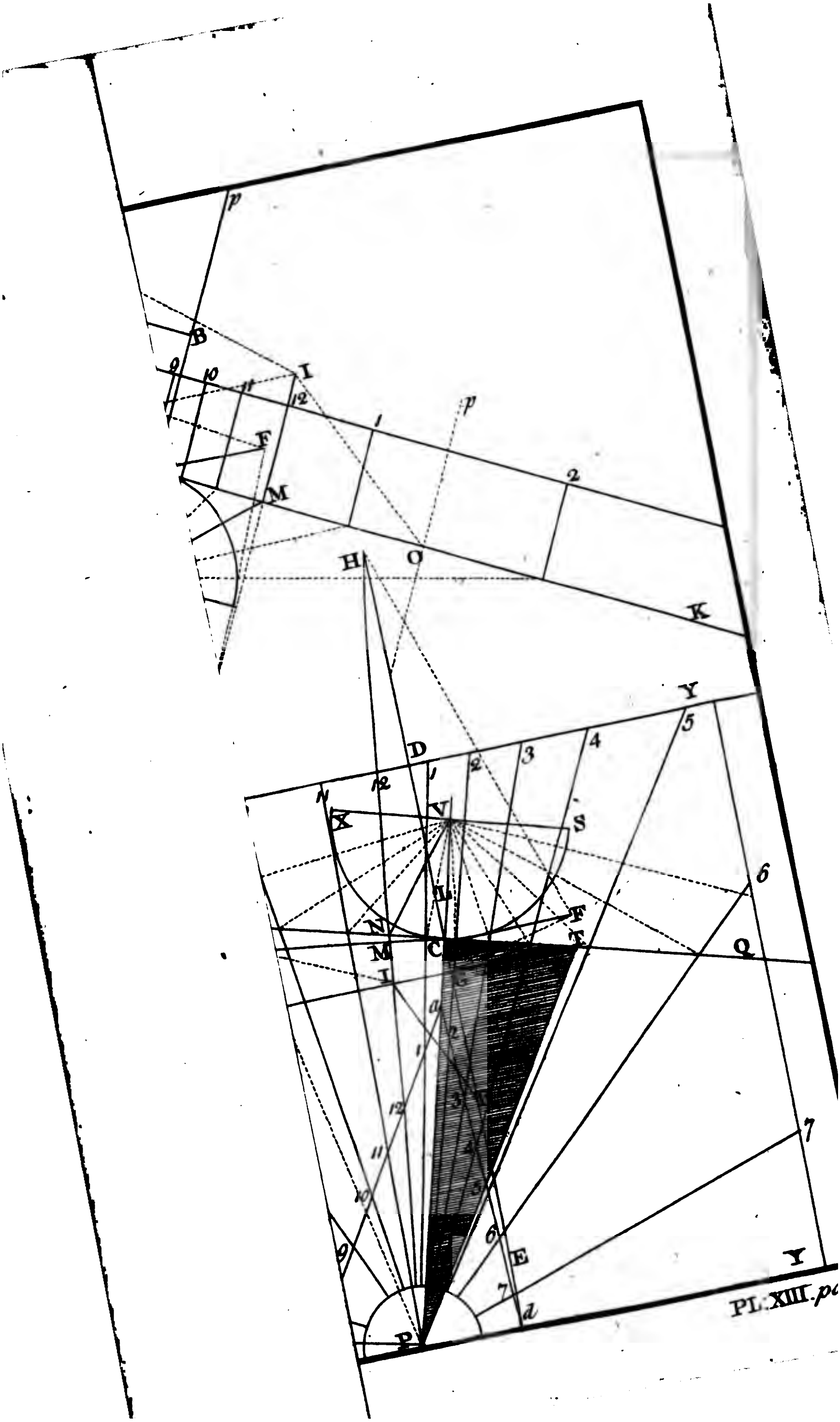
In GD take GR = GF, and make the angle GRI = the declination (60°); to the ſame hand it declines, which is weſt.

Thro' I and H draw the 12 o'clock line IH.

Thro' C draw CMK perpendicular to IH, cutting it in M.

In the line CD ſet CM from C to L, and extend from L to F, and ſet it from M to O, in the line MK, for the dividing center of PH.

Draw OI, and make the angle IOP downwards (being a north plane) equal to the latitude (54½); and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and draw OP, interſecting IH in the pole P, for the center of the dial. Fig. 40.

Thro' P and C, draw PCV for the ſubſtile.

Thro' C, perpendicular to the ſubſtile PC, draw QQ, for the contingent, interſecting the 12 o'clock line PH in N.

Take CT = CF, and draw PT for the edge of the ſtile.

Take the neareſt diſtance from C to PT, and ſet it from C to V in the ſubſtile PCV, and draw VN.

From the center V deſcribe a ſemicircle, which divide into hours as uſual, beginning at VN. Then drawing lines, from V thro' the points in the circle to cut QQ; and then from P thro' the points in QQ; and theſe laſt will be the hour lines. And the hours muſt be numbered from PN, 1, 2, 3, 4, &c. to the left hand, and the midnight hours left out.

2. *By Calculation.*

As *Sin. reclination* (52°)	9.89653
Radius ——	10.
Cotan. declination (60)	9.76143
Tan. height meridian, 36 14,	9.86490

As *Radius* —	10.
Coſ. declination (60)	9.69897
Cotan. reclination (52)	9.89280

Cotan. arch A,	68 40
Colat.	35 30
Arch B	104 10

Here the north pole is elevated. Then,

Cotan.

Fig. 40.

Cotan. arch B (104 10)	9.40212
Coſ. reclination (52)	9.78934
S. declination (60)	9.93753
	19.72687
Tan. ſubſt. diſtance, 64 40 *or rather* 115 20.	10.32475

As *Coſ. ſubſtile's diſt.* (64 40)	9.63132
Coſ. arch B (104 10)	9.38871
Radius —	10.
Coſ. ſtile's height, 55 7,	9.75739

As *Sin. arch* B (104 10)	9.98658
Radius —	10.
Sin. ſubſtile's diſt. (64 40)	9.95608
Sin. dif. longitude, 68 47, *or rather* 111 13.	9.96950

Then make a table of hour arches placing 68 47 againſt 12, and adding or ſubtracting 15 degrees, find the reſt; and from them the hour angles, by the following analogy, which put into the table as uſual. But the midnight hours muſt be left out.

Radius :
S. ſtile's height : :
Tan. any hour arch :
Tan. hour angle from the ſubſtile.

Fig. 40.

Hours.	hour arches.	hour angles.
11	83 47	82 26
12	68 47	64 41
1	53 47	48 15
2	38 47	33 24
3	23 47	19 53
4	8 47	7 13
	Subſtile	
5	6 13	5 6
6	21 13	17 40
7	36 13	31 0
8	51 13	45 36
9	66 13	61 45
10	81 13	79 20

Conſtruction.

From a convenient point P for the center, draw the horizontal line PY. Make the angle YPI = 36 14, the height of the meridian to the right, and draw the 12 o'clock line IPN quite thro' the center. Alſo make the angle NPV = 64 40, the ſubſtile's diſtance, on the left hand, as the declination is weſt, and draw the ſubſtile PV. Then from P draw the hour lines, to make angles with the ſubſtile as in the table; and the hours that are wanting draw thro' the center, leaving out the midnight hours. Make the angle VPT = 55 7 the ſtile's height, and CPT is the ſtile.

3. *By the Scale.*

Find the requiſites by Gunter, or otherwiſe; and draw PY parallel to the horizon. And make the angle YPI = 36 14 the height of the meridian, and draw IPN; and make NPV = 64 40, the ſubſtile's diſtance; and draw the ſubſtile PV; and draw *bPd* perpendicular to PV; and from the line

Fig. 40. line of latitudes take 55 7 the pole's height, and ſet from P to *b* and *d*. Then ſet the whole line of inclination from *b* to *a*, and draw *ba*, *da*. Then having a table of hour arches, take them from the line of inclination, and ſet them from *a* towards *b* and *d*, in the lines *ab*, *ad*; and draw lines from P through theſe points 5, 6, 7, 8, 4, 3, 2, &c. for the hour lines. Make the angle CPT = 55 7, and CPT is the ſtile. The other hour lines that are wanting, muſt be drawn thro' the center.

If A be = the latitude, then B is 90°, and the ſubſtile's diſtance, and plane's dif. longitude are alſo 90°, and the ſtile's height is found by this proportion; *rad*: *S. declination* : : *coſ. reclination* : *coſ. ſtile's height.*

If A be leſs than the latitude; then B, the ſubſtile's diſtance, and plane's dif. longitude, will be leſs than 90.

If A be greater than the latitude, then B and the ſubſtile's diſtance, are greater than 90°; as in this example, where IPV is greater than a right angle.

For inclining dials.

In theſe, the point G will be below C, and H above it; and the angle IOP muſt be taken downwards from I, which repreſents a point of the horizon. The calculation, and projection by the ſcale, are the ſame as in the oppoſite recliners; but the contrary pole will be elevated. Hence we may know, that if we want a north eaſt incliner, we muſt make a ſouth eaſt recliner, and turn it upſide down. Or if we want a north weſt incliner, we muſt make a ſouth weſt recliner, and turn it upſide down: but the hours muſt be numbered the contrary way from the meridian. Alſo any dial being made for a reclining plane; the ſtile and hours produced thro' the plane, will make a dial, Fig. for the oppoſite or inclining ſide of the plane.

I ſhall here add two or three more examples, with the requiſites, hour diſtances, and hour angles; leaving the conſtruction thereof, for the exerciſe of the young ſtudent.

Ex. 1.

Suppoſe a S. E. plane declines 25°, *and reclines* 19°, *in the lat.* 54½.

Height of the meridian	81° 22′
Arch A —	69 12
Subſtile's diſtance	5 59
Stile's height	13 27
Plane's dif. longitude	24 15

Hours	hour arches.		hour angles.	
5	80	45	55	00
6	65	45	27	18
7	50	45	15	53
8	35	45	9	30
9	20	45	5	2
10	5	45	1	20
	Subſtile			
11	9	15	2	10
12	24	15	5	59

Ex. 2.

A N. W. plane declines 55°, *and reclines* 20½; *lat.* 54°½.

Height of the meridian	63° 25′
Arch A —	33 6
Subſtile's diſtance	62 57
Stile's height	36 39
Plane's dif. longitude.	73 3

Hours

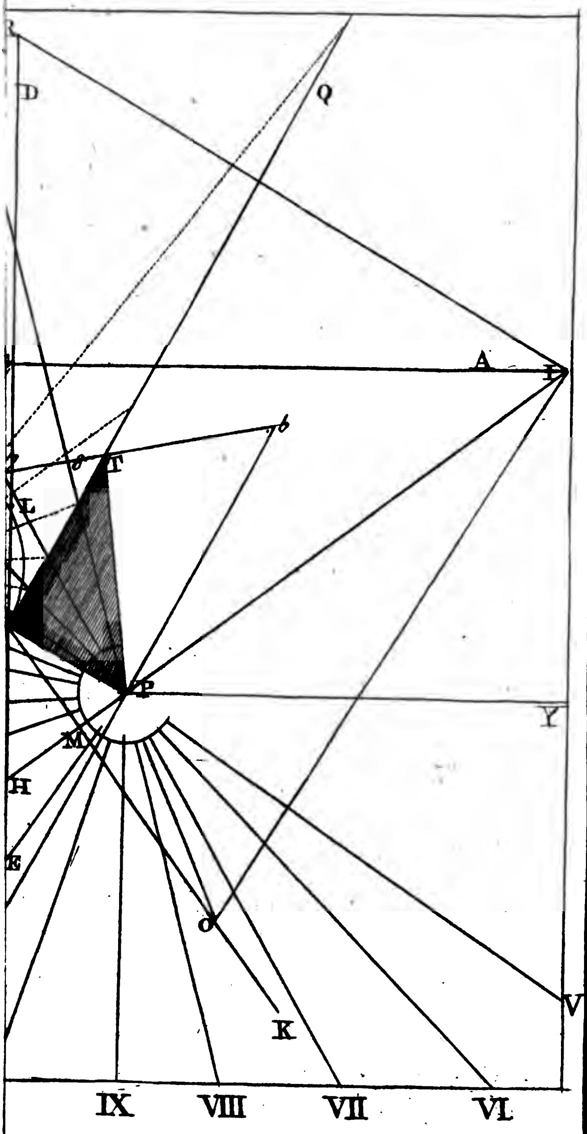
Hours.	hour arches.	hour angles.
2	76 57	68 47
3	61 57	48 15
4	46 57	32 55
5	31 57	20 25
6	16 57	10 19
7	1 57	1 10
	Subſtile	
8	13 3	7 53
9	28 3	17 39

Ex. 3.

A N. E. plane declines 50°, *and reclines* 70°, *in lat.* 54½.

Height of the meridian	41°	46′
Arch A — —	13	10
Subſtile's diſtance —	147	29
Stile's height —	63	13
Plane's dif. longitude	35	32

Fig. 40.



Pl XIV *pa.128.*

Hours.	hour arches.	hour angles.
4	84 28	83 48
5	69 28	67 14
6	54 28	51 20
7	39 28	36 19
8	24 28	22 6
9	9 28	8 28
	Subſtile	
10	5 32	4 19
11	20 32	18 29
12	35 32	32 31
1	50 32	47 19
2	65 32	62 59
3	80 32	79 25

SCHOLIUM.

In all theſe ſorts of dials, the horizontal line is of great uſe, and ought to be kept on, when the other lines are rubbed out; for by the help of that, the dial is ſet up in its true poſition. Alſo in all kinds of dials, that ſhow the hour, &c. by a perpendicular pin, the horizontal line cuts off all ſuperfluous lines in every dial, which can never be touched by the ſhadow of the ſtile. For what is above that line is of no uſe, becauſe the ſun is below the horizon, and cannot then ſhine on the dial, or caſt any ſhade above the horizontal line; except the dial happen to be ſhifted to a new place.

Fig.

PROB. XXII.

To draw any hour line, when the line from the center of the dividing circle, does not cut the contingent line, within the plane.

Let C5 be the hour line to be found, where the
12. dividing line N*n* drawn from the center N of the circle, does not reach the contingent line AB. Try how near the dividing line N*n*, approaches the contingent AB, about the ſide of the plane, as within $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. of NQ. Take ſuch a part of NQ, at which diſtance from Q, draw an obſcure parallel to AB, to cut N*n* in *c*; draw *cbr* parallel to the ſubſtile NQ. Then take the ſame part of QC (as $\frac{1}{2}$, $\frac{1}{3}$, &c.), and ſet it from *b* to *a*, in the parallel *cr*, and thro' *a* draw the hour line C5.

Or thus. When the dividing line N*n* approaches the contingent AB very ſlowly; draw C*r*, N*f* parallel to AB. And ſee what part of NQ (as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c.) the line N*n* is diſtant from N*f*, near the edge of the plane. Set that part from N towards Q, from which point draw an obſcure parallel to AB, to cut N*n* in *c*; then thro' *c* draw *fr* parallel to NQ; and take the ſame part of CQ, and ſet from *r* to *a*; and thro' *a*, draw the hour line C5.

When the end *n* of the line N*n* comes nearer AB than N*f*, the former way is to be uſed; but when it comes nearer N*f*, the latter method.

The reaſon of this operation is, that CQ : QN : : *ba* : *bc* : : *ra* : *fc*, by ſimilar triangles.

PROB. XXIII.

To draw a dial upon the cieling of a room, which will ſhew the hours by reflexion.

Place a ſmall piece of a looking glaſs exactly horizontal in a window, where the ſun ſhines; meaſure

ſure the perpendicular diſtance from the glaſs to the cieling, for the height of the ſtile; and where the perpendicular cuts the cieling, is the foot of the ſtile. Then having the height of the ſtile, and its foot; make a horizontal dial thereto upon the cieling (by Prob. VII.); and that will be your dial. The glaſs is beſt placed pretty high, as upon the tranſom. Fig. 12.

This dial is no more than a horizontal dial, turned upſide down. For let CB be the cieling; G the glaſs, GC perpendicular to the cieling, and equal to CF. Then if AG be a ray of the ſun, reflected from G to the cieling, and FB a direct ray. Then the direct rays AG, FB will be parallel, and therefore the angle CFB = angle CGA, and CGA = CGB, becauſe the angle of incidence is equal to the angle of reflexion. Therefore CFB = CGB, and GC being equal to CF, the direct ray FB and the reflected ray GB will fall upon the ſame point B, of the cieling. Therefore if a dial be made on the plane CB, for the ſtile CF, to ſhew the hour by the direct rays FB, it will ſerve equally for the reflected rays GB. 41.

For the drawing of this dial, a meridian line muſt be had; therefore at 12 o'clock, when the ſun is in the meridian, hang up a line and plummet cloſe by the glaſs, mark its ſhadow in the floor with chalk; transfer this to the cieling, by help of a plumb line, and it will be the 12 o'clock line.

If your window does not face the ſouth, you may draw an eaſt and weſt line, thus. Calculate the ſun's altitude when eaſt or weſt; then obſerve with a quadrant when he has that altitude; and at that moment hold up a line and plummet, cloſe to the glaſs, thick enough to caſt a ſhade to the oppoſite wall, this carried up to the cieling gives the point of eaſt or weſt.

Fig. 41. It will be beſt to draw your dial firſt upon a large paper, and when that is done, draw a line perpendicular to the meridian to cut all the hour lines; and then meaſure the ſeveral diſtances of the hour lines from 12, upon this line, in inches; and alſo the diſtance of this line from the foot of the ſtile. Then draw a line on your cieling as many feet from the foot of the ſtile, on which ſet off the ſeveral hour points in feet, which were meaſured before in inches; thro' which draw the hour lines to the center of the dial. But becauſe the foot of the ſtile is commonly within the wall, and the center without doors; it will be neceſſary to draw two perpendiculars to the meridian, and get the hour points in both, thro' which the hour lines muſt be drawn.

If you would continue the hour lines along the ſides of the room; draw them firſt quite thro' the cieling till they cut the walls. Then a plane paſſing through any hour line and the glaſs, will cut any wall in the ſame hour line. Therefore extend a thread from the glaſs to the extremity of any hour line upon the cieling, which keep fixt there. Then extend another thread acroſs it, from the other end of the ſame hour line, juſt to touch the former thread, and to reach to the oppoſite wall; it will touch that wall in a point, thro' which the ſame hour line is to paſs.

Note, inſtead of a glaſs, you may uſe a little water, which of itſelf, will always have its ſurface horizontal; for if the glaſs be not horizontal, the error by the reflexion will be doubled. And the water being always in motion, by the agitation of the air; makes the point of reflexion on the cieling more eaſily diſtinguiſhed.

Cor. *After the ſame manner, a dial may be made on the floor of a room; to ſhew the hour, by the ſhadow of a black ſpot in the window.*

For let CB be the floor, and F a place in the window upon which you muſt ſtick a patch ſo large as to give a ſhadow in the floor. Then if FC be taken for the height of the ſtile, and C for its foot. And a horizontal dial be made on the floor, to the ſtile CF, whoſe height you muſt meaſure; it is evident, the ſhadow of the point F upon the floor will ſhew the hour of the day. Fig. 41.

PROB. XXIV.

To find what latitude a dial is made for.

Meaſure the angle that the ſtile makes with the plane of the dial, and that will be the latitude, if it was a horizontal dial; or the complement of the latitude, if it was an erect ſouth dial.

But if the dial has no ſtile, as it may happen to be broken off by ſome accident. Then meaſure the angle between the hour lines of 12 and 3, or 12 and 9. Then find the tangent of this angle; ſeek that tangent among the ſines, and the arch belonging to it, is the latitude, if it is a horizontal dial; or the complement of the latitude, if it is a full ſouth dial, or a full north one.

Or in general, meaſure the angle between 12 and any hour line; then take the hour arch belonging; and ſay,

As *Tang. hour arch* :
Tan. hour angle : :
So *Radius* :
S. latitude for a horizontal dial, or *coſ. latitude* for a direct ſouth or north dial.

PROB. XXV.

How to place a dial in a true ſituation.

Before any dial can be truly drawn upon a plane, it is neceſſary to have the ſituation of that plane, Fig. ſuch as the inclination and declination, and the latitude of the place, which are the neceſſary *data* for conſtructing it. And when the dial is drawn, it is equally neceſſary to place it in the ſame ſituation, or elſe it can never ſhew the time truly. If the dial was drawn upon a fixt plane, there is no more to be done with it; but if it is upon a looſe moveable plane, it will require the ſame operations to ſet it up, as were uſed in determining its ſituation.

In the placing any dial, the three following directions muſt be obſerved. 1. That the horizontal line, which is drawn for this purpoſe, be placed parallel to the horizon. 2. That it be placed ſo as to have its proper declination. 3. That it may have its proper degree of reclination or proclination. Theſe rules ſerve generally for all dials, and muſt be exactly performed, if you would expect the dial to go truly.

In a horizontal dial, the meridian muſt be ſet north and ſouth, which may be done by Prob. I. or by help of a line and plummet held up at 12 o'clock. Then the meridian or 12 o'clock line muſt be placed perfectly level; and ſo muſt the 6 o'clock hour lines. And this may be done by help of a level, or with a quadrant.

In a full ſouth dial, the plane of it muſt face the ſouth directly, which may be done by a compaſs, or ſetting it to twelve o'clock, exactly at noon. And the 12 o'clock line muſt be perpendicular to the horizon, which may be ſet by a plumb line. Alſo the plane of the dial muſt be placed perpendicular to the horizon, to be done alſo by a plumb line.

In upright declining dials, the 12 o'clock line muſt be ſet perpendicular by a line and plummet; as alſo the plane of the dial. And it muſt have its proper declination, and this may be done by help of a compaſs, or more exactly by the ſun's altitude Fig. and azimuth, as deſcribed in Prob. III.

And laſtly in declining reclining dials, the horizontal line muſt be ſet parallel to the horizon. And the declination, ſet out by the compaſs, or by the ſun's altitude and azimuth. And the ſame reclination muſt be given to the dial as it was calculated for; the method whereof is deſcribed in Prob. II.

The greateſt difficulty is to ſet a dial to the true declination. Therefore if you have a watch truly ſet, and the dial have its proper poſition in other reſpects. Then if the ſun ſhines, you'll ſee whether the watch and the dial agree or not; if not, alter the declination till they do agree. And if the horizontal line be now parallel to the horizon, and the dial have its proper inclination, and the time agrees with the watch, you may conclude your dial truly ſet; and there it muſt be fixt.

PROB. XXVI.

Any dial being made for one place, how to ſet it up in any other place, to ſhew true time.

If you would ſet it by the rules of this art, you muſt know the latitude it was made for, and the declination and inclination, if it has any. Theſe things being had, its ſituation, in its original place is known. Therefore to ſet it in any other latitude, you muſt place it in a ſituation quite parallel to its firſt ſituation, and the thing is done.

For a horizontal dial, note the difference of latitude between its original place and your place. Then raiſe the north ſide of the dial, if it be carried northward, or the ſouth ſide, if it is carried ſouthward; ſo many degrees, as is the difference of latitude; and there fix it.

Fig. For an upright ſouth or north dial, ſet it to lean towards the ſouth if it is carried northward; or towards the north, if carried ſouthward, as many degrees as is equal to the difference of latitude. And if it reclined or proclined before, north or ſouth; it muſt be made to recline or procline, ſo much more, or ſo much leſs, as the caſe requires.

A full eaſt or weſt upright dial muſt be ſet to face the eaſt or weſt; but the horizontal line muſt be elevated at the north end, if it is carried north; or the ſouth end if carried ſouth, as many degrees as the difference of latitude comes to.

42. For all ſorts of declining dials, new requiſites muſt be found to ſet them up by. Let AB be the meridian, B the place it was made for, A the place it is to ſet up in. Make the angle GBF = declination of the plane, BD the reclination of the plane, and BF its complement. Let F be the pole of the great circle DE, and thro' A draw the parallel circle CA, alſo draw FAE. Then in the ſpherical triangle FBA, there are given FB the comp. reclination, the angle FBA the ſupplement of the declination, and BA the difference of latitude. To find FA the comp. reclination at A; the angle FAB the declination at A; and angle AFB the elevation of the horizontal line of the dial, above the horizon at the place A. And according to theſe new requiſites, when found, the dial muſt be ſet up; and not by the declination and reclination at B, where it was made. And this holds for the oppoſite inclining plane.

For ſince BD is the reclination, if the dial is removed to D, it will be an upright dial, and ſo it will be at E; and when brought to A, and ſet parallel, AE will be its reclination, the ſame as it would have at C, or in any place of the parallel circle AC, for it will always be in the plane of it. Therefore FAB will be the declination at A. And ſince the horizontal line of the dial is parallel to the horizon, when the dial is at C, or any where in the circle FD; therefore when it is removed from C to A (in a quite parallel poſition) being ſtill in the plane of the circle CA, that end of the horizontal line next A, muſt needs be elevated above the horizon at A, to an angle equal to the arch CA or DE, or the angle BFA; or the end next C, depreſſed ſo much. Fig. 42.

It may be noted, that in an eaſt and weſt recliner, FBA will be a right angle. And in an upright decliner, FB will be a quadrant, or BA will coincide with DE, and FA will be greater or leſs than a quadrant, according as the angle FBA is obtuſe or acute.

Otherwiſe.

This Prob. may alſo be done mechanically, after this manner. Stretch a thread, or rather a wire, between two fixt points, ſo that it may be parallel to the earth's axis, or point directly to the pole. Then take your dial, and ſet the edge of the ſtile cloſe to the wire, or at leaſt parallel to it, and continuing it ſo, turn the dial gently about this axis, till the ſun, ſhining on it, ſhews the true hour of the day; which may be known by a clock, or by another dial. There fix your dial; or at leaſt keep it in that poſition, till you take the declination, and reclination of it, as has been ſhewn before; and then the dial may be ſet, from theſe requiſites obtained.

Cor. *Hence you may place a dial, ſo as to ſhew the hour of the day in any given place of the world.*

For if the ſtile be held cloſe to the wire, as before directed, and the dial be turned gently about the wire and ſtile, as an axis; till the ſun ſhews the time ſo much more than the true, as the longitude of the ſaid place is more eaſt than your place; or elſe that it ſhew the time ſo much leſs than the true, as the place differs in longitude weſtward. There fix the dial, and it will always ſhew the true time at that place. Here 15 degrees of longitude is equal to an hour, 30 to two hours, 45 to three hours, &c. Therefore the dial being ſet ſo much faſter for a place of eaſt longitude; or ſo much ſlower for a place in weſt longitude, will conſtantly ſhew the hour of the day at the other place.

PROB. XXVII.

To find the hour of the night by the moon's ſhining upon a ſun dial.

Get the moon's age, or the number of days from either change or full, to the preſent time; and take $\frac{8}{10}$ thereof for the number of hours that the moon is behind the ſun. Add theſe hours to the time ſhewn by the moon, on the dial; throwing out 12 if it exceed, and you have the hour of the night.

Or thus. If you know the time of the moon's ſouthing, count how many hours and minutes the ſhadow on the dial wants of 12 o'clock; ſubtract them from the time of her ſouthing, for the hour of the night. But if the ſhadow be after 12, add theſe hours and minutes (on the dial) to the time of her ſouthing, rejecting 12, if it exceed; and you have the hour of the night.

SCHOLIUM.

Before I leave this ſection, I ſhall put together a few general obſervations, which a dialliſt ought always to remember.

1. In drawing any dial, work with obſcure lines, or ſuch as may be rubbed out again; for we have no farther occaſion for them after the hour lines, and ſubſtile, are obtained.

2. It is beſt to fix the ſtile upon the ſubſtile,

before the dial is drawn; for for it will be difficult to fix it truly after. And it muſt always ſtand perpendicular to the plane of the dial. Fig.

3. In all dials, eſpecially ſuch as are troubleſome to draw; to have it exactly done, firſt make a draught of it upon paper; and then transfer all the lines from thence to the dial plane.

4. In every dial, the thickneſs of the ſtile is to be conſidered; and a ſpace left at the ſubſtile, equal to its thickneſs; juſt as if the dial was to be cut thro' at the ſubſtile, and the two parts ſeparated, to a diſtance equal to the thickneſs of the ſtile. And therefore it ſhould be made ſo at firſt.

5. If the ſun is to ſhine more than 12 hours on the dial; you may get the remaining hours, which you want, by producing the hour lines already drawn, thro' the center.

6. When the center of the circle is found, that is to divide the contingent line, which may be taken on either ſide of it; you may deſcribe a circle with any radius, and it is beſt done by a line of chords; and the hour arches ſet off thereby. For in all circles, that are alike divided, the radii will cut the contingent in the ſame points.

7. If you would have half hours or quarters upon your dial, you muſt divide each hour of your circle in halves or quarters, and draw lines thro' the points to cut the contingent line, as in whole hours. And in the calculation, inſtead of 15 degrees; add continually 7° 30′ for half hours, or 3° 45′ for quarters, and calculate accordingly.

8. In any dial, leave out the midnight hours; and all hours, at which time the ſun can never ſhine on the dial.

9. In any dial, place the numbers for the hours, ſo as to be moſt legible; therefore it would be abſurd to place them upſide down, in regard to the ſpectator.

10. Every

Fig. 10. Every dial muſt be ſo placed, that the upper edge of the ſtile may point directly to the pole; and that the horizontal line be perfectly level; and that it have its proper declination and inclination. And in an upright dial, that the 12 o'clock line be perpendicular to the horizon.

11. When any dial plane paſſes thro' the pole, it requires an equinoctial horizontal dial to be made. And the ſtile will be parallel to the plane.

12. Every dial is too faſt in the morning, and too ſlow in the evening; owing to the refraction of the ſun-beams, by which the ſun is raiſed higher, and the ſhadow brought nearer the ſubſtile.

12. 13. To draw any hour line C5 (fig. 12.) when it does not cut the contingent within the plane. From the centers C, N, draw C*r*, N*f*, parallel to the contingent AB. Try how much the projecting line N*n* advances towards the contingent AB near the edge of the plane at *b*, as $\frac{1}{2}$, $\frac{1}{3}$ or $\frac{1}{4}$, &c. of NQ. Take ſuch a part of NQ in your compaſſes, and running one foot along N*f* (or QB), mind where the other cuts N*n*, as at *c* (the feet of the compaſſes being parallel to NQ). Thro' *c* draw *fr* parallel to the ſubſtile NC. Then take *ra*, (or *ba*) the ſame part of *rb* (as *fc* (or *bc*) is of *fb*, as $\frac{1}{2}$, $\frac{1}{3}$, &c.) Then thro' *a*, draw the hour line C*a*5. See alſo fig. 28.

Or thus. To draw C7; draw *gp* parallel to C2, which is 6 hours from C8, to cut the hour lines C8, C9, in *d* and *b*. Set *db* from *d* to *f*; thro' *f* draw the hour lines C7.

SECT. III.

The Conſtruction of ſome other ſorts of Dials; and drawing the Parallels if Declination, Parallels of Altitude, and other ſuch Furniture upon Dials. A Table of Latitude and Longitude of Places.

PROB. I.

To make a Dial upon the ſurface of a ſphere.

44. THIS is done without any ſtile, only by the line bounding light and darkneſs. Therefore get a ſphere or globe, AEBQ, and mark two points of it, diametrically oppoſite, for the two poles, of which P is one. In the middle between theſe poles deſcribe the equinoctial EQ, which may be done thus; open the compaſſes to a quadrant's diſtance, and ſetting one foot in P, with the other deſcribe the equinoctial EQ, which divide into 24 equal parts, for hours. Then mark theſe diviſions thus; put 6 to the top, and 7, 8, 9, 10, 11, 12, ſucceſſively to the other diviſions on the left hand, and 5, 4, on the right; theſe are to be placed above the equinoctial for the forenoon hours. Again to the ſame point 6, put 6 under the equinoctial, and 5, 4, 3, 2, 1, 12 on the right hand, and 7, 8, on the left; theſe are for the afternoon hours. Then the globe is to be fixed ſo in the ſun, that the pole P may be elevated above the horizon, as much as is the latitude, or ſo that its axis may point

Fig. 45. M for March, A for April, M for May, I for June, I for July, A for Auguſt, S for September. In the middle of this ſpace is a channel for the ſlider, with the hole, to move in.

The next thing is for the winter months. Take the point M for the 12 o'clock line further from the bottom, becauſe the ſun goes low in winter; draw MN parallel to the horizon, make NO = twice the comp. latitude, and draw OP parallel to the horizon; and make the angles PO5, PO4, PO3, &c. equal to the ſun's altitude at 5, 4, 3, &c. in the afternoon, when the ſun is in the equinoctial. Or if *p* or 6 be the place of P, make *p*7, *p*8, *p*9, &c. twice the height at 7, 8, 9, &c. o'clock, in the morning, in the line IZ. In like manner making OQ = 47°, and drawing QR parallel to the horizon, make the angles RQ3, RQ2, RQ1, the ſun's altitude when in the tropic of capricorn, and his depreſſion at 4, 5 and 6; let their places be at 6, 5, 4, &c. in the line *ix*. Then draw the hour lines 12 12, 1 11, 2 10, 39, 48, 57, and 66.

Laſtly, the months muſt be ſet off on the part OQ, as was done upon EF. Let *oq* be that ſpace in the line *iy*. Then twice the ſun's declination, at the beginning of every month, muſt be ſet off from *o* towards *q*; and all completed as in the ſummer months. Theſe two parts *fr* and *qo* muſt be engraven on the outſide of the rim, at FE and QO.

All the lines being drawn, and numbered, the plate *azxy* muſt be bent into the form of a ring, truly circular, and the lines *ay*, *zx*, ſoldered together at A, where a loop of wire muſt be fixt, at which it is to hang by a thread. There is to be made a channel quite round the middle of the plate; in this a thin ſlip of braſs is to be fitted, to move back and forward, having two holes in it, Fig. 45. againſt the parts EF, and OQ. And in theſe places EF and OQ, the channel is cut quite thro' the plate.

The uſe of the dial is this; ſet the proper hole againſt the day of the month whether ſummer or winter, which is eaſily done by moving the ſlip of braſs back and forward. And turning the ſide AE towards the ſun in ſummer, or AD in winter; let the dial hang by the thread at A, then the ſun's ray paſſing thro' the hole, will project a ſpot of light among the hour lines, which ſhows the hour of the day.

But *note*, the hour lines not being parallel; you muſt make the ſpot of light fall at or near the ſide *xy*, when the ſun is at or near either tropic; and near the ſide AZ, when near the equinoctial; whether it be beforenoon or afternoon. And in general, keep further from *az*, as the ſun is further from the equinoctial.

PROB. III.

To make a univerſal ring dial.

46. Such a dial as this will ſhew the hour in any latitude, and is thus made. AECF and ABCD are two braſs circles, the outermoſt AC repreſents the meridian, and the inner one BD the equator. ABCD is divided into 24 equal parts or hours beginning at the edge of the meridian AE; this circle turns upon an axis AC, whoſe ends are fixt in the meridian at A and C; and may be ſet ſquare or perpendicular to AECF, putting it againſt the ſtops at A and C. The points E and F, which are at a quadrant's diſtance from A and C, repreſent the poles; here a flat piece of braſs FG is placed, which turns round upon its axis, going thro' two pieces of braſs fixt at E and F. Along the middle of

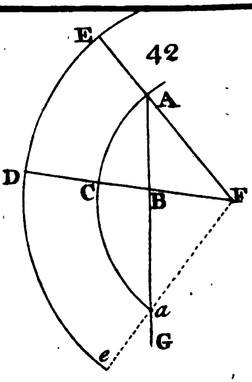
Fig. FG, there is a long ſlit, in which a piece of braſs P, with a ſmall hole in it, is moveable up or down, to be ſet to the day of the month. The quadrant AE is divided into 90 degrees, ſo that the nut N may be ſet to the latitude of the place, by ſliding it along. The circle AC is hollowed out between A and E, and a piece of braſs wire fits into it, and is faſtened beyond A and E; this wire goes thro' a hole in the bottom of the nut N. The piece FG is graduated with the months after this manner; the tangent of the ſun's declination (to the radius of the inner circle), is ſet from the middle towards F, for north declination; and towards G for ſouth. And each month divided into three parts of 10 days each.

The uſe is this; open the inſtrument till the horary circle BD reſt againſt the ſtops at C and A. Move the nut N till the black line in it falls upon the latitude of the place. Move the ſlider P till the hole be againſt the day of the month. Then ſuſpend the dial by the thread at N, and turn the piece FG round its axis till it face the ſun; and move the dial, till the ſun ſhining thro' the hole, caſts the ſpot of light upon the black line, which runs along the middle of the inſide of the hour circle BD; and the numbers on the upper ſide, ſhews the hour. At this time the circle AECF is in the plane of the meridian; and the circle BD parallel to the equinoctial.

PROB. IV.

To make a dial upon a quadrant.

47. Let CAB be the quadrant divided as uſual into 90 degrees, CP a line and plummet. Deſcribe the arch HI at a convenient diſtance from AB, for inſcribing the months, which is done thus. Find the



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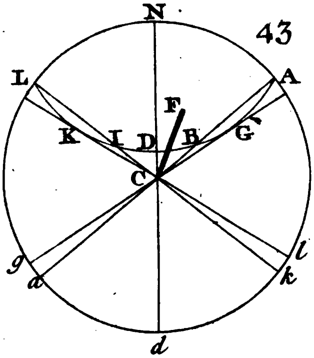

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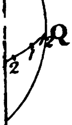

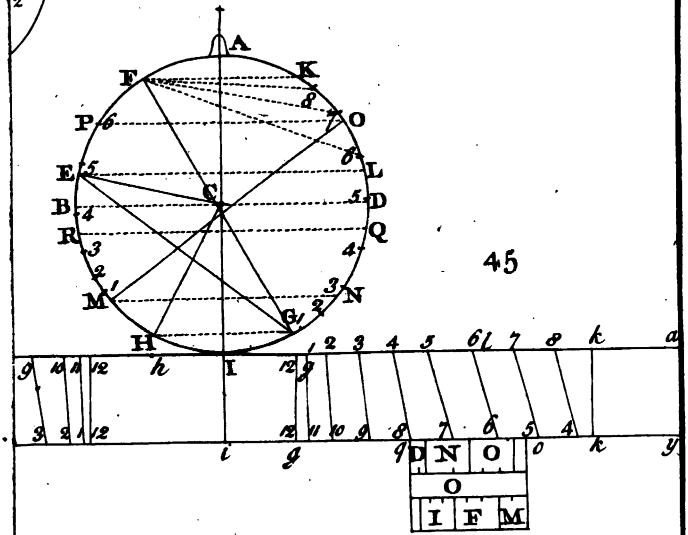

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Pl: XV. pa.146.

the ſun's meridian altitude when he is in the two tropics; lay the line CP over theſe altitudes, by which draw the lines *mn*, *op*. Then all the months are contained in the ſpace *mopn*. Fig. 47.

About the middle of the ſpace CH or CI deſcribe the arch FG. Then lay the line CP to the height of the equinoctial among the degrees, and where it cuts FG, make a mark, as 12. Draw the right lines 12 *m*, and 12 *o*, for the 12 o'clock lines in ſummer and winter, reſpectively.

For inſcribing the months; find the ſun's declination, and from thence his meridian altitude, at the beginning of every month; and laying the line CP ſucceſſively over theſe degrees, draw lines by it, within the ſpace *mopn*, as you ſee in the figure; and theſe lines divide the months from one another, which may be divided into days as you will, and the names of the months written within them, or at leaſt the initial letters.

To draw the reſt of the hour lines; and firſt for the ſummer months. To do this, we muſt firſt find all the hour points upon the line HI. Therefore when the ſun is in the tropic of cancer, by ſpherical trigonometry, find the ſun's altitude for every hour; which mark upon the arch HI at 1, 2, 3, 4, &c. by laying the line upon the ſeveral degrees of the quadrant. Again, find the ſun's altitude for every hour to 6, when he is in the equinoctial. Make marks for theſe in the arch FG, at 11, 10, 9, 8, &c. Then draw the hour lines 1 11, 2 10, 3 9, 4 8, &c. which will be right lines very nearly. Then for the winter months, you muſt in like manner find the ſun's altitude for every hour, when he is in the tropic of capricorn, and make marks along the arch *o*I. From which draw lines to the marks in FG, which will be 1 11, 2 10, 3 9, &c. and theſe will be the hour lines, for the winter.

Fig. 47. But as ſome of the ſummer hour lines do not cut the arch GF; and ſome of the winter hour lines, the arch IH; we muſt find the points where they cut the line GI, at ſun-riſe or ſun-ſet. Therefore for any ſuch hour you have the aſcenſional difference, 15° for 1 hour from 6, 30° for 2 hours, &c. Then from the aſcenſional difference, find the declination; and by that the meridian altitude. Then laying the thread over theſe degrees of altitude, obſerve where it cuts the line 12 12, take the diſtance from that point to the arch HI, and ſet it from I towards G, and you'll have the point in IG, through which that hour line is to be drawn.

If you would have the hour lines drawn very exact, as they deviate a little from right lines; proceed thus. About the middle of the ſpace GI, deſcribe the arch ED. Lay the thread over the point D or *d*, according as it is ſummer or winter, where that arch interſects one of the 12 o'clock lines; and note the degrees cut, which take for the ſun's meridian altitude. From thence find his declination. Then by oblique ſpherical triangles (Caſe 8), find his altitude for any hour, except 12 (which by conſtruction is a right line); lay the thread over theſe degrees of altitude, and where it cuts the arch ED, is a third point to draw the hour line thro'.

If you have a table of the ſun's altitude for every hour, it will ſave you the labour of calculation.

To uſe the quadrant in finding the hour of the day. There is a ſmall bead which ſlips up and down the thread CP. Lay the thread over the day of the month, and ſlip the bead up or down, till it falls on the 12 o'clock line. Then holding the quadrant upright, that the line and plummet may hang at liberty, and play freely; then holding it ſo in the ſun, that the ſhadow of the perpendicular pin at C, may fall on the line CF; then the bead Fig. reſting among the hour lines, will ſhew the hour 47. of the day. And note the hour lines in ſummer run upwards towards the left hand; and in winter, upwards towards the right.

PROB. V.

To draw the parallels of declination upon any dial.

Any circles of the ſphere, whether great or ſmall, are eaſily deſcribed by the rules of the gnomonic projection of the ſphere before delivered; which, if the reader underſtands, he needs no other directions. In theſe ſorts of Problems, I ſhall therefore be as ſhort as poſſible, and deliver what I have to ſay in a general way, which may eaſily be applied to particular caſes. Here all things belonging to the dial muſt be ſuppoſed known, as the height of the ſtile, the place of the ſubſtile, &c. Then the parallels of the ſun's declination will be found, by finding the dividing center of every hour circle; by which the declination is to be ſet off from the equinoctial, upon theſe ſeveral hour circles, or the complement of the declination, from the pole, which is the center of the dial.

Let PC be the ſubſtile, CF the ſtile, and C the 48. foot of it. Draw FO perpendicular to PF, and O is a point of the equinoctial. Thro' O draw the equinoctial QQ perpendicular to PO. Make the angles OFB and OFD equal to the declination of the parallel, north and ſouth, as ſuppoſe $23\frac{1}{2}$. Then B and D will be the points in the ſubſtile, thro' which the north and ſouth tropics paſs; or the vertices of the two hyperbola's repreſenting the tropics. Then to find the points upon the other hour lines, as ſuppoſe on P11. From C the foot of the ſtile, draw CGL perpendicular to the hour

line,

Fig. line, and CH parallel to it, and equal to CF.
48. Then ſet GH from G to L, on either ſide the hour line P 1 1, and L is the dividing center of P 1 1. Draw LE to the point where the hour line interſects the equinoctial; then make the angles ELM and ELN equal to the declination of the parallel (23½); and M and N will be two points where the ſame two parallels paſs thro; and by the like proceſs, the points in every hour line are found. Then two curves drawn thro' all theſe points, as RDS and VBX, will be the parallels required, one ſouth, the other north. And thus all the parallels may be drawn, or thoſe at the beginning of every ſign, having their declinations given; and being marked with the ſigns of Aries, Taurus, &c. the ſhadow of the top of the ſtile F, will ſhew when the ſun enters any of theſe ſigns.

In a horizontal, or a direct north or ſouth dial, having found the points on one ſide the meridian; they may be ſet off on the hour lines on the other ſide. And even in any declining reclining dial, where the ſubſtile falls upon ſome hour line, the points on the hour lines, on one ſide, may be ſet off upon the hour lines equidiſtant, on the other ſide.

The proceſs is juſt the ſame when the dial is a horizontal equinoctial dial, where the hour lines are parallel; and the ſame, in eaſt and weſt dials.

In horizontals under the poles, theſe parallels will be circles, whoſe center is the foot of the ſtile, and radius of any one equal to the co-tangent of the declination; the height of the ſtile being radius.

If you make uſe of a triangular ſtile POT, you muſt cut a notch out of the edge at F, for the ſun to ſhine thro'; which will do the ſame as the ſhadow of the point F. And the point F, for the

notch, will be found by erecting CF perpendicular Fig.
to the ſubſtile PO. 48.

Cor. 1. *Hence you may mark upon the dial, the time of ſun riſing or ſun ſetting, or the length of the day, when the ſun enters into any of the ſigns.*

For if the horizontal line be drawn on your dial, the interſection thereof with any parallel of the ſigns, will ſhew among the hours, the time of ſun riſing or ſetting, which may be marked upon theſe ſigns in the meridian, or on the ſide of the dial, or elſe the length of the day or night; that is on one ſide of the equinoctial. And if a parallel be drawn to the horizontal line on the other ſide of O, and at the ſame diſtance from O, its interſection with the parallels of the ſigns will ſhew the hours on the other ſide of the equinoctial. But if the horizontal line does not cut ſome parallel circle, then the time of ſun riſe muſt be calculated by ſpherical trigonometry, having the declination of that parallel given.

Cor. 2. *Hence the dividing center* L *of any hour line* PE, *is diſtant from the center of the dial* P, *the length of the axis or ſtile* PF. *And this gives an eaſier method of finding the dividing center of an hour line. For all theſe centers will be in the circumference of a circle deſcribed from* P *with the radius* PF.

For PE is 90 degrees = angle PLE; and $PL^2 = PG^2 + GL^2 = PG^2 + GH^2 = PG^2 + GC^2 + CH^2 = PC^2 + CH^2 = PC^2 + CF^2 = PF^2$.

Fig. 48.

PROB. VI.

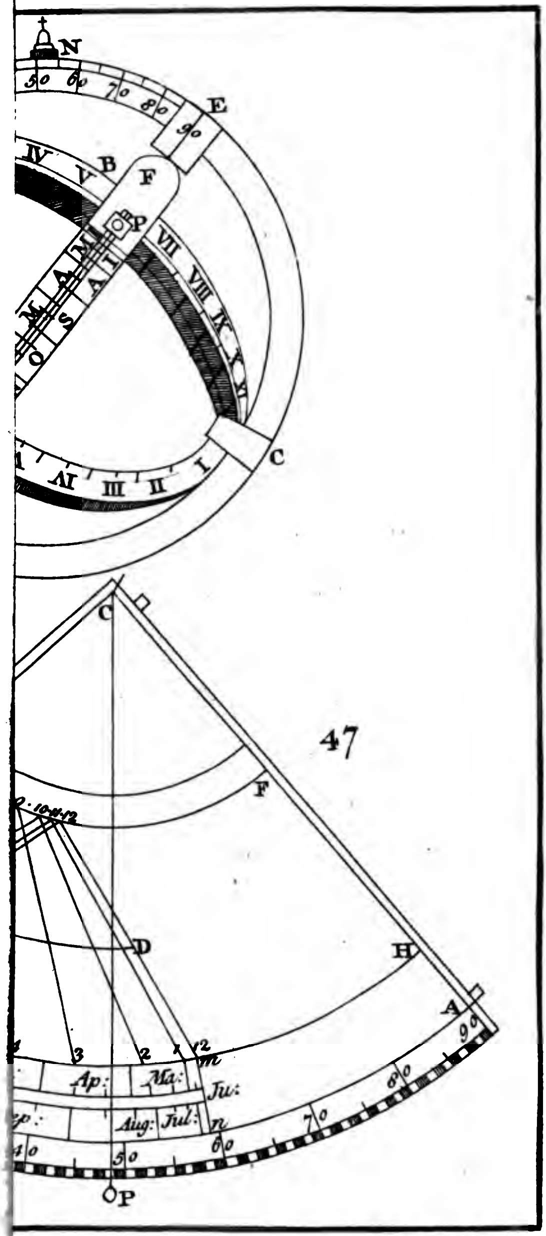
To deſcribe ſuch lines upon a dial, as will ſhew the riſing and ſetting of the ſun; or the length of the days.

To do this, you muſt find what declination the ſun has when he riſes at 4, 5, 7, 8, &c. of the clock; which is eaſily done by right angled ſpherical triangles, having the aſcenſional difference given; or even by projection. Then having the declination of each hour, you muſt deſcribe ſo many parallels for theſe ſeveral declinations, by the laſt Prob. ſuch as RDS, VBX, &c. which done, write upon each of them the hour of ſun riſe, correſponding to that declination. Or you may write the hour of ſun riſe on one ſide, and that of ſun ſet on the other ſide of the dial; and the length of the day, where each parallel cuts the meridian; or after any other manner, which you think proper. Where thoſe above the equinoctial ſerve for the winter; and thoſe below for the ſummer. Then at any time, when the ſhadow of the top of the gnomon F, traces out any of theſe parallels, you'll have by inſpection the time of ſun riſe, or the length of the day.

Cor. *By a like method you may inſert in a dial any holydays, or remarkable days in the year.*

For you have no more to do but deſcribe the ſun's parallels for theſe days, and write upon them reſpectively, the names of the days; and on theſe days the extremity of the ſhadow will fall upon the reſpective parallels,

PROB.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PL:XVI. pa:152

PROB. VII.

To deſcribe the azimuths and circles of altitude upon any dial.

The azimuths being great circles, are projected into right lines; and the parallels of altitude are projected into conic ſection, by the gnomonic projection of the ſphere. In order to deſcribe theſe, we muſt have given us the place and height of the ſtile, the horizontal line, the zenith, the dividing center of the horizon: but we have all theſe from the conſtruction of the dial. And we muſt have beſides the dividing center of every azimuth, by help of which we muſt ſet off the diſtance of each parallel upon it. 49.

Let DE be a vertical line, AGB the horizontal line, CF the ſtile, H the nadir, or rather the zenith, becauſe the ſhadow is always oppoſite to the ſun; R the dividing center of the horizontal line; HI the meridian or 12 o'clock line. From the center R, divide the horizontal line in parts of 10 or 20 degrees, beginning at the meridian, that is, at RI; and through the points of diviſion draw the azimuths H*k*, H*l*, H*m*, H*n*, &c. from the zenith H.

Then for drawing the parallels of altitude, we have F the dividing center of the azimuth DH. Therefore at F make the angle GF*a* equal to the diſtance of the parallel above the horizon, ſuppoſe 10 degrees, or HF*a* its zenith diſtance; and *a* will be the point thro' which the parallel is to paſs; and theſe parallels muſt be drawn below the horizon AB, as the ſhadow is oppoſite to the ſun. Then to find the point it muſt paſs thro', in any other azimuth H*q*; firſt find the dividing center, thus; draw C*kr* perpendicular to H*q*. Set CF from

Fig. from *b* to *t*, and extend from *t* to C, and ſet that
49. extent from *b* to *r*, and *r* is the dividing center of H*q*. Therefore at *r* make the angle *qrd*, equal to the parallel's diſtance from the horizon, or H*rd* its zenith diſtance, and *d* is the point for the parallel to paſs thro'. Alſo O was the dividing center of the meridian HI. Therefore make the angle IO*b* equal the parallel's diſtance (10°), and *b* is its point in the meridian. And the points being thus found in all the azimuths, draw the curve *xdaby*, which will be the parallel of altitude of 10 degrees. And whenever the ſhadow of the point F of the ſtile, touches that parallel *xay*, the ſun is 10 degrees high. And thus all the parallels may be drawn, at every 10 degrees diſtance, up to the zenith.

If the azimuths and parallels of altitude, were to be deſcribed on a horizontal dial; then the azimuths are right lines drawn from the foot of the ſtile C, making equal angles with one another. And the parallels of altitude are circles, and the radius of any one is the tangent of the zenith diſtance, the ſtile CF being radius, and C the center. For here the zenith falls in C the foot of the ſtile.

If the dial be an erect one, then the zenith H is at an infinite diſtance; and therefore all the azimuths are parallel to one another, and perpendicular to the horizon. And the horizontal line goes thro' C the foot of the ſtile; and the dividing centers of the azimuths are all in the horizontal line; and the parallels of altitude will be hyperbola's.

The azimuths are to be numbered from 12 o'clock; the weſt azimuths towards the eaſt; and the eaſt towards the weſt. And if the tropics are deſcribed in your dial, you need only draw ſuch parts of the azimuths as are contained between the tropics; or between the lower tropic and the horizontal line; for the horizon cuts off all the ſuperfluous parts.

Cor. *Here we may uſe a ſhorter way for finding the dividing center* r, *of any azimuth; like that for dividing the meridians; which holds for all great circles meeting in a point, as* H. Fig. 49.

About H*q* deſcribe a ſemicircle, or only a ſmall part thereof beſide *r*; with the extent HF, and one foot in H, croſs it at *r*, for the center.

For H*rq* being a right angle, it is inſcribed in a ſemicircle, whoſe diameter is H*q*; and H*r* is equal to HF, which is proved as before, in Cor. 2. Prob. V.

PROB. VIII.

To draw the Babylonian hours in a dial.

The *Babylonian hours* are reckoned from ſun riſing to ſun riſing, and are 24 equal hours, nearly of the ſame length as the common hours; only they are differently numbered. 50.

The way of deſcribing theſe hours is the ſame for all ſorts of dials; ſo that being ſhewn for one, it will ſerve for all. Therefore in a ſouth dial, let FC be the perpendicular ſtile, P the center. Draw FO perpendicular to FP, to cut the meridian P12 in O; thro' O draw the equinoctial AB perpendicular to P12. Then find what preciſe hour the ſun riſes at, when neareſt either tropic; as ſuppoſe at 4 in ſummer, and at 8 in winter; and in the equator he riſes, in all places, at 6: then find his declination, when he riſes at theſe times; and by Prob. V. draw two parallels for that declination, *mm*, and *nn*, on either ſide the equinoctial. Draw alſo the two tropics *cc*, and *dd*, which are only of uſe to terminate the hour lines, when drawn.

Obſerve all the points where the hour lines of the dial cut the ſouth parallel *mm*; then ſince the Babylonian hours proceed from 1 to 24; and in this parallel he riſes at 8; therefore write

Fig. 50. 24 at that point of the parallel, where the eight o'clock line paſſes; and write 1 at 9, 2 at 10, 3 at 11, &c.

Alſo note where the hour lines cut the equinoctial AB; and ſince the ſun then riſes at 6, call that 24; then 7 is 1, and at 8 write 2, at 9 write 3, at 10 write 4, &c.

Again obſerve all the points where the hour lines of the dial cut the northern parallel *nn*. And ſince here the ſun riſes at 4, call that 24, and 5 call 1, 6 call 2, 7 call 3, 8 call 4, 9 call 5, or at 9 write 5, at 10 write 6, at 11 write 7, at 12 write 8, &c. for the Babylonian hours.

This done, you have nothing to do but draw lines thro' theſe points where the figures are the ſame, as 22, 33, 44, 555, 666, 777, 888, 999, 10 10, &c. and theſe are the Babyloniſh hours; which are right lines, becauſe they repreſent great circles of the ſphere; as will be evident at ſight, by looking on a globe.

We may obſerve, if the horizontal line was drawn thro' the foot of the ſtile C, it would paſs thro' the interſection of the parallel *mm*, with the 8 o'clock line; becauſe the ſun riſes at 8; and alſo thro' its interſection with the 4 o'clock line, becauſe it ſets at 4.

All this being done, then at any time when the ſun ſhines, the top F of the ſtile, will ſhew the Babylonian hour of the day, among the Babylonian hours; as well as the true hour, among the hour lines of the dial.

PROB.

PROB. IX.

To deſcribe the Italian hours upon a dial.

The *Italian hours* begin at ſun ſet, and are numbered to 24 at ſun ſet next day; they are equal hours, nearly the ſame as common hours. Theſe hours are drawn the ſame way from ſun ſet, as the Babylonian hours were drawn from ſun riſe. 51.

Having deſcribed upon your dial, the equinoctial line AB, the two tropics, *cc* and *dd*, and the two parallels *mm*, *nn*, for the ſun's ſetting at 4 and 8 o'clock; obſerve where the ſouthern parallel *mm* cuts the 4 o'clock hour line, and mark it 24. Alſo obſerve where the other hour lines cut it, and write 23 at 3, 22 at 2, 21 at 1, &c.

Alſo obſerve where the hour lines cut the equinoctial, and reckon 6 (in the evening) 24, then 5 is 23, and at 4 write 22, at 3 write 21, at 2 write 20, &c.

And obſerve where the northern parallel *nn* is cut by the hour lines, and call 8 (at night) 24, and 7 call 23, 6 call 22, 5 call 21, 4 call 20, 3 call 19, or at 3 write 19, at 2 write 18, at 1 write 17, &c. Then lines drawn thro' the points with the ſame numbers, will be the hour lines; as 14 14, 15 15 15, 16 16 16, &c. Then when the ſun ſhines, the end of the ſtile F will ſhew the Italian or common hours, among their reſpective hour lines.

Cor. *If both the Babylonian and Italian hours were deſcribed upon one dial; the hour lines of both, will interſect in the parallel circles, where the ſun riſes at* 4, 5, 6, 7, 8, *o'clock, and their half hours. Or wherein the day is* 8, 9, 10, *&c. to* 16 *hours long.*

For it is evident they interſect in the parallels *mm* and *nn*; and for the ſame reaſon the other interſections

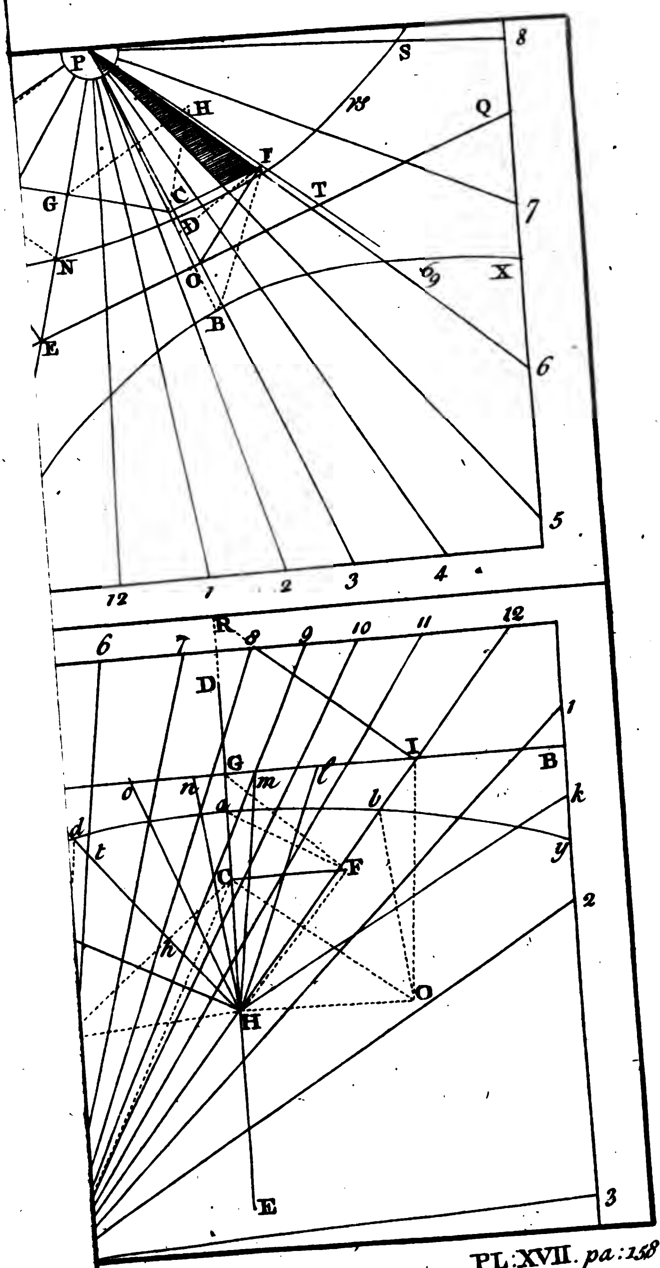
Fig. terſections would fall in ſome other parallels. Suppoſe a Babylonian hour drawn from 16 to 20, it
51. will croſs 3 Italian hour lines, in its way; and ſo there will be 3 parallels between 6 and 8 o'clock, or between 4 and 6; that is, one for every half hour. And they will anſwer reſpectively to theſe half hours, becauſe the parallels *mm*, *nn*, anſwer to the whole hours 4 and 8. Therefore they ſhew ſun riſe or ſun ſet, every half hour; or if you will, the length of the day, in whole hours, by doubling the time of ſun ſet.

PROB. X.

To draw the Jewiſh hours in a dial.

The *Jewiſh hours*, otherwiſe called the *old*, *unequal*, *planetary hours*, are reckoned from ſun riſe; and the day from ſun riſe to ſun ſet, is ſuppoſed to be divided into 12 equal parts or hours; and therefore theſe hours in any one day will be equal to one another, but not equal to the hours of another day. For when the days are long, as in ſummer, the hours will be long; but in winter, when the days are ſhort, the hours are alſo ſhort. The nights are alſo ſuppoſed to be divided into 12 hours; but theſe hours are longeſt in winter, and ſhorteſt in ſummer. But the hours of the day are never equal to the hours of the night, but at the equinoxes; and then the Jewiſh hours are equal to the common hours.

52. Let P be the center of the dial, CF the perpendicular ſtile; EQ the equinoctial, which is ſuppoſed to be drawn; if not, draw FA perpendicular to PF, and thro' A, draw the equinoctial EQ perpendicular to the ſubſtile PA. Set AF from A to D, ſo is D the dividing center of the equinoctial.



PL:XVII. *pa*:158.

Find the declination of the ſun, when he riſes at ſome even hour after 6 in ſummer; and as much before 6 in winter; as ſuppoſe at 4 in ſummer, and 8 in winter. Draw two parallels to the equinoctial, on each ſide, for that declination, as *mm* and *nn*. Then when the ſun is in the north parallel *nn*, the day is 16 hours long, which anſwers to 240 degrees, which divided into twelve parts, gives 20 degrees to a Jewiſh hour at that time. Therefore with the center D, deſcribe the circle SAW, which divide into parts of 20 degrees each, beginning at the line DN, where the 12 o'clock line cuts the equinoctial, for our 12 o'clock is the ſame as the Jewiſh 6th hour. Draw lines from D thro' all theſe points, to cut the equinoctial; and thro' the points in the equinoctial draw lines from P, to cut the parallel *nn* in the points where the hour lines paſs. Then mark theſe points on the parallel thus, write 6 at the 12 o'clock line, and to the other points, towards the weſt 5, 4, 3, 2, 1; and towards the eaſt 7, 8, 9, 10, 11, 12. Fig. 52.

Again, when the ſun is in the parallel *mm*, the day is but 8 hours long, which anſwers to 120 degrees, and that divided by 12, gives 10 degrees to a Jewiſh hour. Therefore from the ſame center D deſcribe another circle; or rather, from a center as far on the other ſide of the equinoctial EQ, deſcribe a circle, which divide into parts of 10 degrees each, beginning at the interſection of the 12 o'clock, and the equinoctial as before; and draw lines from the center thro' theſe points to cut the equinoctial; and from P, thro' theſe points in the equinoctial line, to cut the parallel *mm*, in points for the hour lines. Or you may perform the ſame thing, with the other ſemicircle S*a*W oppoſite to SAW; for drawing the line ND*r*, begin at *r*, to divide the ſemicircle into equal parts of 10 degrees; from which draw lines thro' D, to cut the equinoctial;

Fig. noctial; to which points in the equinoctial draw
52. lines from P, to cut the parallel *mm* in points for the hours. Then mark the point in the 12 o'clock line with 6, and the other points towards the weſt, 5, 4, 3, 2, 1; and towards the eaſt, 7, 8, 9, 10, 11, 12; or as many of them as are wanted.

Alſo upon the equinoctial at 12 write 6, at 11 write 5, at 10 write 4, &c. and at 1 write 7, at 2 write 8, at 3 write 9, and ſo on. Then drawing lines thro' all the three points that have the ſame figures, theſe will be the hour lines for the Jewiſh hours; which will be nearly ſtreight lines. But if you would be more exact, take two more parallels between theſe and the equinoctial, and find the points for the hours as before.

Note, the two tropics ought to be drawn for terminating theſe hour lines; but here is not room in this ſmall ſcheme to draw all the lines required.

Otherwiſe thus.

The Jewiſh hours may alſo be deſcribed by help of the common hours, already drawn upon the dial. For ſince in the parallel *mm*, a Jewiſh hour is $\frac{2}{3}$ a common hour; and in the parallel *nn*, $\frac{4}{3}$ a common hour. Therefore dividing every hour into 3 parts, ſet off from 12, in the parallel *mm*, firſt $\frac{2}{3}$, then $1\frac{1}{3}$, then 2, then $2\frac{2}{3}$, then $3\frac{1}{3}$, then 4 hours both ways; and mark the points. Then on the parallel *nn*, ſet off alſo from 12 both ways; $1\frac{1}{3}$, $2\frac{2}{3}$, 4, $5\frac{1}{3}$, $6\frac{2}{3}$, and 8 hours, and mark the points. Then drawing lines thro' the correſpondent points, and thro' the points of the common hour lines, on the equinoctial; and theſe are the hour lines required, and muſt be numbered as before directed.

PROB. XI.

To draw meridian lines in a dial, to ſhew when it is noon at any particular places on the earth.

The meridian of any place is eaſily drawn in a dial, if you know the longitude of that place, reckoning from yours, either eaſt or weſt. For allowing 15 degrees to an hour; by reducing the longitude to time; you will find what hour and minute it correſponds with. Therefore reckon ſo many hours and minutes from 12, upon your dial; towards the weſt, if the place lies eaſt; or towards the eaſt, if weſt; and thro' that point draw a line from the center. Then when the ſhadow falls upon that line, the ſun is in the meridian of the ſaid place. Or having the difference of longitude, or the hour arch, you'll find the hour angle, by the common proportion in dialling.

Suppoſe *Quebec*, whoſe longitude from London is 82° weſt; which is now $5\frac{1}{2}$ hours. Therefore thro' $5\frac{1}{2}$ hours, draw a line from P, and you have the meridian of Quebec. And the like for other places, as you ſee in the figure. 53.

Many other things of like ſort might be inſcribed upon dials, as the circles of the 12 cœleſtial houſes, and curves ſhewing what ſign aſcends or deſcends; curves ſhewing the latitude of particular places, or when the ſun is vertical at ſuch places, &c. But as theſe things are of little conſequence here, I ſhall ſpend no more paper about them.

SCHOLIUM.

Some people may deſire to know, how a dial plane is to be painted; for if it be made of wood, it will not endure the weather without painting. The ground or firſt painting, muſt be with Spaniſh

Fig. brown, and the laſt with white lead; and are to be ground with linſeed oil boiled. To grind any colour, put only a little oil to it at firſt, after it have been ground ſometime without; then grind the colour and oil together, adding oil by degrees, to make it like an ointment, and thinner ſtill as you grind the longer. It will be apt to work off the ſtone, if you do not ſometimes ſcrape it together with a wooden knife or lat. Your plane muſt be coloured ſeveral times over, to abide the weather, letting it dry between the times.

18. For glueing the joints together, boil your glue in blue or old milk; boil the milk firſt, and take off all the ſcum, before you put in your glue. This holds better than glue boiled in water.

As ſome people may want to know the latitude and longitude of ſome remarkable places, I ſhall therefore annex the following table.

A Table of the Latitude and Longitude of Places; the Longitude reckoned from London. Thoſe places that are in South Latitude, are particularly marked with S; and thoſe in Weſt Longitude, with W.

	Lat.		Long.	
	°	′	°	′
A.				
Aberdeen	57	20	1	50W
Agra	26	29	79	12
Aleppo	36	12	37	40
Alexandria	30	39	31	15
Algiers	36	50	3	27
Amſterdam	52	20	4	30
Arda (Ardra)	5	5	4	10
B.				
Bagdat	33	26	43	40
Belfaſt	54	38	6	15W
Belgrade	45	10	21	20
Bergen	60	5	6	21
Berlin	52	30	14	10
Berwick	50	49	1	40W
Boſton (America)	42	26	71	4W
Breſlaw	51	20	16	56
Briſtol	51	28	2	30W
C.				
Cadiz (Cales)	36	30	6	40W
Cairo (grand)	30	15	32	12
Cambaia	23	30	72	21
Cambridge	52	16	0	0
Canterbury	51	16	1	15
Cardigan	52	12	4	32W
Carliſle	55	0	2	40W
Chambala	57	0	139	0
Charles-town	32	30	79	10W
Chaxumo	15	0	37	0
Cheſter	53	15	3	2W
Chicheſter	50	50	0	48W
Coca	41	29	3	28W
Conſtantiople	41	29	28	58
Copenhagen	55	40	12	50
Cork	51	45	7	30W
Coventry	52	29	1	27W
Cracow	50	0	19	30
D.				
Dantzick	53	38	18	35
Darlington	54	31	1	39W
Davids (Saint)	52	0	5	4W
Dreſden	51	12	13	40
Dublin	53	12	6	55W
Duncola	17	20	33	40
Dundalk	53	52	7	2
Dundee	56	26	2	48W
Dunkirk	51	2	2	27
Durham	54	54	1	40W
E.				
Edenburgh	55	56	3	0W
Exeter	50	43	3	30W
F.				
Ferro (iſle)	27	48	17	40W
Fez	34	10	5	20W
Florence	43	34	12	24
G.				
Geneva	46	31	6	12
Genoa	44	25	8	41
Glaſcow	55	56	4	12W
Gloceſter	52	0	2	10W
Goa	15	36	73	53
Gombroon	27	49	55	45
H.				
Hague	52	12	4	20
Hamburg	53	41	10	38
Hartford	51	50	0	10W
Hanover	52	29	10	0
Hereford	52	10	2	36W
Hull	53	50	0	23W
I.				
Jeruſalem.	31	46	32	31
Iſpahan	32	34	50	15
K.				
Kildare	53	0	7	25W
Kinſale	51	32	8	26W
L.				
Lancaſter	54	10	2	40W
Landaf	51	35	3	15W
Lima	12	2S	75	52W

Limeric

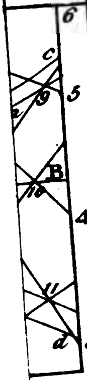
	Lat.		Long.	
	°	′	°	′
Limeric	52	55	6	48W
Lincoln	53	14	0	34W
Liſbon	38	45	9	7W
London	51	32	0	0
M.				
Madrid	40	25	3	39W
Marſeilles	43	18	5	27
Mexico	20	15	103	12W
Mongul	66	10	126	40
Monomotapa	25	08	26	0
Morocco	31	56	9	12W
Moſcow	55	47	38	14
N.				
Nanchang (Nankin)	32	10	118	38
Naples	41	51	14	45
Newcaſtle	55	0	1	42W
Nice	43	54	7	20
Norwich	52	42	1	7
Nottingham.	53	2	1	10W
Nuremburg	49	40	11	12
O.				
Oxford	51	45	1	10W
P.				
Paris	48	50	2	25
Pegu	17	29	97	12
Pekin	40	15	111	10
Pembroke	51	45	5	0W
Peterborough	52	33	0	20W
Peterſburg	60	00	36	6
Portroyal	17	32	77	5W
Prague	50	0	14	28
Q.				
Quebeck	47	35	74	10W

	Lat.		Long.	
R.	°	′	°	′
Reading	51	28	0	53W
Rhodes	36	24	20	0
Rocheſter	51	30	0	32
Rome	41	47	15	46
S.				
Saliſbury	51	4	1	52W
Sterling	56	30	3	45W
Stockholm	59	22	19	30
Suſa	45	5	7	10
T.				
Tenerif	28	17	15	29W
Tombute	14	20	11	12W
Troy	39	36	26	38
Tunis	36	26	10	16
Turin	44	56	7	16
V.				
Venice	45	46	13	12
Verſailles	48	42	2	20
Vienna	48	29	16	24
Villa rica	20	10	100	16W
W.				
Warſaw	52	21	21	10
Warwick	52	23	1	33W
Waterford	52	7	7	52W
Wexford	52	18	6	56W
Wincheſter	51	7	1	16W
Worceſter	52	18	2	10W
Y. Z.				
York	53	58	0	56W
Zuenziga	25	0	1	30W

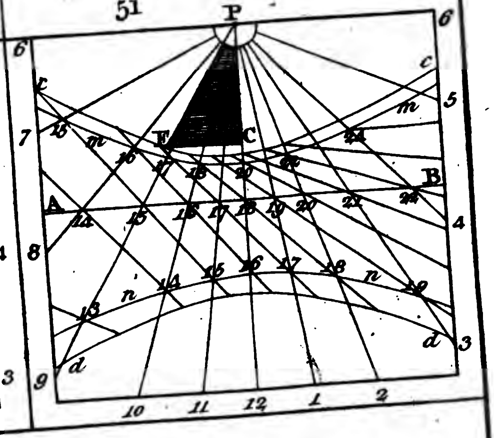
FINIS.

ERRATUM, Dialling.

Page 9, line 15, read, *tag*, his rays.



51



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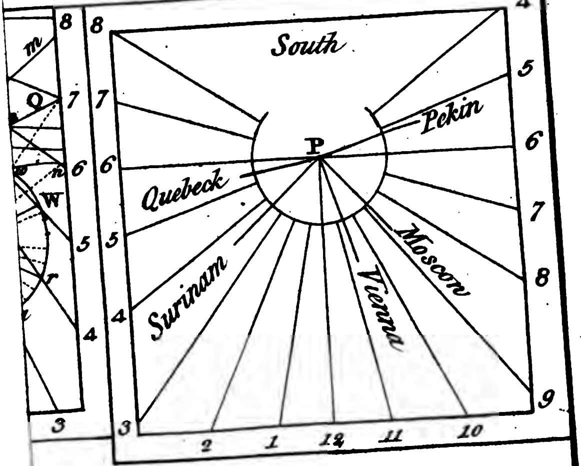

Pl. XVIII *the end*.